

# Problem Set 5: Due Wednesday, February 22

For this problem set, please e-mail Problem 1 and Problem 2 as separate documents.

## Problem 1

Submit a 2-3 page outline for your Final Paper project. The syllabus, on the bottom half of page 1, describes five questions that you should answer in your project. Do your best to address these questions. In about a week and a half, for Problem Set 7, a classmate will look over your proposal and assess your answer to these questions, and give a short ten minute presentation based on your write-up.

## Problem 2

The purpose of this exercise is to familiarize you with the Oaxaca-Blinder decomposition, a method that can be used to partition changes in the mean of a variable (e.g. workers' wages) that can be attributed to i) changes in observable differences (e.g., education, work experience) across the two points in time, versus ii) changes in the outcome conditional on these observable differences (e.g., wages conditional on education status, experience). This decomposition is a key building block of the Firpo, Fortin, and Lemieux decomposition (which tries to decompose changes over time in other distributional statistics, beside the mean) which we will discuss in Wednesday's class.

Assume, as an example, that wages at in 1980, wages are given by

$$\log w_{i,1980} = X_i \cdot \beta_{1980} + \epsilon_{i,1980}, \quad (1)$$

and that wages in 2010 are given by

$$\log w_{i,2010} = X_i \cdot \beta_{2010} + \epsilon_{i,2010} . \quad (2)$$

The covariates in Equation 1 and 2 include categorical variables for education—i) high school dropouts; ii) high school graduates; iii) some college; iv) college graduate; v) postgraduate education— and categorical variables of potential labor market experience—i) 0-10 years of experience; ii) 11-20 years of experience; iii) 21-30 years of experience; and iv) 31+ years of experience.

The decomposition of observed wage changes between the two periods is given begins

with the following equations:

$$\begin{aligned}
\Delta_O &\equiv \mathbb{E} [\log w_{i,2010} - \log w_{i,1980}] \\
&= \mathbb{E}_{2010} [\mathbb{E} [\log w_{i,2010} | X]] - \mathbb{E}_{1980} [\mathbb{E} [\log w_{i,1980} | X]] \\
&= \mathbb{E}_{2010} [X] \cdot \beta_{2010} - \mathbb{E}_{1980} [X] \cdot \beta_{1980}
\end{aligned}$$

In the previous equation  $\mathbb{E}_{2010}$  signifies that the expectation is over the values that the covariate could take in 2010 (and similarly for  $\mathbb{E}_{1980}$  and 1980).

With additional manipulations

$$\begin{aligned}
\Delta_O &= \mathbb{E}_{2010} [X] \cdot \beta_{2010} - \mathbb{E}_{1980} [X] \cdot \beta_{1980} \\
&= \mathbb{E}_{2010} [X] \cdot \beta_{2010} - \mathbb{E}_{1980} [X] \cdot \beta_{2010} + \mathbb{E}_{1980} [X] \cdot \beta_{2010} - \mathbb{E}_{1980} [X] \cdot \beta_{1980} \\
&= \underbrace{(\bar{X}_{2010} - \bar{X}_{1980}) \cdot \beta_{2010}}_{\equiv \Delta_X} + \underbrace{\bar{X}_{1980} \cdot (\beta_{2010} - \beta_{1980})}_{\equiv \Delta_S} \tag{3}
\end{aligned}$$

The "trick", here, is to add an subtract  $\mathbb{E}_{1980} [X] \cdot \beta_{2010}$ , the counterfactual wage that would hold if individuals with experience/education of the 1980s were paid according to wage structure in 2010. The first term in Equation 3,  $\Delta_X$ , (which Fortin, Firpo, and Lemieux and others call the composition effect) reflects changes in wages due to differences in experience, education, etc... The second term,  $\Delta_S$ , (the wage structure effect) reflects the part of the wage differential unexplained by compositional differences.

1. Using your data set from Problem Set 4, compute  $\Delta_O$ ,  $\Delta_X$  and  $\Delta_S$ . In your initial regression let the "base" education group be workers with high school education and 11-20 years of potential experience. For this entire problem use only male workers. Use the same sample and data cleaning choices you made for Problem Set 4.
2. Note that one can further break down Equation 3 as

$$\begin{aligned}
\Delta_O &= (\bar{X}_{2010} - \bar{X}_{1980}) \cdot \beta_{2010} + \bar{X}_{1980} \cdot (\beta_{2010} - \beta_{1980}) \\
&= (\bar{X}_{2010, \text{education}} - \bar{X}_{1980, \text{education}}) \cdot \beta_{2010, \text{education}} \\
&\quad + (\bar{X}_{2010, \text{experience}} - \bar{X}_{1980, \text{experience}}) \cdot \beta_{2010, \text{experience}} \\
&\quad + \bar{X}_{1980, \text{education}} \cdot (\beta_{2010, \text{education}} - \beta_{1980, \text{education}}) \\
&\quad + \bar{X}_{1980, \text{experience}} \cdot (\beta_{2010, \text{experience}} - \beta_{1980, \text{experience}}) + \beta_{2010, 0} - \beta_{1980, 0}
\end{aligned}$$

Compute the individual components of  $\Delta_X$  and  $\Delta_S$ .

3. Recompute your answer to part 2 with post-graduate education as the base education group. Are the magnitudes of the composition and wage structure effects robust to the omitted category?
4. Note an alternate (perhaps equally reasonable) decomposition to Equation (3) is

$$\begin{aligned}
\Delta_O &= \mathbb{E}_{2010} [X] \cdot \beta_{2010} - \mathbb{E}_{1980} [X] \cdot \beta_{1980} \\
&= \mathbb{E}_{2010} [X] \cdot \beta_{2010} - \mathbb{E}_{2010} [X] \cdot \beta_{1980} + \mathbb{E}_{2010} [X] \cdot \beta_{1980} - \mathbb{E}_{1980} [X] \cdot \beta_{1980} \\
&= \underbrace{(\bar{X}_{2010} - \bar{X}_{1980}) \cdot \beta_{1980}}_{\equiv \Delta_X} + \underbrace{\bar{X}_{2010} \cdot (\beta_{2010} - \beta_{1980})}_{\equiv \Delta_S}
\end{aligned} \tag{4}$$

Recompute the Oaxaca-Blinder decomposition using Equation 4. Is the Oaxaca-Blinder decomposition robust to the base year?

As a side note: This decomposition was originally devised (in Oaxaca, 1973, "Male-Female Wage Differentials in Urban Labor Markets") to understand differences in male-female wage differences. For example using the same methodology, one may examine the extent female wages lower due to less educational attainment, or other observable differences? To address this question, we could re-label the above equations (for example with 1980 instead of male, and 2010 instead of female.)