Problem Set 3: Due Monday, February 6

This week, we'll discuss Oberfield and Raval. The subsequent questions consider the draft of the paper given at

http://economics.mit.edu/files/9861 .

Note: In case you get stuck at any point, I've posted the solutions to this problem. But do your best to work through as much as you can, on your own.

Also, Problem Sets 4 and 5 are somewhat long (especially compared to Problem Set 3.) You may want to take a look at these problem sets. If you have extra time now, it might be a good idea to start working on Problem 1 of each of these two problem sets (especially Problem 1 of Problem Set 5).

1. What is the contribution of this paper? What gap in the literature does this paper fill?

2. In this part of the question, we will derive Equation (8) of the paper. Recall the set-up. Industries (n) which are composed of plants (i). There is a representative consumer who has nested CES preferences. Consider, for this problem, only the "within-industry" component. The utility that the consumer gets from consuming output from plants in industry n, is given by:

$$Y_n = \left[\sum_{i \in I_n} D_{ni}^{\frac{1}{\varepsilon_n}} Y_{ni}^{\frac{\varepsilon_n - 1}{\varepsilon_n}} \right]^{\frac{\varepsilon_n}{\varepsilon_n - 1}},$$

where ε_n is the elasticity of substitution across varieties within and industry. Each plant has the following production function

$$Y_{ni} = \left[(A_{ni} K_{ni})^{\frac{\sigma_n - 1}{\sigma_n}} + (B_{ni} L_{ni})^{\frac{\sigma_n - 1}{\sigma_n}} \right]^{\frac{\sigma_n}{\sigma_n - 1}}$$

The D's are preference weights.

(a) Given the preferences of the representative consumer, and using $P_n = \left[\sum D_{ni} P_{ni}^{1-\varepsilon_n} \right]^{\frac{1}{1-\varepsilon_n}}$ to refer to the ideal price index industry n's output, write out the demand curve faced by plant i (relative to other plants in the industry); you should have a relatively simple expression for $\frac{Y_{ni}}{Y_n}$.

(b) Using your answer from part (a), write out the profit maximization problem of plant i, who takes P_n as given.

(c) Let K_n and L_n refer to the amount of capital and labor employed by industry n and r and w to refer to the factor prices. We are interested in writing out an expression for $\sigma_n^{\text{industry}} \equiv 1 + \frac{\partial \log(K_n r / L_n w)}{\partial \log(w/r)}$ in terms of the plant level production elasticity, σ_n, the plant demand elasticity ε_n and dispersion in plants' capital shares. Define $\alpha_{ni} = \frac{rK_{ni}}{rK_{ni} + wL_{ni}}$ as the capital share of plant i, and θ_{ni} as the $\frac{rK_{ni} + wL_{ni}}{rK_n + wL_n}$ as the expenditure share of plant i, within industry n. Then $\alpha_n = \sum_i \alpha_{ni}\theta_{ni}$ is the capital share of industry n. Note that $\frac{\partial \log(K_n r / L_n w)}{\partial \log(w/r)} = \frac{1}{K_n r / L_n w} \frac{\partial (K_n r / L_n w)}{\partial (w/r)}$. Use this fact and the relationship $K_n r / (L_n w + K_n r\ ) = \alpha_n = \sum_i \alpha_{ni}\theta_{ni}$ to re-write σ^{industry}. You should now have a term $\frac{\partial \left(\sum_i \alpha_{ni}\theta_{ni}\right)}{\partial \log(w/r)}$

(d) Use plant i's cost-minimization conditions to explain why $\frac{\partial \alpha_{ni}}{\partial \log(w/r)} = \alpha_{ni} \left(1 - \alpha_{ni}\right) \left(1 - \sigma_n\right)$.

(e) Explain why $\frac{\partial \log \theta_{ni}}{\partial \log(w/r)} = (\varepsilon_n - 1) \cdot (\alpha_{ni} - \alpha_n)$. Hint: Write $\frac{\partial \log \theta_{ni}}{\partial \log(w/r)} = \frac{\partial \log \theta_{ni}}{\partial \log\left(\frac{p_{ni}}{p_n}\right)} \times \frac{\partial \log\left(\frac{p_{ni}}{p_n}\right)}{\partial \log(w/r)}$. What do each of the two terms represent?

(f) Plug the relationships from parts (d) and (e) into your expression of $\frac{\partial \left(\sum_i \alpha_{ni}\theta_{ni}\right)}{\partial \log(w/r)}$ from part (c). Re-arrange until you get the desired result.