

# Problem Set 1: Due Wednesday, January 25

Note: For this problem set, and all future problem sets, email me your solution by 9:00 am on the due date. No need to print anything out.

## Problem 1

In this problem, we will re-evaluate some of the results presented in Heathcote, Perri, and Violante (2010). In particular, we will update their CPS-based results using data spanning the Great Recession.

**Preliminary Steps:** The easiest way to retrieve the data is to download them from the IPUMS website (see <https://cps.ipums.org/cps/>). There, you'll find an easy-to-use website through which you can access the data. Download the March supplements, beginning in 1976 and extending to 2016. For the first part of the sample (up to 2006), feel free to download every other year or every third year if you want to economize on memory. From 2007 to 2016, download every year. The variables you should download are: year, serial, hwtsupp, cpsid, region, statefip, aseclflag, hflag, metro, hhincome, housret, cpi99, month, pernum, cpsidp, wtsupp, relate, age, sex, race, marst, educ, higrade, educ99, empstat, labforce, occ1990, ind1990, wkswork1, wkswork2, srcearn, srceduc, screti1, screti2, eitcred, fedtax, fica, statetax, migsta1, fullpart, ftotval, inctot, incwage, incbus, incfarm, incss, incwelldr, incgov, incidr, incaloth, incretir, incssi, incdrt, incint, incunemp, incwkcom, incvet, incsurv, incdisab, incdivid, incrent, inceduc, incchild, incalim, incasist, incother, earnweek, incdisa1, incdisa2, inclongj, increti1, increti2, incsurv1, incsurv2, oincbus, oincfarm, oincwage, srcdisa1, srcdisa2, uhrsworkly, ahrsworkt.

Approximately fifteen minutes after submitting the download request, you should get an e-mail with a link to the dataset. From my website you can also find a *.do* file which will clean the data and construct the main household income variables. The first few questions pertain to some of the details in the code that I have posted.

1. *Top coding* refers to the censoring of certain observations to maintain the confidentiality of survey respondents. In our context the CPS income variables greater than some variable-year-specific threshold are censored.
  - (a) What distributional assumption is made on each income variable to impute the true income of the censored observations?
  - (b) What parameter, according to this distribution, does the coefficient from the logy vs. logy regression represent?

- (c) Why is it necessary to run these regressions separately for each year?
  - (d) What is the significance of restricting the sample of this regression to be greater than the "x1" local macro variable?
2. Use one paragraph to describe how you would modify the code if you thought the tails of the income variables were log-normally distributed.

The next few questions ask you to produce some figures and tables. When making the figures and tables, make sure to include a short description describing how it was constructed. In the figures, label each axis and data series, using a sensible name (e.g., "Wage Income" and not "incwage").

3. How has equivalized household income inequality (according to the following measures: labor earnings, net asset income, pre-government income, and pre-tax income) evolved since 2005? Plot the 90-50 and 50-10 ratios of the aforementioned income measures from 1976 to 2014. What has changed since the beginning of the Great Recession?
4. Define  $y_{lt}$  as equivalized household earnings,  $y_{kt}$  as equivalized household asset income,  $y_{k+l,t}$  as the sum of the two,  $\bar{y}_{it}$  as the year-t average income (for  $i \in \{l, k, k+l\}$ ), and  $s_{kt} \equiv \frac{\bar{y}_{kt}}{\bar{y}_{k+l,t}}$ . Note the approximation

$$\text{Variance} \left( \log \left( \frac{y_{k+l,t}}{\bar{y}_{k+l,t}} \right) \right) \approx \underbrace{(s_{kt})^2 \cdot \text{Variance} \left( \log \left( \frac{y_{kt}}{\bar{y}_{kt}} \right) \right)}_{\text{Term 1: Asset Income}} + \underbrace{(1 - s_{kt})^2 \cdot \text{Variance} \left( \log \left( \frac{y_{lt}}{\bar{y}_{lt}} \right) \right)}_{\text{Term 2: Labor Earnings}} + \underbrace{2 \cdot s_{kt} \cdot (1 - s_{kt}) \cdot \text{Covariance} \left( \log \left( \frac{y_{lt}}{\bar{y}_{lt}} \right), \log \left( \frac{y_{kt}}{\bar{y}_{kt}} \right) \right)}_{\text{Term 3: Covariance btw. Asset and Labor Income}} \quad (1)$$

- (a) Note that Equation 1 doesn't hold exactly; it would if the log income terms were replaced by income, in levels. Plot the three components of the right-hand side of Equation 1 for the 1976 to 2014 period. What, if anything, does this exercise tell you about the sources of earnings-plus-capital income inequality?
5. David Autor and co-authors<sup>1</sup> argue that part of the increase in 90-50 inequality is due to a reduction in the demand due to "middle-skill" occupations, in particular occupations that are centered around routine tasks, such as clerical work and goods production. To explore this hypothesis, rank occupations (using the occ1990 variable)

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<sup>1</sup>See, for example, Acemoglu and Autor (2011) and Autor and Dorn (2013).

according to skill (and use the mean hourly wage paid in 1976 to proxy for skill). For each occupation skill percentile, compute and plot the percentage change in a) hours worked and b) hourly wages in the following intervals i) 1976 to 2007, ii) 2007 to 2016, and iii) 1976 to 2016. These figures should, in form, resemble Figure 1 of Autor and Dorn (2013). Describe, in a paragraph, your findings from these figures.

## Problem 2

On Wednesday, we'll start discussing Ngai and Pissarides. The subsequent questions consider the draft of the paper given at

[http://eprints.lse.ac.uk/4468/1/Structural\\_Change\\_in\\_a\\_Multi-Sector\\_Model\\_of\\_Growth.pdf](http://eprints.lse.ac.uk/4468/1/Structural_Change_in_a_Multi-Sector_Model_of_Growth.pdf)

I recognize that we haven't gone over this paper, at all, yet. A goal of this exercise is to become familiar with the paper before we discuss it in class.

1. What is the contribution of this paper? What gap in the literature does this paper fill?
2. Our goal in this problem and the next is to derive some of the optimality conditions given in page 5. Write out the *Hamiltonian*.<sup>2</sup>

$$\mathcal{H}(t) = e^{-\rho t} v(c_1, \dots, c_m) + \lambda(t) [A_m F(n_m k, n_m) - c_m - (\delta + \nu) k(t)] \quad (\text{A})$$

Here we have already employed the assumptions given in Equation 7. Also, if it makes the math easier for you, free to use the assumption that  $F(n_i k_i, n_i) = k_i^\alpha n_i$ .

To derive the static optimization condition given in Equation (10), substitute the  $c_i$  out of the first term in the right hand side of our Hamiltonian. Then take derivatives with respect to  $n_i$ ,  $k_i$ ,  $n_m$  and  $k_m$ . Using Equation (8), you should be able to derive, now, Equation (10) of the paper.

3. Now, the dynamic efficiency condition. Begin with the intertemporal optimality condition:

$$\frac{\partial \mathcal{H}(t)}{\partial k(t)} = -\dot{\lambda}(t)$$

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<sup>2</sup>If you are unfamiliar, or need a refresher, [http://krebs.vwl.uni-mannheim.de/fileadmin/user\\_upload/krebs/pdf/Hamiltonian.pdf](http://krebs.vwl.uni-mannheim.de/fileadmin/user_upload/krebs/pdf/Hamiltonian.pdf) derives the Hamiltonian function. Todd Keister provides some nice intuition of the Hamiltonian method in Section 2 of <http://www.toddkeister.net/pdf/optimal-growth-notes.pdf>

The above equation implies that

$$-A_m F_K(n_m k, n_m) + (\delta + \nu) = \frac{\dot{\lambda}(t)}{\lambda(t)} \quad (\text{B})$$

Now, take first order conditions of Equation A with respect to  $c_m(t)$  and differentiate this first order condition with respect to time, to get an expression for  $\frac{\dot{c}_m}{c_m}$ . Substitute out the  $\frac{\dot{\lambda}(t)}{\lambda(t)}$  term, using equation B, to arrive at Equation (6) of the paper.