**Voting**

ALVARO SANDRONI, JONATHAN POGACH, MICHELA TINCANI, ANTONIO PENTA, DENIZ SELMAN
University of Pennsylvania, Philadelphia, USA

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**Glossary**

**Arrow's impossibility theorem** Arrow's Impossibility Theorem states that there does not exist a complete social ranking over alternatives that meets minimum impositions of egalitarianism and efficiency, No Dictatorship and Pareto Optimality, respectively. Consequently, there is no voting mechanism that can simultaneously satisfy basic notions of egalitarianism and efficiency.

**Cost of voting** Any sacrifice in utility that voting entails, such as cognitive costs, time cost of going to the polls, etc.

**Collective or social choice problem** A collective or social choice problem is a setting in which a group of individuals must jointly decide on a single alternative from a set. The outcome of such a problem potentially affects the welfare of all individuals.

**Common values** A common values problem is one in which agents share preferences over the alternatives, but may differ on their information regarding the attributes of alternatives.

**Condorcet jury theorem** In a common values setting, the result that under majority voting the correct candidate will (almost always) be elected so long as the following assumptions are satisfied: (1) each voter's belief is correct with a probability higher than half, (2) each voter votes according to her belief and (3) there is a large number of voters. In political science this result is sometimes referred to as wisdom of crowds.

**Condorcet paradox** See voting cycle.

**Downsian model of political competition** A model in which candidate strategically position themselves on a unidimensional policy space in order to win an election.

**Efficiency** Efficiency is a broad criterion that may be used to evaluate the social value of alternative outcomes by demanding that society make use of all valuable resources. Examples of efficiency measures are utilitarianism and Pareto Optimality.

**Egalitarianism** Egalitarianism is a broad criterion used to evaluate the social value of alternative outcomes by demanding that welfare or resources are evenly distributed across the population.

**Gibbard-Satterthwaite theorem** The Gibbard-Satterthwaite Theorem states that, under some assumptions, every non-dictatorial social choice function is manipulable.

**Majority voting** Majority voting is a voting rule that stipulates that each agent vote for a single alternative and an alternative that receives more than half of all votes is the collective choice.

**Manipulability of a social choice function** A social choice function is said to be manipulable if there is some agent who, given the social choice function, prefers to misreport his true preferences. In a voting context, this translates to voting for an alternative different from that which is most preferred.

**Median voter theorem** The median voter theorem states that if individuals' preferences are single peaked, then there exists an alternative that beats all others in a pairwise majority vote. Single peaked requires that each individual has a bliss point (most preferred alternative) and alternatives are less preferred the further they are from the bliss point. The selected alternative is then the median bliss point and the voter who has this bliss point is the median voter.

**No Dictatorship** The No Dictatorship criterion of Arrow's desiderata demands that there is no single individual whose preferences always determine those of society. Any egalitarian arrangement must satisfy the no dictatorship criterion, though arrangements that satisfy no dictatorship need not be egalitarian.

**Pairwise majority voting rule** A pairwise majority voting rule compares each pair of alternatives in a majority vote. Depending on agents preferences and votes, this rule may lead to a voting cycle.

**Paradox of voting** The puzzle of why there is high voter turnout in large elections, when the probability of any single vote to be determinant for the outcome is extremely small.
Pareto optimality or pareto efficiency  An outcome is Pareto Optimal or Pareto Efficient if it is not possible to increase the welfare of one individual without lessening the welfare of another.

Political ignorance  A state in which voters are not well informed about the issues and/or candidates they must vote on.

Social choice function  A social choice function is a mapping of all individuals’ preferences into an alternative, the social choice.

Strategic abstention  The tactical decision of an uninformed citizen to abstain in order to allow informed citizens with the same preferences determine the outcome.

Strategic voting  Voting by agents who aim to maximize their utility and might do so by misreporting their true preferences over electoral outcomes.

Utilitarianism  Utilitarianism is a conception of efficiency that evaluates outcomes on aggregate utility.

Utility of voting  Benefit to citizens from voting, usually divided into two components: a non instrumental component which includes utility derived from the mere act of voting and not related to the actual outcome of the election, and an instrumental component given by the utility of the outcome a voter would induce if determining the outcome, weighted by the probability that his vote determines the outcome.

Voting cycle or Condorcet’s paradox  A voting cycle or Condorcet’s Paradox results when every feasible alternative is beaten by another in a pairwise majority vote. As such, any collective choice is less preferred to some other alternative by more than half of the population.

Voting rule  A voting rule is a mapping of votes into a collective choice. Examples of different voting rules include majority voting and plurality voting. The collective choice may vary under alternative voting rules.

Definition of the Subject

Voting is a fundamental mechanism that individuals use to reach an agreement on which one of many alternatives to implement. The individuals might all be affected by the outcome of such a process and might have conflicting preferences and/or information over alternatives. In a voting mechanism, preferences and information are aggregated as individuals submit votes and a voting rule maps the compilation of votes into the alternative that is to be selected. The use of voting as a means of making a group decision dates back at least to ancient Greece, though French Revolutionary contemporaries Condorcet’s and Borda’s works are among the pioneers of voting theory. Meanwhile, welfare economists such as Bentham suggested formal definitions of socially desirable outcomes. As voting theory and welfare economics evolved, Arrow’s result in the middle twentieth century showed that no mechanism, voting or otherwise, can produce outcomes consistent with some of welfare economists’ definitions of socially desirable states. Moreover, results in the early 1970s suggested that voters have incentives to misrepresent their true preferences in elections. Contemporary voting theory has developed new models of strategic behavior to address questions on how political agents behave and which outcomes voting might produce.

Introduction

Voting is one of the most commonly used ways of making collective decisions. Two issues arise when a group of people must find an agreement on a choice that will potentially affect the welfare of all. First, individuals might have conflicting preferences over the set of alternatives they are choosing from. An interesting question is what is the best way to aggregate individual’s preferences in order to reach a common decision when preferences conflict. Second, even if agents share the same preferences, they might possess different information about the alternatives. In this situation, an interesting question is what is the best way to aggregate people’s conflicting information so as to make the right choice.

The optimal ways to aggregate preferences and information are among the most important questions that the literature on voting has tried to answer. Since voting concerns societies, as opposed to single individuals, this literature relies on works in the field of welfare economics. This branch of economics is primarily concerned with the analysis and the definition of the welfare of a society. There is no agreement among social scientists on a single definition of welfare of a society. However, two concepts are prominent: efficiency, which concerns the minimal waste scarce resources, and egalitarianism, which concerns the equal distribution of those resources. Using these concepts to define the welfare of a society, social scientists have attempted to answer questions on preference and information aggregation in making a collective choice.

Early works on different voting techniques date back to at least the late eighteenth century, with the works of French mathematicians such as Borda [6] and Condorcet [9]. Their works were among the first to analyze voting procedures with formal mathematical tools, which have since been prominently used in the literature on voting. Continuing in this tradition, a fundamental result in voting theory is Arrow’s Impossibility Theorem. This is
a formal treatment of the problem of aggregation of preferences that reached a striking result: there is no way, under some assumptions, to aggregate individuals’ preferences that is minimally egalitarian and minimally efficient.

Unlike the problem of aggregation of conflicting preferences, studies on the aggregation of conflicting information obtain positive results. In particular, the Condorcet Jury Theorem finds a mathematical justification for the phenomenon that is known in Political Science as the wisdom of the crowds, that is, the observation that democracies seem to be better at making decisions than single individuals. However Condorcet’s result relies on the assumption that people vote sincerely and subsequent works on voters’ behavior suggest that this is not always the case. A fundamental theoretical result, known as Gibbard–Satterthwaite theorem, shows formally that, under some assumptions, in every election at least one individual has an incentive to vote non sincerely, i.e. to vote strategically.

Gibbard and Satterthwaite’s result is a starting point for a branch of the voting literature that deals with strategic voting. Numerous works analyze strategic voting though the use of mathematical tools such as Game Theory. The latter has been used not only to explain voters’ behavior, but also to describe competing candidates’ behaviors in an election. The research in this game theoretic literature focuses on the outcome of political competition and on the development of a theory of turnout. The former is analyzed through the use of a model of political competition. A main result is that in a two party election, both parties choose the same political platform in order to maximize the probability of winning an election. The latter is motivated by the paradox of voting, which refers to the empirical observation that citizens vote even when the probability that their vote determines the outcome of the election is negligible, such as in large elections. At present, there is no widely accepted theory to explain this phenomenon.

As with the paradox of voting, the voting literature has still many unanswered interesting questions. As will be mentioned in the section on future directions, the voter’s behavior is still only partially understood and much needs to be explained. On a broader level, the study of the historical evolution of democracies needs further developments. Much has been written on the subject of voting, and some interesting results have been obtained, but much still needs to be explained.

The Collective Choice Problem

The basic framework for understanding voting is a collective or social choice problem: a group of agents must reach an agreement on which alternative to select, and this decision potentially affects the welfare of everyone. Such problems could range from “what should be taught in public schools?” to “who should be president?” Consider two possible alternatives, A and B, and a group of individuals who must jointly choose one of the two. It might be the case that some individuals in society prefer A and some prefer B. These conflicting preferences pose a challenge in determining the appropriate social choice. Alternatively, consider a scenario where A and B are different characteristics that two candidates running for a public office may possess. Suppose that all individuals agree that A is more desirable than B, i.e. this is a common values setting. However, individuals differ in their information; some think that candidate 1 possesses trait A, while others think candidate 2 does. In this case, the challenge is finding the best way to balance the conflicting information in arriving to a collective choice.

There are several ways to resolve the problems of conflicting preferences and information. For example, people can bargain to reach an agreement, or they can fight. A collective choice problem might also be solved through a dictatorship, or even through the toss of a coin. A fundamental mechanism used to resolve these conflicts is an election in which people submit votes on the feasible alternatives and a voting rule maps the collection of votes into a social choice. As can be seen in the next section, there exists a multitude of voting rules.

Before discussing voting rules, one should note the following distinction between elections: those in which the alternatives are policy and those in which the alternatives are candidates. The former is known as direct democracy, a common example of which is a referendum. In a referendum, citizens vote on a particular proposal, such as the adoption of a new law. The outcome of such an election is then to implement or not implement the proposal. This is in contrast to the case, known as representative democracy, where the election is over candidates. The collective choice in such a system is an agent or group of agents with the responsibility of choosing policy. The following voting rules apply to both situations.

Voting Rules

Majority voting is one of the most commonly used voting rules. The rule prescribes that each citizen vote for a single alternative and an alternative becomes the social choice if it receives more than half of all votes. Clearly, when there are more than two alternatives, majority voting does not necessarily produce a social choice.
To ensure a comparison between alternatives, one can resort to pairwise majority voting rule, in which alternatives are voted over pair by pair with a majority vote. That is, a majority vote is held between every pair of feasible alternatives and for each majority vote, the winner is deemed socially preferable to the loser. However, this voting rule might generate an intransitive social preference in which society chooses $x$ to $y$, $y$ to $z$, but $z$ to $x$. Consider a three-individual committee comprised of voters 1, 2 and 3 who must choose one of three alternatives, $x$, $y$ and $z$. Individual preferences are such that voter 1 prefers $x$ to $y$ to $z$, voter 2 prefers $z$ to $x$ to $y$, and voter 3 prefers $y$ to $z$ to $x$. By pairwise majority voting $x$ beats $y$, which in turn beats $z$, which in turn beats $x$. This intransitivity over alternatives is known as a voting cycle or Condorcet’s Paradox.

In the presence of a voting cycle, pairwise majority voting might not produce an overall winner, as each alternative might be beaten by another. So, an agenda setter could end a cycle by specifying the order in which the alternatives are to be compared in a pairwise vote. However, pairwise majority voting then places all the decision power in the hands of the agenda setter. Returning to the previous example, suppose the agenda prescribes that voting be carried out between alternatives $x$ and $y$ first and the winner is then to be compared with $z$. In the first round of voting $x$ beats $y$ and $z$ then beats $x$, so $z$ becomes collective choice. However, the agenda setter could instead choose an initial comparison between $y$ and $z$. Since $y$ beats $z$ in the first round and $x$ beats $y$ in the second, $x$ would then be the collective choice. Hence, the agenda setter decides the outcome of the election by choosing the order of the pairwise voting.

A plurality voting rule is an alternative way to make a collective choice: agents each vote for one alternative and the alternative with the most votes is chosen.

Other voting rules require agents to submit scores or rankings of all available alternatives, rather than just voting for a single one. In a Borda Count, agents rank all alternatives assigning the larger numbers to those that are more preferred. The voting rule sums the scores for each alternative across individuals and the alternative with the highest sum is the social choice.

A supramajority voting rule stipulates that the ‘status quo’ alternative is chosen unless another alternative receives at least some specified percentage of the vote larger than fifty percent. In the limit, there might be a unanimity rule that mandates one hundred percent of the electorate vote for an alternative for it to be chosen against the status quo. Examples of supramajority rules include the passing of constitutional amendments in the United States, where the current constitution is the status quo. Unanimity rules are commonly found in the judicial system in which all jurors must agree on the guilt of the defendant to override the status quo, the presumption of innocence.

Voting rules might also grant veto power to one or more agents. For example, each of the five permanent members of the fifteen member United Nations Security Council has the power to veto resolutions on particular matters. Any collective choice must therefore have the approval of all five permanent members.

The different voting rules are not simply different methods of arriving at the same social choice. Rather, the result of an election depends critically on the voting rule that is used. In fact, an alternative that is the social choice according to one voting rule might be the least preferred under another. For instance, consider alternatives $x$, $y$, and $z$ and seven voters, three who prefer $x$ to $y$ to $z$, two who prefer $y$ to $z$ to $x$, and two who prefer $z$ to $y$ to $x$. By pairwise majority voting, $x$ loses to both $y$ and $z$. In contrast, $x$ beats both $y$ and $z$ in a plurality vote. This suggests that in order to determine which voting rule to use, one must carefully analyze the various resulting outcomes.

To this end, it is useful to identify criteria that allow one to discriminate among the different outcomes produced under various voting rules. There are two main methods of doing this: efficiency and egalitarianism.

Welfare Economics

The analysis of efficiency and egalitarianism of a social state is among the objectives of a discipline called welfare economics. The first criterion for efficiency used by welfare economists dates back at least to Jeremy Bentham [2] and is known as utilitarianism. According to utilitarianism, the social interest is judged in terms of the total utility of a community. For example, if by moving from arrangement A to arrangement B, Mr. 1 benefits more than Ms 2 suffers, then the movement from A to B is judged as a social welfare improvement. Notice that in order to implement this criterion, the satisfaction intensities of different individuals must be comparable. In the 1930s this criterion was criticized by Lionel Robbins [35] and other welfare economists who claimed that the comparison of utilities across individuals has no scientific basis. In the 1940s a new criterion was developed which required no comparison of individual utilities: the Pareto criterion. A social outcome is said to be Pareto Optimal (Pareto Efficient) if there is no other outcome that would benefit at least one individual without hurting anyone else. Consider a scenario where ten dollars must be split among two individuals who value money and there are two alternatives: either person 1 receives five dollars, person 2 four and the re-
maining dollar is thrown away, or both receive five dollars. Clearly, the first alternative is not Pareto Optimal because person 2 can be made better off and person 1 would remain as well off if 2 is given the dollar that is being thrown away. Notice that the first alternative is also non utilitarian; in fact, the sum total of utilities can not be maximized when valuable resources are thrown away. However, a drawback of this efficiency criterion is that there exist multiple non comparable Pareto Optimal outcomes: any division of the ten dollars among the two individuals is Pareto Optimal as long as no money is thrown away, since to make one person better off one would have to take resources away from the other person. Hence, the Pareto Optimality criterion does not allow one to distinguish among multiple outcomes. Finally, notice that Pareto Optimal outcomes can be extremely non egalitarian: person 1 receiving ten dollars and person 2 zero is an unequal but Pareto Efficient division.

An alternative criterion often used to discriminate among social outcomes is egalitarianism, which focuses on the distribution of welfare across members of a society. One of the abstract principles behind egalitarianism is the *veil of ignorance* [18,19,33]. Consider a situation in which two persons must share a cake. Pareto Optimality does not help in selecting a division: as in the ten dollars example, any division of the cake is Pareto Optimal. Suppose that one of the two persons sharing the cake is asked to cut it in two without knowing a priori which piece she will receive. Her ignorance about which piece she will receive makes her cut the cake in two equal shares, an egalitarian division. Notice that there are a number of ways to define egalitarian outcomes. For example, Rawls’s [33] *maximin rule* suggests that the social objective should be to maximize the welfare of the worst-off individual. Finally, notice that an egalitarian outcome might be extremely inefficient. Returning to the ten dollar example, an arrangement where person 1 is given nine dollars and person 2 one dollar is not as egalitarian as one where both are given two dollars, though the latter does not make use of more than half of the available resources.

**Arrow’s Impossibility Theorem**

Ideally, one would like to use the normative criteria of social efficiency and egalitarianism to discriminate between different voting rules and select the best one. However a general result known as *Arrow’s Impossibility Theorem* [1] shows mathematically that this is impossible. In his seminal work, Arrow shows that there is no voting mechanism that generates a social consensus on the ordering of the different alternatives while satisfying a number of axioms, among which are the weakest forms of efficiency and egalitarianism: Pareto Optimality and No Dictatorship. The latter, which states that no individual always determines preferences of society, is a weak form of egalitarianism. While a non-dictatorial society can be quite unequal, any egalitarian society must be non-dictatorial.

A number of possibility results have been obtained by the relaxation of some of Arrow’s axioms. For example, pairwise majority voting with a particular restriction on individual tastes, which violates what Arrow called Unrestricted Domain, satisfies Pareto Optimality and No Dictatorship, while generating an ordering of the social alternatives.

Black [4] noticed that pairwise majority voting produces an outcome that is not subject to Condorcet’s paradox when individual preferences are single-peaked: every individual must have a most preferred alternative (bliss point) and between any two alternatives he prefers the one that is closer to his bliss point. An important result in voting theory, called the median voter theorem, shows that when individuals’ preferences satisfy this condition, the bliss point of the median voter beats any other alternative by pairwise majority voting. The median voter is found by ordering voters according to their bliss points. The importance of this theorem derives from its ability to describe how democracies work in practice. It is commonly observed that candidates try to appeal to voters who are politically moderate, or “in the middle”; this is consistent with the theory, which suggests that these are the preferences that will eventually prevail in a democratic system.

**Political Ignorance and the Condorcet Jury Theorem**

As mentioned earlier, voting is not only a way to aggregate conflicting preferences but it is also a way to aggregate individual, possibly conflicting, information when preferences are partially or totally aligned. This is the case in *common value* settings. When people would agree on the best choice if given the same information on alternatives, but differ in the information they actually receive, a natural question is which is the voting mechanism that aggregates information in a way that maximizes the probability of the right decision being made. A result called *Condorcet Jury Theorem* [9] shows that among all the possible voting rules, simple majority rule guarantees that the right decision is made under three crucial assumptions: that each voter has a correct belief with a probability higher than 50%, the voter votes according to his belief, and that the electorate is very large. Before going into the details of the theorem, it must be mentioned that this result has a practical importance. A number of works document that voters
are ignorant over both policy and candidate alternatives over which they are to vote. [7] claim that "many people know the existence of few if any of the major issues of policy", while [12] discusses "mass political ignorance" and "mass political apathy" as playing key roles throughout the history of American politics. More recently, the 2004 American National Election Study found that Americans performed extremely poorly when asked simple questions about the political system and the leaders in charge of it. This evidence is in favor of what is called political ignorance. In a setting where voters are politically ignorant, the Condorcet Jury Theorem provides a valuable insight as to how much political information matters in determining the outcome.

Consider a committee who has to elect an administrator out of two candidates, one "good" and one "bad." Assume that all the members of the committee share the same preferences: they all prefer the good administrator to be selected. Individuals differ, however, in the information they have about which candidate is the good one. Suppose that each voter has a belief about which is the good candidate and votes according to his belief. If each voter has a correct belief with probability higher than 50%, then by the Law of Large Numbers as the number of voters becomes very large the probability that more than half of the electorate votes for the right candidate goes to one. Hence, under simple majority the probability that the right choice is made goes to one. Condorcet’s conclusion is that in a common value setting a democratic decision is superior to an individual decision, because each voter makes the wrong decision with a non-negligible positive probability, whereas the population as a whole makes the right decision almost always. As far as political ignorance is concerned, this result shows that it is not necessary for an electorate to be well informed for it to make the right decision. So long as each voter has a correct belief with a probability higher than a half, the electoral outcome will almost always be identical to one in which the electorate was perfectly informed. Therefore, Condorcet’s result implies that the ignorance of individual voters is overcome by the aggregation of information in an election.

Gibbard–Satterthwaite Theorem

Thus far, citizens have been treated as if they disregard any tactical considerations when faced with a voting decision. However, a citizen might find it worthwhile to misrepresent his true preferences in order to achieve a social outcome more preferred than the one that would result if he voted naively. Consider an election in which a status quo will be replaced if a simple majority agrees on one of three candidates. Suppose the status quo is a conservative government, and the three alternative candidates to be voted for are a conservative, a moderate and a liberal. Imagine that there are only three voters, and two votes have already been cast: one is for the moderate candidate, one is for the conservative one. Suppose that the last individual who is called to vote is politically liberal. He knows that if he votes for his truly most preferred candidate, the liberal one, there would be a tie and the status quo conservative government would not be replaced. However, by voting for his second most preferred alternative, the moderate candidate, he would break a tie and the status quo government would be replaced with a moderate one. A liberal voter prefers this outcome to the one where conservatives win. Therefore he has an incentive to misrepresent his true preferences and vote tactically for his second-best alternative.

A powerful result in voting theory called Gibbard–Satterthwaite theorem [17,36] shows formally that in most electoral settings at least one citizen has an incentive to vote tactically. Define a social choice function as a mapping of all individuals’ preferences into a collective choice. The theorem states that there is no social choice function that is Non Dictatorial and Non Manipulable, i.e. such that no agent has an incentive to vote tactically. Consequently, for every voting rule there is at least one agent with an incentive to misrepresent her preferences.

It should be mentioned that the Gibbard–Satterthwaite theorem places also other technical restrictions. Furthermore, in a setting such as a majority vote, a misrepresentation of one’s true preferences coincides with voting for an alternative which the voter does not rank top. In the original formulation of their theorem, however, Gibbard and Satterthwaite dealt with mechanisms where agents are required to submit a ranking over all alternatives, and a misrepresentation of tastes in their framework does not coincide necessarily with a misrepresentation of only the most preferred alternative (see [26]).

This result suggests that for a deep understanding of voting one should not focus only on the assumption that citizens vote sincerely. Tactical voting is not only an abstract possibility, it is also an actual behavior that must be considered in any voting analysis. The following sections explore how the literature on voting has dealt with strategic voting.

Political Competition and Strategic Voting

The early works of Downs [10] and Tullock [39] initiated the analysis of political issues within a strategic framework, where voters and/or candidates are assumed
to be rational decision makers. Political competition describes a situation in which candidates strategically position themselves in order to win an election, whereas strategic voting refers to individuals’ decision to vote so to maximize utility by sometimes misreporting true preferences. This game theoretical framework developed due to positive arguments such as Gibbard–Satterthwaite theorem, which suggests that voters have an incentive to behave strategically, and is also due to spread of game theory as a dominant tool in economic analysis.

Political Competition

The classic model analyzing the candidates’ choice of positioning on the political spectrum is that of Downs [10], who adapted the classical Hotelling model [21] to the analysis of the choice of political platforms by candidates. In the Downsian model of political competition, there is a unidimensional policy space, representing the political spectrum. There are two candidates who position themselves on this policy space. Each voter has a most preferred point on this space and prefers points closer to this point than those further away, i.e. each voter has single-peaked preferences. Downs argues that strategic candidates concerned only with winning position themselves at the point that is most preferred by the median voter. If candidate A were positioned anywhere else, say to the left of the median voter, candidate B could get the majority of votes by positioning himself between candidate A and the median voter; all the agents to the right of B, constituting more than half of the voters, would prefer B to A. Given this scenario, candidate A (for the same reason) would then have an incentive to position himself between B and the median voter, and so on. Hence, the result is that both candidates position themselves at the policy platform most preferred by the median voter. An interesting implication of this result is that under a democracy with two parties, both parties act identically, and therefore there are only as many positions (just one) taken by political parties as there would be in a dictatorship. However, it is crucial that two parties exist so that the competition between them can allow the chosen policy point to represent the preferences of the voters. This is in contrast to a dictatorship in which the ruling party can implement its own preferred policy without voter approval.

Notice that the example above assumes a two-party system (for models that allow for more than two parties see [3,29]). [11] posits that in a representative democracy with a plurality voting rule, only two parties compete in the elections. This theory, known as Duverger’s Law, states that a proportional representation system, in which parties gain seats proportional to the number of votes received, fosters elections with numerous parties. In contrast, a plurality system marginalizes smaller parties and results in only two parties entering into political competition.

The Decision to Vote: the Paradox of Voting

Another issue raised by Downs, one which focuses on voters’ behavior rather than candidates’, is known as the paradox of voting. It refers to the fact that in a large election, the probability that any single vote determines the outcome is vanishingly small. If every person only votes for the purpose of influencing the outcome of the election, even a small cost of voting would be sufficient to dissuade anyone from voting. Yet, it is commonly observed that turnout is very high, even in large elections. From the large empirical literature on turnout in elections, some facts seem to be acquired knowledge: (1) turnout is higher in more important elections (e.g. Presidential election in the US have a significantly higher turnout than Gubernatorial elections), (2) turnout is generally higher in close elections (i.e. with smaller margins of victory), and (3) turnout rates are different among groups with different demographic characteristics. For instance, from the thorough work by [41], it emerges that education has a substantial effect on the probability that one will vote. Income has less of an effect once it has been controlled for the impact of other variables. After education, the second most important variable is age, which appears to have a strong positive relationship with turnout. Other socio-economic variables are also important; in particular, racial minorities appear to be less likely to vote. Finally, turnout seems to be significantly influenced by factors such as the weather conditions on the day of the election and voters’ distance from the polls (see [8]).

Such comparative statics suggest that it is appropriate to model voters’ behavior as a rational choice problem within a standard utility maximization framework. The modern theory of voting applies the classic utilitarian framework to the voting problem, positing that agents decide whether or not to vote by comparing the cost of voting with the utility of voting. The traditional starting point for the modern theory of voting is [34], who formalize the insights of [10] and [39] in a simple utilitarian model of voting. The cost of voting comprises any sacrifice in utility that voting entails. The utility of voting is usually divided into two components: a non-instrumental component and an instrumental component. The non-instrumental component includes utility derived from the mere act of voting and not related to the actual outcome of the election. It may include, for instance, the sense of civic duty. There is
considerable evidence that voters are motivated by a sense of civic duty (see, for example, [5]). The instrumental component is the utility of the outcome a voter induces if her vote determines the outcome, weighted by the probability that her vote actually determines the outcome.

The instrumental component of the utility of voting has attracted most of the attention in the literature. It is typically analyzed through the rational theory of voting, which is motivated by an empirical observation: there exists a strong positive correlation between turnout rate and closeness of the election. This fact suggests that, ceteris paribus, voters are more likely to vote if their vote is more likely to make a difference. The main theoretical problem is to endogenize the probability that each voter is pivotal, i.e. that his vote is determinant for the outcome of the election.

Ledyard [23,24] are among the early works in the literature on game theoretical models of the pivotal-voter. In these models, voters infer the probability of being pivotal from the equilibrium strategies of other voters. Subsequently, they decide whether or not to vote, trading off the cost of voting with the expected (instrumental) utility of voting. Although Ledyard did not focus on the magnitude of turnout in a strategic model, this question is addressed by Palfrey and Rosenthal ([30,31]) who model elections with uncertainty about the total number of voters. Voters strategically choose whether or not to vote for their favorite alternative amongst two candidates. However, Palfrey and Rosenthal’s theories do not explain high turnout in large elections, when the cost of voting is not very (and unrealistically) low.

Ultimately, the game-theoretic approach to costly voting could not escape the paradox of voting. Since the probability of being pivotal is very small in large elections, the individual incentives to vote cannot justify high turnouts unless the cost of voting is sufficiently small. Conversely, regardless of how small the cost of voting is, the theory posits that there should be low turnout as the election becomes arbitrarily large, which is in contrast to empirical evidence. The puzzle that remains open is how to reconcile the evidence of high turnout in large elections with the responsiveness of turnout levels to the closeness of the election.

**Mobilization and Group-Based Notion of Welfare**

Two strands of the literature try to overcome the paradox of voting by focusing on groups of like-minded people rather than on individual agents. These are models of mobilization and models incorporating a group based notion of welfare.

In models of mobilization, the population of voters is assumed to be divided into groups, each of which has a leader who has the same preferences as all agents in the group and coordinates their behavior. The turnout decision within each group is determined by how the leaders allocate costly resources to voters. It is as if leaders buy the votes of the agents in their group, compensating for the agents’ costs of voting. Since leaders influence a large number of voters, their decisions have a non-negligible impact on the probability of affecting the electoral outcome and consequently, on the individual instrumental benefit from voting. [37,38] test group based models and provide some empirical support for the mobilization thesis. (see also [27,28,40]). The problem for models of mobilization is that it is not clear how leaders affect the individual behavior of voters.

Models of group based welfare consider groups of like-minded individuals whose actions are intended not to maximize their individual utilities, but rather that of the group (see [22,25]). In this case, there is no leader who prescribes behavior as in mobilization models, but instead there is an implicit understanding among agents in the group on appropriate behavior. This idea is developed by [16], who appeal to [20] Group Rule-Utilitarian Theory to endogenize the non-instrumental component of the utility from voting in a way that preserves the positive relation between closeness of the election and incentive to vote typical of the classic pivotal-voter models. In this model, agents derive utility from “doing their part”: in the spirit of [20], this is understood to mean following the rule that, when followed by all the agents in a given group, would maximize some measure of the group’s utility. The outcome is a set of rules for each group, which are mutually optimal (from the point of view of the group) given that individuals follow the rules within their group. Abstention still occurs because for some agents (those with higher costs of voting) the rule prescribes not to vote, since their contribution to increase the group’s utility from the election’s outcome does not compensate the increase in the group’s total cost of voting. [8] provide some empirical support to the group rule.

**The Common Value Setting with Strategic Agents**

Feddersen and Pesendorfer [13,14] consider the Condorcet Jury Theorem in a strategic setting and reach a different conclusion than Condorcet. They model a voting problem in an almost common value setting as a game, that is, as a situation where agents interact strategically. The following simple example provides the basic insights of their model: suppose that there are three voters, 1, 2
and 3, and two candidates, A and B. Suppose that voter 1 is an A-partisan, meaning he prefers candidate A in all states of the world. Agents 2 and 3 instead prefer candidate A in state $s_A$ and candidate B in state $s_B$. Let $p$ be the probability of state $s_A$ and $1 - p$ the probability of $s_B$. Now, suppose that agent 2 is informed, i.e., he knows the state of the world before voting, while agent 3 is not. Finally, suppose that there is no cost of voting and that the election is decided by simple majority rule. In this situation, agent 1 votes for A, agent 3 for B, and agent 2 for A if he observes $s_A$ and for B if he observes $s_B$. To understand why the uninformed agent 3 votes for B, notice that by doing so the outcome is that A is always selected in state $s_A$, while B is always selected in state $s_B$. This is clearly the best outcome for agent 3, and in all states this is also the best outcome for the majority of the population. In fact, if the true state is $s_A$, A is selected, which is preferred to B by all voters; if the true state is $s_B$, B is selected, which is preferred to B by two voters out of three.

The uninformed agent in the example votes for B no matter what the prior probability $p$ is, even if $p$ is close to one, that is, even if he is almost sure that A is the right candidate. By voting for B individual 3 counterbalances the A-partisan’s vote, thereby allowing the informed voter (2) to induce the “right” outcome with probability one. In Condorcet’s argument, if voters vote according to their belief about the state of the world, information is aggregated in a way that induces the right social choice to be made. In this setting, where preferences are only partially aligned, information is aggregated so as to always generate the decision that is preferred by the majority of the population only if uninformed voters vote strategically. In this setting, if voters vote sincerely, as is assumed by Condorcet, the result that the aggregation of information taking place during an election delivers the “right” social choice does not hold. In Feddersen and Pesendorfer’s framework it is strategic voting that induces the “correct” social choice. This is the major difference with Condorcet’s result, which was driven by the assumption of sincere voting.

Recognizing the strategic incentives that voters may have in an election, Feddersen and Pesendorfer [15] apply a similar analysis to the unanimity rule in juries. They find that in the context of a common value setting, unanimity voting might result in convicting the innocent more often than other rules because strategic jurors consider the probability of being pivotal (like voter 3 in the example) and make their decisions conditional on being pivotal.

Now suppose there is another voter, 4, who is uninformed and shares the same preferences as 2 and 3. In this setup, even with a zero cost of voting agent 4 would abstain, so that the informed voter is pivotal with probability one. Voter 4’s behavior is known as strategic abstention: the act of abstaining by uninformed voters not because voting is costly, but because by doing so they allow the informed voters to be pivotal. By abstaining, uninformed voters in effect delegate the decision to the informed voters. In the example, voter 4’s strategic abstention allows for information equivalence to arise: making voter 4 informed would not change the outcome of the election, as long as 4 strategically abstains so that the informed voter determines the outcome. Notice that this result is in the same spirit as Condorcet’s Jury Theorem’s result: there, having each voter’s belief accurate with a probability slightly higher than 50% or substantially higher than 50% does not make a difference. As long as voters vote according to their belief, the outcome is the same no matter what the underlying belief accuracy is (as long as it is greater than a half). Although for different reasons, in both Condorcet’s setting and Feddersen and Pesendorfer’s model (under some circumstances) the aggregation of information that takes place during an election ensures that the outcome of the election does not vary if the electorate is made more informed.

**Future Directions**

There are a number of open questions in the literature on voting. As mentioned in earlier sections, at the time of writing, there is no universally accepted theory of turnout. In particular, no theory delivers all of the relevant comparative statics observed empirically, that is, the variability of turnout across elections of difference size, importance, and closeness. Broadly speaking, one may say that the behavior of voters is still only partially understood. Whether voters vote based on strategic considerations or vote without regard to how other citizens might be voting is an unsettled issue. Furthermore, individuals’ choices in hypothetical and real situations might differ. The act of voting in large elections is almost a hypothetical choice, in that the likelihood that a vote determines the outcome is negligible. An open question then is whether voters choose candidates as they would in a real situation or as they would in a hypothetical situation. As far as empirical work is concerned, little seems to be known of the empirical impact of political ignorance on the outcome of elections. Also, an issue that is both empirical and theoretical and has not been satisfactorily addressed is what types of election rules are best suited for different decisions. Finally, although much has been written on the origins and evolution of democracies, there is no general consensus as to what are the possible reasons of democracies historical evolution. As is clear from this section, many interesting questions in
politics have not yet been satisfactorily answered, leaving space for future research.

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