

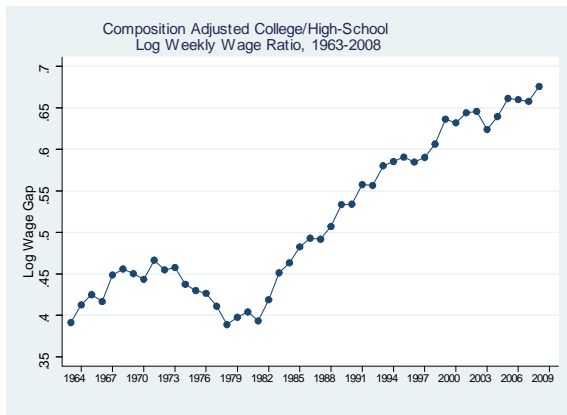
Questions for the next couple of weeks

1. What are the trends of labor income inequality?
 - ▶ ... in the relationship between education and income? between experience and income?
 - ▶ ... in between-group vs. within-group (residual) inequality?
2. What are the sources of these trends?
 - ▶ Institutional factors
 - ▶ Unionization
 - ▶ Minimum wage
 - ▶ Supply and demand factors
 - ▶ Trade and offshoring
 - ▶ Automation/computerization

Outline for the next couple of weeks

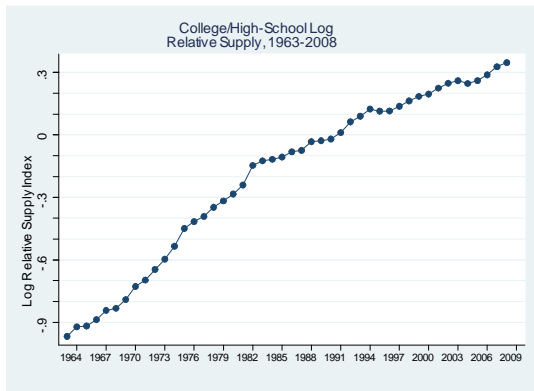
- ▶ Acemoglu and Autor, Autor and Dorn:
 - ▶ "Canonical" model can help rationalize changes in the skill premium, but...
 - ▶ ... a task-based model of the labor market is necessary to match other trends in the wage distribution.
- ▶ Burstein, Morales, Vogel: Computerization explains much of the changes in "between-group" inequality.
- ▶ Statistical decompositions of the wage distribution

Acemoglu and Autor (2011; Figure 1)



- ▶ College premium increases from 50 percent (1963) to 60 percent (1973),
- ▶ ... decreases to 50 percent (1979), then up to 95 percent by 2010.

Acemoglu and Autor (2011; Figure 2)



- ▶ Fraction of hours worked by college workers increases continuously, decelerating in 1982

"Canonical Model"

- ▶ Designed to rationalize things like the last two figures.
- ▶ Workers, i , come in one of two skill types.
 - ▶ Let $L = \int_{i \in \mathcal{L}} l_i di$ refer to the aggregate supply of low-skilled workers
 - ▶ Let $H = \int_{i \in \mathcal{H}} h_i di$ refer to the aggregate supply of high-skilled workers
- ▶ Aggregate output is a combination of low-skilled, high-skilled-workers.

$$Y = \left[(A_H H)^{\frac{\sigma-1}{\sigma}} + (A_L L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

"Canonical Model"

- ▶ Designed to rationalize things like the last two figures.
- ▶ Workers, i , come in one of two skill types.
 - ▶ Let $L = \int_{i \in \mathcal{L}} l_i di$ refer to the aggregate supply of low-skilled workers
 - ▶ Let $H = \int_{i \in \mathcal{H}} h_i di$ refer to the aggregate supply of high-skilled workers
- ▶ Aggregate output is a combination of low-skilled, high-skilled-workers.

$$Y = \left[(A_H H)^{\frac{\sigma-1}{\sigma}} + (A_L L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- ▶ Factor markets are competitive:

$$w_L = \frac{\partial Y}{\partial L} = [A_L]^{\frac{\sigma-1}{\sigma}} \cdot L^{-\frac{1}{\sigma}} \left[(A_H H)^{\frac{\sigma-1}{\sigma}} + (A_L L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}$$

$$w_H = \frac{\partial Y}{\partial H} = [A_H]^{\frac{\sigma-1}{\sigma}} \cdot H^{-\frac{1}{\sigma}} \left[(A_H H)^{\frac{\sigma-1}{\sigma}} + (A_L L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}$$

"Canonical Model"

Implications:

1. Take the $\frac{\partial w_L}{\partial(H/L)}$ derivative \Rightarrow Increases in the supply of high skilled (versus low skilled workers) increase the wages of low-skilled workers

"Canonical Model"

Implications:

1. Take the $\frac{\partial w_L}{\partial(H/L)}$ derivative \Rightarrow Increases in the supply of high skilled (versus low skilled workers) increase the wages of low-skilled workers
2. Take the $\frac{\partial w_L}{\partial A_L}$ and $\frac{\partial w_L}{\partial A_H}$ derivatives \Rightarrow Improvements in technology raise wages of both types of workers.

"Canonical Model"

Implications:

1. Take the $\frac{\partial w_L}{\partial(H/L)}$ derivative \Rightarrow Increases in the supply of high skilled (versus low skilled workers) increase the wages of low-skilled workers
2. Take the $\frac{\partial w_L}{\partial A_L}$ and $\frac{\partial w_L}{\partial A_H}$ derivatives \Rightarrow Improvements in technology raise wages of both types of workers.
3. Combine the two FOCs from the last slide

$$\log \frac{w_H}{w_L} = \frac{\sigma - 1}{\sigma} \log \frac{A_H}{A_L} - \frac{1}{\sigma} \log \frac{H}{L}$$

"Canonical Model"

Implications:

1. Take the $\frac{\partial w_L}{\partial(H/L)}$ derivative \Rightarrow Increases in the supply of high skilled (versus low skilled workers) increase the wages of low-skilled workers
2. Take the $\frac{\partial w_L}{\partial A_L}$ and $\frac{\partial w_L}{\partial A_H}$ derivatives \Rightarrow Improvements in technology raise wages of both types of workers.
3. Combine the two FOCs from the last slide

$$\log \frac{w_H}{w_L} = \frac{\sigma - 1}{\sigma} \log \frac{A_H}{A_L} - \frac{1}{\sigma} \log \frac{H}{L}$$

- ▶ $\frac{H}{L}$ and $\frac{w_H}{w_L}$ have been increasing \Rightarrow Either $\sigma > 1$ and $\frac{A_H}{A_L}$ is growing or $\sigma < 1$ and $\frac{A_H}{A_L}$ is shrinking.

"Canonical Model"

- ▶ From last slide, adding time subscripts:

$$\log \frac{w_{Ht}}{w_{Lt}} = \frac{\sigma - 1}{\sigma} \log \frac{A_{Ht}}{A_{Lt}} - \frac{1}{\sigma} \log \frac{H_t}{L_t}$$

- ▶ Suppose

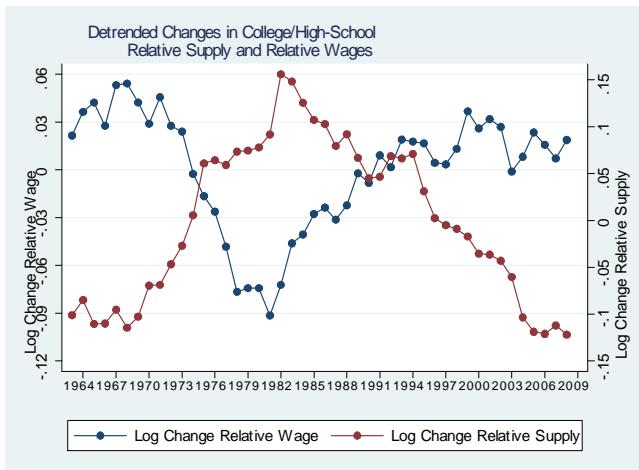
$$\log \frac{A_{Ht}}{A_{Lt}} = \gamma_0 + \gamma_1 t$$

- ▶ Then

$$\log \frac{w_{Ht}}{w_{Lt}} = \frac{\sigma - 1}{\sigma} \gamma_0 + \frac{\sigma - 1}{\sigma} \gamma_1 t - \frac{1}{\sigma} \log \frac{H_t}{L_t}$$

- ▶ An OLS regression, using data from 1963-87, of the past previous equation $\Rightarrow \sigma \approx 1.6$ and $\gamma_1 \approx 0.07$

"Canonical Model"



"Canonical Model"

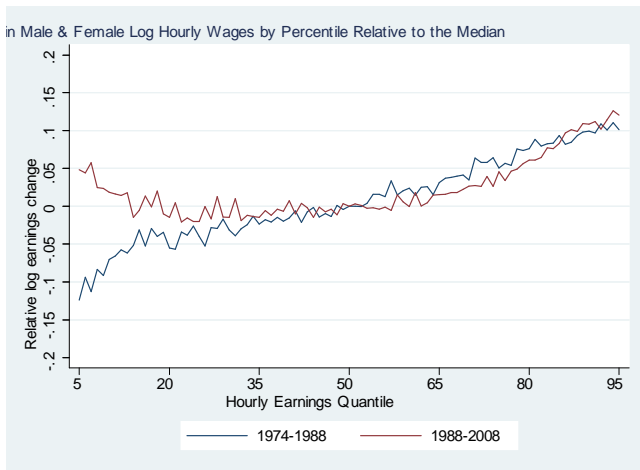
Summary

1. Changes in the skill premium are due to changes in factor-augmenting technology and to the supply of workers of different skilled types.
2. Fits well the decrease in the skill premium of the 70s, increase in the 60s and 80s. (Overpredicts the skill premium increase of the 90s and 00s)

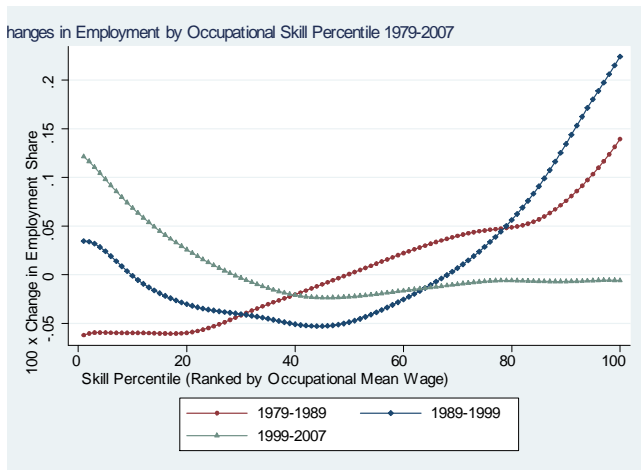
Areas where it has trouble fitting the data

1. Has trouble explaining why wages would decline for some types of workers.
2. Cannot speak to wage polarization, or occupational polarization, which seems to have been prevalent in the 90s and/or 00s.

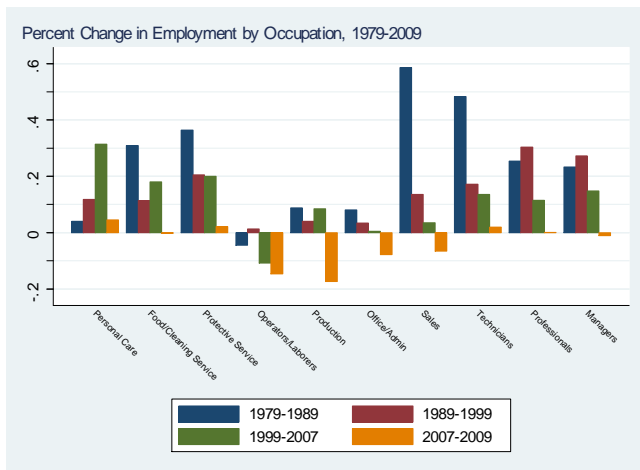
Wage polarization



Occupational polarization



Occupational polarization



Notes on Autor and Dorn (2013): "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market"

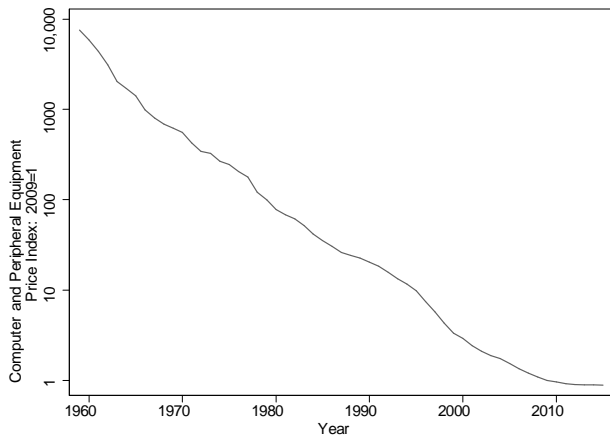
Overview

- ▶ Research question: To what extent is job polarization due to a decline in the demand for routine tasks?
 - ▶ Underlying driving force: Decline in the price of computers (substitutable with labor in the production of routine tasks)

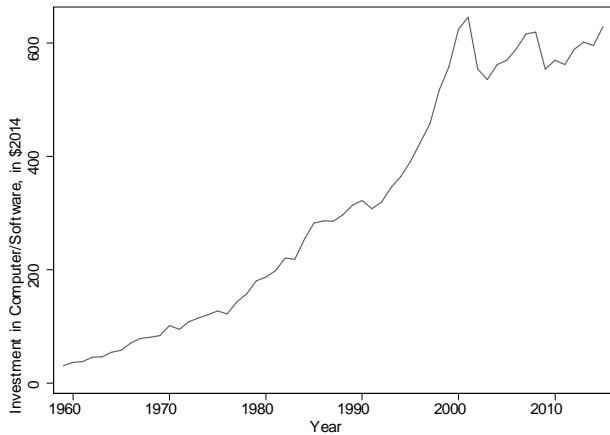
Two types of evidence and a model.

- ▶ National evidence:
 - ▶ Growth in the bottom of the distribution is increasingly in service occupations. Without this growth, statistically, there would be no wage polarization.
 - ▶ Routine occupations are in the 2nd quintile.
- ▶ Cross-sectional evidence:
 - ▶ Areas with higher initial routine-occupation shares have larger service occupation growth rates, more polarization.

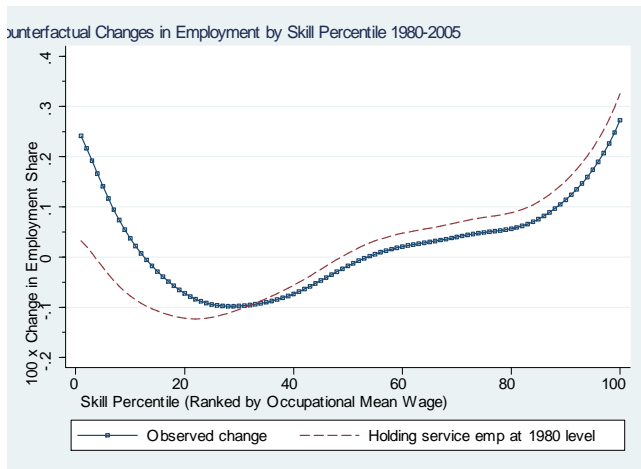
Price of computers and peripheral equipment



Investment in computers and software



Without the growth of services, job polarization looks much less pronounced



Reweighting procedure

- ▶ Due originally to DiNardo, Firpo, Lemieux (1996)
- ▶ Designed to answer things like: What would a counterfactual employment distribution look like if the distribution of service occupations vs. other occupations stayed as in 1980?
- ▶ Mechanics: Pool sample of (many) workers in 1980 and 2005.
 - ▶ Let π = share of observations from 1980.

Reweighting procedure

- ▶ Due originally to DiNardo, Firpo, Lemieux (1996)
- ▶ Designed to answer things like: What would a counterfactual employment distribution look like if the distribution of service occupations vs. other occupations stayed as in 1980?
- ▶ Mechanics: Pool sample of (many) workers in 1980 and 2005.
 - ▶ Let π = share of observations from 1980.
 - ▶ Run a regression

$$\mathbf{1}_{\text{year}=1980} = \Lambda(\beta_0 + \beta_1 \cdot \mathbf{1}_{\text{occupation}_t=\text{service}} + \varepsilon_i)$$

Reweighting procedure

- ▶ Due originally to DiNardo, Firpo, Lemieux (1996)
- ▶ Designed to answer things like: What would a counterfactual employment distribution look like if the distribution of service occupations vs. other occupations stayed as in 1980?
- ▶ Mechanics: Pool sample of (many) workers in 1980 and 2005.
 - ▶ Let π = share of observations from 1980.
 - ▶ Run a regression

$$\mathbf{1}_{\text{year}=1980} = \Lambda(\beta_0 + \beta_1 \cdot \mathbf{1}_{\text{occupation}_t=\text{service}} + \varepsilon_i)$$

- ▶ Generate predicted values p_i

Reweighting procedure

- ▶ Due originally to DiNardo, Firpo, Lemieux (1996)
- ▶ Designed to answer things like: What would a counterfactual employment distribution look like if the distribution of service occupations vs. other occupations stayed as in 1980?
- ▶ Mechanics: Pool sample of (many) workers in 1980 and 2005.
 - ▶ Let π = share of observations from 1980.
 - ▶ Run a regression

$$\mathbf{1}_{\text{year}=1980} = \Lambda(\beta_0 + \beta_1 \cdot \mathbf{1}_{\text{occupation}_i=\text{service}} + \varepsilon_i)$$

- ▶ Generate predicted values p_i
- ▶ Weight 2005 observations $w_i = \frac{p_i}{1-p_i} \cdot \frac{1-\pi}{\pi}$.

Reweighting procedure

- ▶ Due originally to DiNardo, Firpo, Lemieux (1996)
- ▶ Designed to answer things like: What would a counterfactual employment distribution look like if the distribution of service occupations vs. other occupations stayed as in 1980?
- ▶ Mechanics: Pool sample of (many) workers in 1980 and 2005.
 - ▶ Let π = share of observations from 1980.
 - ▶ Run a regression

$$\mathbf{1}_{\text{year}=1980} = \Lambda(\beta_0 + \beta_1 \cdot \mathbf{1}_{\text{occupation}_i=\text{service}} + \varepsilon_i)$$

- ▶ Generate predicted values p_i
- ▶ Weight 2005 observations $w_i = \frac{p_i}{1-p_i} \cdot \frac{1-\pi}{\pi}$.
- ▶ Weight 1980 observations $w_i = 1$

Reweighting procedure

- ▶ Due originally to DiNardo, Firpo, Lemieux (1996)
- ▶ Designed to answer things like: What would a counterfactual employment distribution look like if the distribution of service occupations vs. other occupations stayed as in 1980?
- ▶ Mechanics: Pool sample of (many) workers in 1980 and 2005.
 - ▶ Let π = share of observations from 1980.
 - ▶ Run a regression

$$\mathbf{1}_{\text{year}=1980} = \Lambda(\beta_0 + \beta_1 \cdot \mathbf{1}_{\text{occupation}_i=\text{service}} + \varepsilon_i)$$

- ▶ Generate predicted values p_i
 - ▶ Weight 2005 observations $w_i = \frac{p_i}{1-p_i} \cdot \frac{1-\pi}{\pi}$.
 - ▶ Weight 1980 observations $w_i = 1$
-
- ▶ Since $\beta_1 < 0$: for i in a service occupation $p_i < \pi \Rightarrow w_i < 1$

Reweighting procedure

Where does this procedure come from?

- ▶ Suppose we observe (Y, X) from two periods (or from two groups).
- ▶ The marginal cdf of Y in period 1 is

$$F_{Y_1}(y) = \int F_{Y_1|X}(y|x) dF_{X_1}(x)$$

(integrate over the covariates that are observed in period 1).

- ▶ Similarly for period 2:

$$F_{Y_2}(y) = \int F_{Y_2|X}(y|x) dF_{X_2}(x)$$

Reweighting procedure

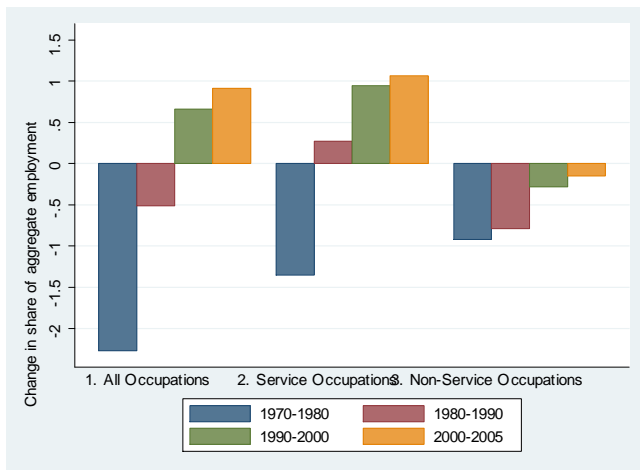
Where does this procedure come from?

- ▶ Our ("holding services constant") exercise was doing something like:

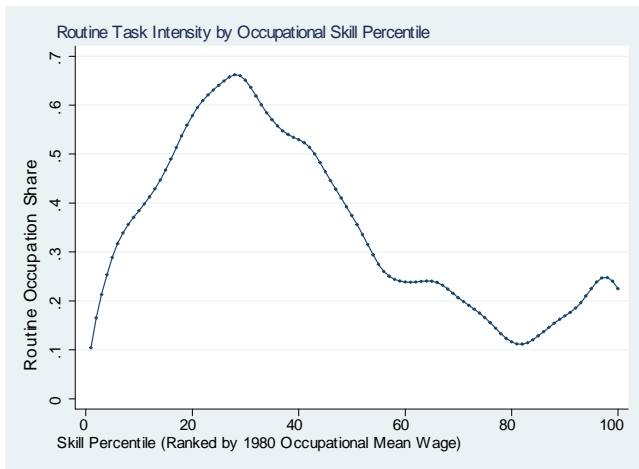
$$\begin{aligned} F_{Y_2^C}(y) &= \int F_{Y_2|X}(y|x) dF_{X_1}(x) \\ &= \int F_{Y_2|X}(y|x) \cdot \frac{dF_{X_1}(x)}{dF_{X_2}(x)} \cdot dF_{X_2}(x) \end{aligned}$$

- ▶ $\frac{dF_{X_1}(x)}{dF_{X_2}(x)}$ was the weighting factor that we computed in the last slide.

Beginning in the 1980s, service occupation (unlike other low-type occupations) tend to grow.



Routine tasks are concentrated in the second quintile of the "skill" distribution



Measures of Occupational Content

Dictionary of Occupational Titles

- ▶ Updated periodically (first version in 1939, last version in 1991)
 - ▶ see <http://www.icpsr.umich.edu/icpsrweb/ICPSR/studies/8942> for the 1977 version.
- ▶ For each occupation multiple measures, including:
 1. Direction, Control, and Planning of activities
 2. Mathematical ability
 3. Ability to Set limits, Tolerances, or Standards
 4. Finger Dexterity
 5. Eye Hand Coordination.
- ▶ Autor, Levy, and Murnane (2003) first used these as measures of interpersonal tasks (1), analytical tasks(2), routine manual tasks (3-4), and nonroutine manual tasks (5).
- ▶ In Autor and Dorn, the routineness measure equals the difference between 3, 4 and 1, 2, 5

Measures of Occupational Content

Dictionary of Occupational Titles

- ▶ Alternatively: Michaels, Rauch, Redding (2016) use text from the descriptions of each occupation.
- ▶ Description of an economist:

Plans, designs, and conducts research to aid in interpretation of economic relationships and in solution of problems arising from production and distribution of goods and services: Studies economic and statistical data in area of specialization, such as finance, labor, or agriculture. Devises methods and procedures for collecting and processing data, utilizing knowledge of available sources of data and various econometric and sampling techniques....

- ▶ Groups verbs (e.g., "plans", "design", "studies", "devises") according to the type of task (analytical? interpersonal?)

Measures of Occupational Content

O*NET

- ▶ First version in 1998, updated periodically. Based on interviews of workers (plus expert opinion) across a much broader set of work elements:

13. Persuasion Persuading others to change their minds or behavior.

A. How important is PERSUASION to the performance of *your current job*?

Not Important* Somewhat Important Important Very Important Extremely Important

① ————— ② ————— ③ ————— ④ ————— ⑤

* If you marked Not Important, skip LEVEL below and go on to the next skill.

B. What level of PERSUASION is needed to perform *your current job*?

Solicit donations for a charity Convince a supervisor to purchase a new copy machine Change the opinion of the jury in a complex legal case

① ————— ② ————— ③ ————— ④ ————— ⑤ ————— ⑥ ————— ⑦

Highest Level

- ▶ Top occupations (importance/level): 1) Sales Engineers, 2) Sales Representatives, 3) Chief Executives/ 1) Chief Executives, 2) Arbitrators, 3) Lawyers.
- ▶ 68 questions like this on skills/requirements, 98 questions on work activities/contexts.

Measures of Occupational Content

O*NET

A common approach is to take certain questionnaire items as measures of routineness, manual vs. cognitive, etc...

- ▶ Routine manual:
 - ▶ Pace determined by speed of equipment
 - ▶ Controlling machines and processes
 - ▶ Spend time making repetitive motions
- ▶ Non-routine cognitive: Analytical
 - ▶ Analyzing data/information
 - ▶ Thinking creatively
 - ▶ Interpreting information for others
- ▶ Non-routine cognitive: Interpersonal
 - ▶ Establishing and maintaining personal relationships
 - ▶ Guiding, directing and motivating subordinates
 - ▶ Coaching/developing others

Task-based model

1. Workers have different abilities to perform different tasks, sort into different tasks based on their comparative advantage.
2. Model can be consistent with job/wage polarization, technology growth \Rightarrow declines in wages for some workers.
3. Generalization of the "canonical" model from earlier.
4. We'll work out a closed-economy model, but consider different values of β

Key elements of the task-based model

- ▶ High-skilled, H (produce abstract/analytic tasks).
- ▶ Low-skilled workers, U (produce either routine or manual tasks).
 - ▶ have heterogeneous ability η to produce routine tasks, pdf $f(\eta) = e^{-\eta}$
- ▶ Two sectors:
 - ▶ Goods

$$Y_g = \underbrace{\left(L_a \right)^{1-\beta}}_{\text{"abstract tasks"}=1} \cdot \underbrace{\left[(\alpha_r L_r)^\mu + (\alpha_k K)^\mu \right]}_{\text{"routine tasks"}}^{\frac{\beta}{\mu}}$$

Key elements of the task-based model

- ▶ High-skilled, H (produce abstract/analytic tasks).
- ▶ Low-skilled workers, U (produce either routine or manual tasks).
 - ▶ have heterogeneous ability η to produce routine tasks, pdf $f(\eta) = e^{-\eta}$
- ▶ Two sectors:
 - ▶ Goods

$$Y_g = \underbrace{\left(L_a\right)^{1-\beta}}_{\text{"abstract tasks"}=1} \cdot \underbrace{\left[\left(\alpha_r L_r\right)^\mu + \left(\alpha_k K\right)^\mu\right]}_{\text{"routine tasks"}}^{\frac{\beta}{\mu}}$$

- ▶ Elasticity of substitution between computers and routine workers in producing routine tasks is $\frac{1}{1-\mu}$

Key elements of the task-based model

- ▶ High-skilled, H (produce abstract/analytic tasks).
- ▶ Low-skilled workers, U (produce either routine or manual tasks).
 - ▶ have heterogeneous ability η to produce routine tasks, pdf $f(\eta) = e^{-\eta}$
- ▶ Two sectors:
 - ▶ Goods

$$Y_g = \underbrace{\left(L_a \right)^{1-\beta}}_{\text{"abstract tasks"}=1} \cdot \underbrace{\left[(\alpha_r L_r)^\mu + (\alpha_k K)^\mu \right]}_{\text{"routine tasks"}^{\frac{\beta}{\mu}}$$

- ▶ Elasticity of substitution between computers and routine workers in producing routine tasks is $\frac{1}{1-\mu}$
- ▶ Over time, price of capital is declining at rate δ

Key elements of the task-based model

- ▶ Two sectors:
- ▶ Services (everything that is not a goods occupation)

$$Y_s = \underbrace{L_m}_{\text{"manual tasks"}}$$

Key elements of the task-based model

- ▶ Two sectors:
- ▶ Services (everything that is not a goods occupation)

$$Y_s = \underbrace{L_m}_{\text{"manual tasks"}}$$

- ▶ Goods are used either as consumption or to produce capital:
 $c_g + e^{-\delta t} \theta K = Y_g$

Key elements of the task-based model

- ▶ Two sectors:
- ▶ Services (everything that is not a goods occupation)

$$Y_s = \underbrace{L_m}_{\text{"manual tasks"}}$$

- ▶ Goods are used either as consumption or to produce capital:
$$c_g + e^{-\delta t} \theta K = Y_g$$
- ▶ Utility over goods and services is CES with elasticity σ

Allocation of low-skilled workers

- ▶ Suppose η_i is the cutoff unskilled worker, where $1 - L_m$ workers are to the right of η_i .
 - ▶ $\eta_i = -\log(1 - L_m)$
- ▶ The effective labor of routine tasks equals the integral to the right of η_i

$$\begin{aligned}L_r &= \int_{\eta_i}^{\infty} \eta e^{-\eta} \cdot d\eta \\ &= \int_{-\log(1-L_m)}^{\infty} \eta e^{-\eta} \cdot d\eta \\ &= (1 - L_m) (1 - \log [1 - L_m])\end{aligned}$$

Planner's problem

$$\begin{aligned} & \max_{K, L_m} \left[(c_s)^{\frac{\sigma-1}{\sigma}} + (c_g)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ & = \max_{K, L_m} \left[L_m^{\frac{\sigma-1}{\sigma}} + (Y_g - p_k K)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ & = \max_{K, L_m} \left[L_m^{\frac{\sigma-1}{\sigma}} + \left(L_a^{1-\beta} [(\alpha_r L_r)^\mu + (\alpha_k K)^\mu]^{\frac{\beta}{\mu}} - p_k K \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Use

$$L_r = (1 - L_m) (1 - \log[1 - L_m])$$

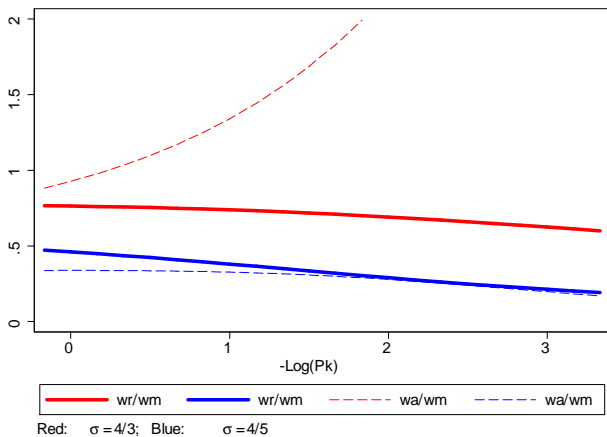
from the last slide

Use this model to ask: What happens to $\frac{L_r}{L_m}$, $\frac{w_r}{w_m}$, $\frac{w_a}{w_m}$ as $p_k \rightarrow 0$?
How does this depend on σ , β , μ ?

What is the importance of σ ?

- ▶ σ measures the substitutability between goods (where H workers work, and where some U workers work) and services (where U workers work).
- ▶ If U skilled workers end up working in manual occupations (we will give conditions for this in a second), what happens to $\frac{w_a}{w_m}$ when $p_k \rightarrow 0$?
 - ▶ When $p_k \rightarrow 0$, if $\sigma > 1$: consumers are relatively happy to substitute to the increasingly cheaper goods \Rightarrow labor demand for H workers increases relative to that for U workers $\Rightarrow \frac{w_a}{w_m}$ increases.
 - ▶ Opposite result for $\frac{w_a}{w_m}$ when $\sigma < 1$.

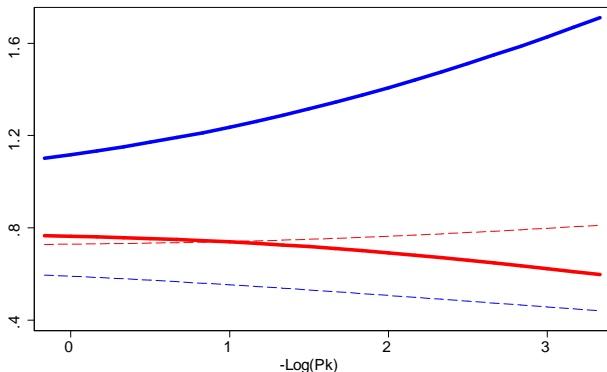
When σ is >1 , $\frac{w_a}{w_m}$ increases as $p_k \rightarrow 0$.



What is the importance of σ , β , μ ?

- ▶ If μ is large, computers and workers are substitutable in producing routine tasks. For a given decline in p_k , labor demand (for producing routine tasks) falls more
- ▶ If σ is large, consumers are happy to substitute between goods and services. Declining price of computers (and hence goods) means that the demand for workers producing goods (either routine workers or high-skilled "a" workers) increases more.
- ▶ Long-run ($p_k \rightarrow 0$) allocation of workers and $\frac{w_r}{w_m}$ follows
 - ▶ If $\frac{1}{\sigma} > 1 - \frac{\mu}{\beta}$ then $L_m \rightarrow 1$ and $\frac{w_r}{w_m} \rightarrow 0$.
 - ▶ If $\frac{1}{\sigma} < 1 - \frac{\mu}{\beta}$ then $L_m \rightarrow 0$ and $\frac{w_r}{w_m} \rightarrow 1$.

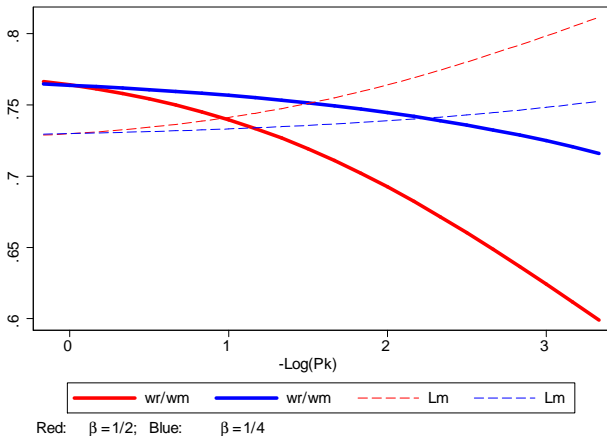
Path of $\frac{w_r}{w_m}$ depends on $\frac{1}{\sigma}$ compared to $\frac{\beta-\mu}{\beta}$



Red: $\sigma = 4/3$; Blue: $\sigma = 2$

Legend: $\frac{w_r}{w_m}$ (solid line), L_m (dashed line)

"Polarization is more pronounced when β is large

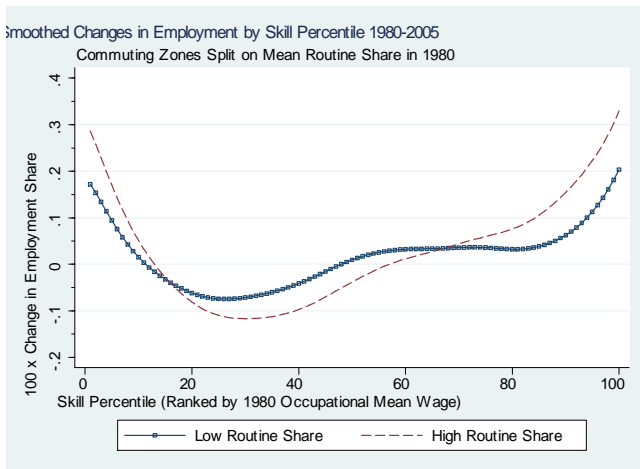


Summarizing the model predictions

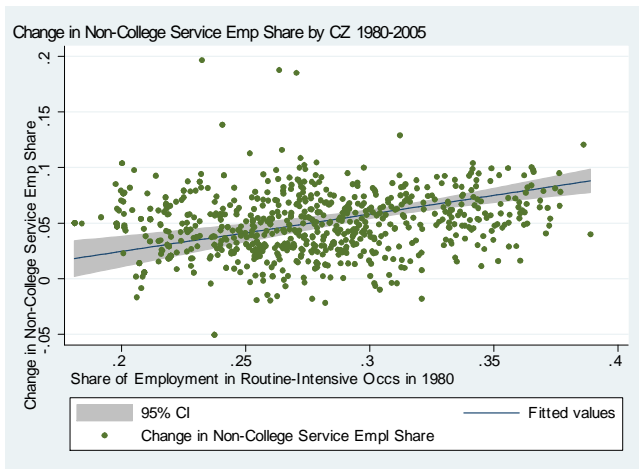
Cross-sectional implications:

- ▶ Regions differ according to β_j .
- 1. IT adoption coincides with replacement of labor from routine tasks, into service occupations
- 2. With greater IT adoption (which happens in high routine-labor-share areas), greater shifts of low-skilled labor into service occupations.

Areas with a greater share of routine-intensive occupations in 1980 had "more polarization"



Areas with a greater share of routine-intensive occupations in 1980 had "more polarization"



Summary

Open question so far: How much of the observed change in the service occupation share comes because of lower computer prices?
From other sources?

- ▶ How informative is the cross-regional variation from the previous slide?

Notes on Burstein, Morales, Vogel (2016):
"Changes in between-group inequality:
computers, occupations, and international trade"

Research Question

- ▶ To what extent can skill-biased technical change account for a higher skill premium? (Or more generally the changes in between-group inequality which we have observed)
- ▶ Others (including Krusell et al. 2000) have addressed this in the past.
 - ▶ Add structures, equipment to our "canonical model" production function

$$G(K_{st}, K_{et}, H_t, L_t) = K_{st}^\alpha \left[\mu L_t^{\frac{\sigma-1}{\sigma}} + (1-\mu) \times \left(\lambda K_{te}^{\frac{\rho-1}{\rho}} + (1-\lambda) H_t^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1} \frac{\sigma}{\sigma-1}} \right]^{1-\alpha \frac{\sigma}{\sigma-1}}$$

Research Question

- ▶ To what extent can skill-biased technical change account for a higher skill premium? (Or more generally the changes in between-group inequality which we have observed)
- ▶ Others (including Krusell et al. 2000) have addressed this in the past.
 - ▶ Add structures, equipment to our "canonical model" production function

$$G(K_{st}, K_{et}, H_t, L_t) = K_{st}^\alpha \left[\mu L_t^{\frac{\sigma-1}{\sigma}} + (1-\mu) \times \left(\lambda K_{te}^{\frac{\rho-1}{\rho}} + (1-\lambda) H_t^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1} \frac{\sigma}{\sigma-1}} \right]^{1-\alpha \frac{\sigma}{\sigma-1}}$$

- ▶ Allow for $H_t = \tilde{H}_t \psi_{ht}$ to be a composite of hours worked by high-skilled workers and unobserved labor quality/utilization (similarly for low-skilled labor)

More on Krusell et al. 2000

- ▶ With appropriate (competitive) assumptions, this setup leads to the following equation:

$$\log \left(\frac{w_{Ht}}{w_{Lt}} \right) = \lambda \left[\frac{\sigma - \rho}{\sigma(\rho - 1)} \right] \left(\frac{K_{et}}{K_{st}} \right)^{\frac{\rho-1}{\rho}} + \frac{1}{\sigma} \log \left(\frac{\tilde{H}_t}{\tilde{L}_t} \right) + \frac{\sigma - 1}{\sigma} \log \left(\frac{\psi_{ht}}{\psi_{lt}} \right)$$

- ▶ Key to Krusell et al.: $\sigma > \rho$:
 - ▶ If high-skilled labor and equipment are relatively complementary, then lower prices of equipment push up the demand for skilled labor.
 - ▶ In their benchmark specification, assume that $\frac{\psi_{ht}}{\psi_{lt}}$ is constant.
- ▶ Main finding (after estimating σ , ρ , among other parameters) is that changes in $\frac{K_{et}}{K_{st}}$ can account for most of the changes in $\frac{w_{Ht}}{w_{Lt}}$.

Approach

- ▶ Overall goal: Use individual-level data from different points in time to recover sources of between-group premia changes.
- ▶ Workers of different types (college vs. high-school educated; women vs. men; experienced vs. inexperienced)
 - ▶ tend to use computer capital at different intensities;
 - ▶ tend to be employed in different occupations (which may be growing or shrinking for non-computer related reasons);
 - ▶ may be experiencing different trends in unobserved (quality); and
 - ▶ may have growing/shrinking observed labor supply
- ▶ Construct a general equilibrium model in which these four forces can lead to changes in
 - ▶ which occupations workers work in.
 - ▶ the wages of different types of workers
- ▶ ... to back out the importance of supply; occupational demand; unobserved labor quality; increased computer capital in generating wage premia.

October CPS has a supplement on Computer/Internet Usage

- ▶ Ask things like:
 - ▶ Does the worker "directly use computer at work?"
 - ▶ Is a "computer used at work for electronic mail?"
 - ▶ Is a "computer used at work for programming?"
 - ▶ Is a "Computer used at home for household record keeping, taxes, etc.?"

- ▶ Every few years between 1989 (in the paper, 1984) and 2003. More frequently after.

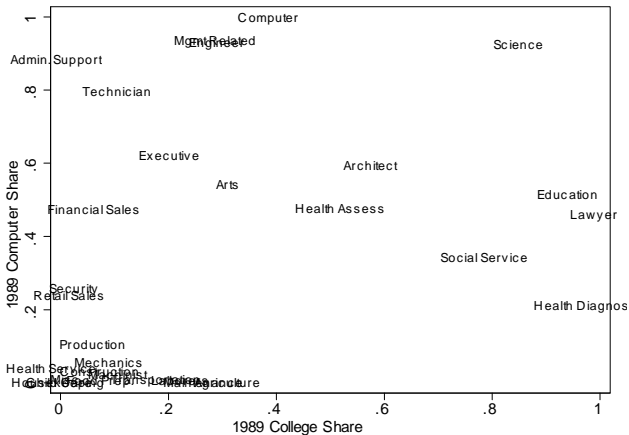
Occupations differ considerably in their workers, computer usage

		Computer	Female
1	Executive, Admin.	57	34
2	Mgmt. Related	80	49
3	Architect	46	12
4	Engineer	79	8
5	Life, Physical Science	76	29
6	Computer/Math	96	36
7	Social Services	38	47
8	Lawyer	47	23
9	Education	47	65
10	Arts	49	41
11	Health diagnosing	33	17
12	Health treatment	51	82
13	Technician	67	43
14	Financial Sales	50	35
15	Retail Sales	36	45

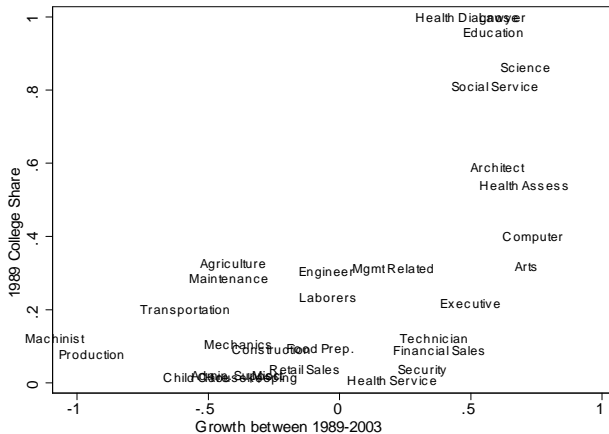
Occupations differ considerably in their workers, computer usage

		\leq HS	Some College	Collge+
1	Executive, Admin.	44	28	28
2	Mgmt. Related	32	36	32
3	Architect	7	42	52
4	Engineer	24	42	34
5	Life, Physical Science	10	19	71
6	Computer/Math	28	32	40
7	Social Services	20	19	61
8	Lawyer	3	3	94
9	Education	7	12	81
10	Arts	40	26	34
11	Health diagnosing	2	2	95
12	Health treatment	14	33	53
13	Technician	48	31	20
14	Financial Sales	54	27	19
15	Retail Sales	62	24	14

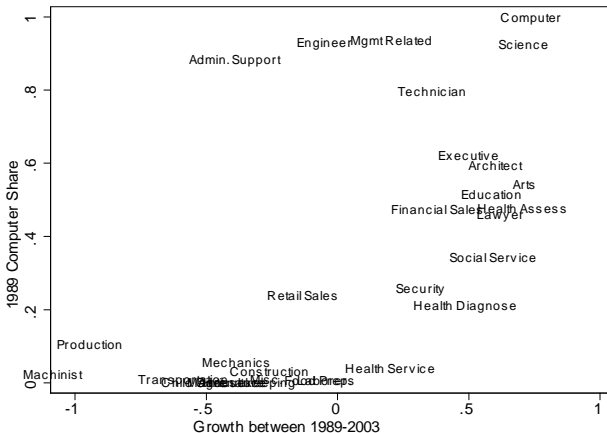
College+ Share and Computer Share Are Moderately Positively Correlated



College+ occupations (as of 1989) grew between 1989 and 2003



Computer-heavy occupations (as of 1989) grew between 1989 and 2003

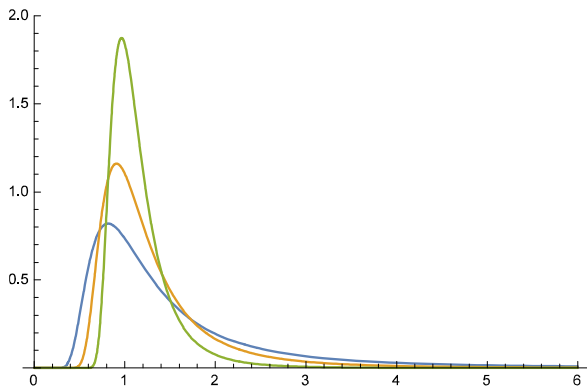


Reasons why college premium could increase/decrease:

1. Demand for occupations (those with many college workers) has been increasing
2. Price of computers has been decreasing.
3. Labor quality of college workers has been increasing relative to other workers.
4. Supply of college workers has been increasing

Background on the Frechet Distribution

- ▶ $G(\varepsilon) = \exp\{-\varepsilon^{-\theta}\}$
- ▶ Pdf of the Frechet distribution for $\theta \in \{2, 3, 5\}$:



- ▶ Less dispersion when θ is large.

Background on the Frechet Distribution

- ▶ Suppose we have a sample of N Frechet independently distributed random variables, $\phi_1, \dots, \phi_N \dots$

Background on the Frechet Distribution

- ▶ Suppose we have a sample of N Frechet independently distributed random variables, $\phi_1, \dots, \phi_N, \dots$
- ▶ And for each of the $n \in \{1, \dots, N\}$ there is a shifter T_n .

Background on the Frechet Distribution

- ▶ Suppose we have a sample of N Frechet independently distributed random variables, $\phi_1, \dots, \phi_N, \dots$
- ▶ And for each of the $n \in \{1, \dots, N\}$ there is a shifter T_n .
- ▶ What is $\pi_n \equiv \Pr \{T_n \phi_n > \max_{m \neq n} T_m \phi_m\}$?

Background on the Frechet Distribution

- ▶ Suppose we have a sample of N Frechet independently distributed random variables, $\phi_1, \dots, \phi_N, \dots$
- ▶ And for each of the $n \in \{1, \dots, N\}$ there is a shifter T_n .
- ▶ What is $\pi_n \equiv \Pr \{ T_n \phi_n > \max_{m \neq n} T_m \phi_m \}$?
- ▶ Let's start with an easier problem.

$$\begin{aligned} \Pr \left\{ \max_{m \neq n} T_m \phi_m < x \right\} &= \prod_{m \neq n} \Pr \left\{ \max_{m \neq n} \phi_m < \frac{x}{T_m} \right\} \\ &= \prod_{m \neq n} \exp \left\{ - \left(\frac{x}{T_m} \right)^{-\theta} \right\} \\ &= \exp \left[- \sum x^{-\theta} T_m^\theta \right] \exp \left\{ x^{-\theta} T_n^\theta \right\} \end{aligned}$$

Background on the Frechet Distribution

- ▶ From last slide:

$$\Pr \left\{ \max_{m \neq n} T_m \phi_m < x \right\} = \exp \left[- \sum x^{-\theta} T_m^\theta \right] \exp \left\{ x^{-\theta} T_n^\theta \right\}$$

Background on the Frechet Distribution

- ▶ From last slide:

$$\Pr \left\{ \max_{m \neq n} T_m \phi_m < x \right\} = \exp \left[- \sum x^{-\theta} T_m^\theta \right] \exp \left\{ x^{-\theta} T_n^\theta \right\}$$

- ▶ Back to our original problem:

$$\begin{aligned} \pi_n &= \int_0^\infty \Pr \left\{ \max_{m \neq n} T_m \phi_m < x \right\} dG(x) \\ &= \int_0^\infty \exp \left[-x^{-\theta} \sum T_m^\theta \right] \exp \left\{ x^{-\theta} T_n^\theta \right\} \cdot dG(x) \\ &= \theta T_n^\theta \int_0^\infty \exp \left[-x^{-\theta} \sum T_m^\theta \right] x^{-1-\theta} dx \\ &= \theta T_n^\theta \frac{\exp \left\{ - \sum T_m^\theta x^{-\theta} \right\}}{\sum T_m^\theta \cdot \theta} \Bigg|_{x=0}^\infty \\ &= \frac{T_n^\theta}{\sum T_m^\theta} \end{aligned}$$

Background on the Frechet Distribution

- ▶ Summary

$$\pi_n = \frac{T_n^\theta}{\sum_m T_m^\theta}$$

- ▶ Heterogeneity in idiosyncratic draws (θ is small) means that π is less sensitive to the T 's.
- ▶ Another formula. Back in the Frechet case:

$$\mathbb{E} \left[\max_m T_m \phi_m \right] = \left[\sum T_m^\theta \right]^{1/\theta} \cdot \Gamma \left[\frac{\theta - 1}{\theta} \right]$$

- ▶ Formula for the choice probability resembles that in the Gumbel (type 1 extreme value) distribution.
 - ▶ $G(x) = \exp \{-\exp \{-x\}\}$.
 - ▶ In this case the probabilities would look like:

$$\pi_n = \frac{\exp \{\theta T_n\}}{\sum_m \exp \{\theta T_m\}}$$

Model Overview

- ▶ Aggregate output is a CES composite of occupation (ω) output, can be used for consumption or capital investment

$$Y_t = \left[\sum_{\omega} \underbrace{\mu_t(\omega)}_{\text{demand shifter}} Y_t(\omega)^{\frac{\rho-1}{\rho}} \right]^{\rho/(\rho-1)}$$
$$= C_t + \underbrace{p_t(\kappa=0) Y_t(\kappa=0)}_{\text{non computer capital inv.}} + \underbrace{p_t(\kappa=1) Y_t(\kappa=1)}_{\text{computer capital investment.}}$$

Model Overview

- ▶ Aggregate output is a CES composite of occupation (ω) output, can be used for consumption or capital investment

$$Y_t = \left[\sum_{\omega} \underbrace{\mu_t(\omega)}_{\text{demand shifter}} Y_t(\omega)^{\frac{\rho-1}{\rho}} \right]^{\rho/(\rho-1)}$$
$$= C_t + \underbrace{p_t(\kappa = 0) Y_t(\kappa = 0)}_{\text{non computer capital inv.}} + \underbrace{p_t(\kappa = 1) Y_t(\kappa = 1)}_{\text{computer capital investment.}}$$

- ▶ Workers z come of different types, λ (education; gender; experience).

Model Overview

- ▶ Aggregate output is a CES composite of occupation (ω) output, can be used for consumption or capital investment

$$Y_t = \left[\sum_{\omega} \underbrace{\mu_t(\omega)}_{\text{demand shifter}} Y_t(\omega)^{\frac{\rho-1}{\rho}} \right]^{\rho/(\rho-1)}$$
$$= C_t + \underbrace{p_t(\kappa = 0) Y_t(\kappa = 0)}_{\text{non computer capital inv.}} + \underbrace{p_t(\kappa = 1) Y_t(\kappa = 1)}_{\text{computer capital investment.}}$$

- ▶ Workers z come of different types, λ (education; gender; experience).
 - ▶ Workers have idiosyncratic productivity in different occupations, capital types, $\varepsilon(z, \kappa, \omega)$ is distributed according to a Frechet distribution.

Model Overview

- ▶ Occupation output is the combination of capital and labor of different types. A worker of type λ with l units of efficiency labor, using capital k of type κ , produces in occupation ω :

$$k(\kappa)^\alpha \cdot l(\lambda)^{1-\alpha} \cdot T(\lambda, \kappa, \omega)^{1-\alpha}$$

Model Overview

- ▶ Occupation output is the combination of capital and labor of different types. A worker of type λ with l units of efficiency labor, using capital k of type κ , produces in occupation ω :

$$k(\kappa)^\alpha \cdot l(\lambda)^{1-\alpha} \cdot T(\lambda, \kappa, \omega)^{1-\alpha}$$

- ▶ $T_t(\lambda, \kappa, \omega)$ is the productivity in producing occupation output when combining capital κ and labor type λ

Model Overview

- ▶ Things that are exogeneous
 - ▶ supply of workers of type λ
 - ▶ $T(\lambda, \kappa, \omega)$
 - ▶ demand for occupations $\mu(\omega)$
 - ▶ price of capital $p(\kappa)$
- ▶ What we want to solve for
 - ▶ Occupational choice and wages of different types of workers
 - ▶ Choice on computer vs. non-computer capital.

Occupational Choice

- ▶ Suppose the price of ω 's output is $p(\omega)$ and of capital κ is $p(\kappa)$. A profit maximizing firm is maximizing

$$p(\omega) \cdot k(\kappa)^\alpha \cdot l(\lambda)^{1-\alpha} \cdot T(\lambda, \kappa, \omega)^{1-\alpha} - \underbrace{v(\lambda, \kappa, \omega)}_{\text{efficiency wage, tbd}} \cdot l(\lambda) - p(\kappa) k$$

Occupational Choice

- ▶ Suppose the price of ω 's output is $p(\omega)$ and of capital κ is $p(\kappa)$. A profit maximizing firm is maximizing

$$p(\omega) \cdot k(\kappa)^\alpha \cdot l(\lambda)^{1-\alpha} \cdot T(\lambda, \kappa, \omega)^{1-\alpha} - \underbrace{v(\lambda, \kappa, \omega) \cdot l(\lambda)}_{\text{efficiency wage, tbd}} - p(\kappa) k$$

- ▶ Plug in profit maximizing choice of capital to get indirect profit function as a function of a firm. Then compute marginal productivity of labor to get

$$v(\lambda, \kappa, \omega) = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} \cdot p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} T(\lambda, \kappa, \omega)$$

Occupational Choice

- ▶ Suppose the price of ω 's output is $p(\omega)$ and of capital κ is $p(\kappa)$. A profit maximizing firm is maximizing

$$p(\omega) \cdot k(\kappa)^\alpha \cdot l(\lambda)^{1-\alpha} \cdot T(\lambda, \kappa, \omega)^{1-\alpha} - \underbrace{v(\lambda, \kappa, \omega) \cdot l(\lambda)}_{\text{efficiency wage, tbd}} - p(\kappa) k$$

- ▶ Plug in profit maximizing choice of capital to get indirect profit function as a function of a firm. Then compute marginal productivity of labor to get

$$v(\lambda, \kappa, \omega) = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} \cdot p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} T(\lambda, \kappa, \omega)$$

- ▶ Important point. Efficiency wage is just a function of its type.

Occupational Choice

- ▶ Suppose the price of ω 's output is $p(\omega)$ and of capital κ is $p(\kappa)$. A profit maximizing firm is maximizing

$$p(\omega) \cdot k(\kappa)^\alpha \cdot l(\lambda)^{1-\alpha} \cdot T(\lambda, \kappa, \omega)^{1-\alpha} - \underbrace{v(\lambda, \kappa, \omega)}_{\text{efficiency wage, tbd}} \cdot l(\lambda) - p(\kappa) k$$

- ▶ Plug in profit maximizing choice of capital to get indirect profit function as a function of a firm. Then compute marginal productivity of labor to get

$$v(\lambda, \kappa, \omega) = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} \cdot p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} T(\lambda, \kappa, \omega)$$

- ▶ Important point. Efficiency wage is just a function of its type.
- ▶ Workers choose occupations to maximize

$$v(\lambda, \kappa, \omega) \cdot \varepsilon(z, \kappa, \omega)$$

Occupational Choice

- ▶ From last slide, the value of a worker of type λ in occupation ω with capital type κ :

$$v(\lambda, \kappa, \omega) = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} \cdot p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} T(\lambda, \kappa, \omega)$$

Occupational Choice

- ▶ From last slide, the value of a worker of type λ in occupation ω with capital type κ :

$$v(\lambda, \kappa, \omega) = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} \cdot p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} T(\lambda, \kappa, \omega)$$

- ▶ The probability that a type λ worker chooses κ, ω is

$$\pi(\lambda, \kappa, \omega) = \frac{\left[p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} \cdot T(\lambda, \kappa, \omega) \right]^{\theta}}{\left[\sum_{\kappa', \omega'} p(\kappa')^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega')^{\frac{1}{1-\alpha}} T(\lambda', \kappa', \omega) \right]^{\theta}} \quad (1)$$

Occupational Choice

- ▶ From last slide, the value of a worker of type λ in occupation ω with capital type κ :

$$v(\lambda, \kappa, \omega) = (1 - \alpha) \alpha^{\frac{-\alpha}{1-\alpha}} \cdot p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} T(\lambda, \kappa, \omega)$$

- ▶ The probability that a type λ worker chooses κ, ω is

$$\pi(\lambda, \kappa, \omega) = \frac{\left[p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} \cdot T(\lambda, \kappa, \omega) \right]^\theta}{\left[\sum_{\kappa', \omega'} p(\kappa')^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega')^{\frac{1}{1-\alpha}} T(\lambda', \kappa', \omega) \right]^\theta} \quad (1)$$

- ▶ And the wage of a worker of type λ

$$w(\lambda) = \Gamma \left[\frac{\theta - 1}{\theta} \right] \cdot \left[\sum_{\kappa, \omega} v(\lambda, \kappa, \omega)^\theta \right]^{1/\theta} \quad (2)$$

Goods market clearing

For each occupation, expenditures equals payments to factors in that occupation

$$\mu_t(\omega) \cdot p_t(\omega)^{1-\rho} \cdot E_t = \frac{1}{1-\alpha} \cdot \sum_{\lambda, \kappa} w_t(\lambda) \cdot L_t(\lambda) \cdot \pi_t(\lambda, \kappa, \omega) \quad (3)$$

- ▶ LHS: Total expenditure (E_t) multiplied by expenditure share of occupation ω .
- ▶ RHS: The sum equals payments just to labor. The $\frac{1}{1-\alpha}$ indicates that a fraction α of payments go to capital, so to get revenues scale up labor costs by $(1-\alpha)^{-1}$.

Equations (1)-(3) characterize and equilibrium.

Effect of...

- ▶ Decrease in the price of computers (or equivalently an increase in the productivity of working with computers, through $T(\lambda, \kappa, \omega)$):
 - ▶ Raises the wage of workers who use computers intensively.
 - ▶ Reduces the price of occupations which use computers.
 - ▶ Lowers the wage of workers in these occupations who don't use computers.

- ▶ Increase in the demand for an occupation (or equivalently an increase in the productivity of workers in an occupation):

Effect of...

- ▶ Decrease in the price of computers (or equivalently an increase in the productivity of working with computers, through $T(\lambda, \kappa, \omega)$):
 - ▶ Raises the wage of workers who use computers intensively.
 - ▶ Reduces the price of occupations which use computers.
 - ▶ Lowers the wage of workers in these occupations who don't use computers.

- ▶ Increase in the demand for an occupation (or equivalently an increase in the productivity of workers in an occupation):
 - ▶ Raises the wage of individuals disproportionately in this occupation.

Effect of...

- ▶ Decrease in the price of computers (or equivalently an increase in the productivity of working with computers, through $T(\lambda, \kappa, \omega)$):
 - ▶ Raises the wage of workers who use computers intensively.
 - ▶ Reduces the price of occupations which use computers.
 - ▶ Lowers the wage of workers in these occupations who don't use computers.
- ▶ Increase in the demand for an occupation (or equivalently an increase in the productivity of workers in an occupation):
 - ▶ Raises the wage of individuals disproportionately in this occupation.
- ▶ Increase in the supply of workers of type λ

Effect of...

- ▶ Decrease in the price of computers (or equivalently an increase in the productivity of working with computers, through $T(\lambda, \kappa, \omega)$):
 - ▶ Raises the wage of workers who use computers intensively.
 - ▶ Reduces the price of occupations which use computers.
 - ▶ Lowers the wage of workers in these occupations who don't use computers.
- ▶ Increase in the demand for an occupation (or equivalently an increase in the productivity of workers in an occupation):
 - ▶ Raises the wage of individuals disproportionately in this occupation.
- ▶ Increase in the supply of workers of type λ
 - ▶ Reduces the wages of λ

Effect of...

- ▶ Decrease in the price of computers (or equivalently an increase in the productivity of working with computers, through $T(\lambda, \kappa, \omega)$):
 - ▶ Raises the wage of workers who use computers intensively.
 - ▶ Reduces the price of occupations which use computers.
 - ▶ Lowers the wage of workers in these occupations who don't use computers.
- ▶ Increase in the demand for an occupation (or equivalently an increase in the productivity of workers in an occupation):
 - ▶ Raises the wage of individuals disproportionately in this occupation.
- ▶ Increase in the supply of workers of type λ
 - ▶ Reduces the wages of λ
 - ▶ Reduces the prices of occupations ω which use λ intensively, wages of workers who work in same occupations as λ .

Backing out the exogenous shifters

- ▶ We are interested in the *changes* of wages, worker allocation as a function of *changes* in labor supply, the price of capital, occupational demand, and productivity
- ▶ Assume a particular functional form for how T changes over time

$$T_t(\lambda, \kappa, \omega) = T_t(\lambda) \cdot T_t(\kappa) \cdot T_t(\omega) \cdot T(\lambda, \kappa, \omega)$$

$T(\lambda, \kappa, \omega)$ will be unobserved. Use a hat to refer to a change:
 $\hat{X}_t \equiv \frac{X_{t+\tau}}{X_t}$. So:

$$\hat{T}_t(\lambda, \kappa, \omega) = \hat{T}_t(\lambda) \cdot \hat{T}_t(\kappa) \cdot \hat{T}_t(\omega)$$

Backing out the exogenous shifters

- ▶ Can't tell apart differences in capital productivity and exogenous capital prices. Define a variable which combines their effect:

$$\hat{q}_t(\kappa) = [\hat{p}_t(\kappa)]^{-\frac{\alpha}{1-\alpha}} \cdot \hat{T}_t(\kappa)$$

- ▶ Similarly for occupation prices:

$$\hat{q}_t(\omega) = \hat{p}_t(\omega)^{\frac{1}{1-\alpha}} \cdot \hat{T}_t(\omega)$$

- ▶ And occupational output:

$$\hat{a}_t(\omega) = \hat{\mu}_t(\omega)^{\frac{1}{1-\alpha}} \cdot \hat{T}_t(\omega)^{(1-\alpha)(\rho-1)}$$

Backing out the exogenous shifters

- ▶ Changes in $\hat{L}(\lambda) / \hat{L}(\lambda_1)$ can be read off of data.
- ▶ With the variable re-definitions from the previous slide, Equations (1) to (3) imply that when comparing two periods:

$$\left(\frac{\hat{q}(\kappa)}{\hat{q}(\kappa_1)} \right)^\theta = \frac{\hat{\pi}(\lambda, \kappa, \omega)}{\hat{\pi}(\lambda, \kappa_1, \omega)}$$

The right hand side is observable, which (if θ were known) would allow us to back out changes in capital productivity.

- ▶ Similarly one can compute changes in occupational productivity

$$\left(\frac{\hat{q}(\omega)}{\hat{q}(\omega_1)} \right)^\theta = \frac{\hat{\pi}(\lambda, \kappa, \omega)}{\hat{\pi}(\lambda, \kappa, \omega_1)}$$

Backing out the exogenous shifters

- ▶ Finally, for a given value of ρ one can back out shocks to occupational output

$$\frac{\hat{a}(\omega)}{\hat{a}(\omega_1)} = \frac{\sum_{\lambda, \kappa} w(\lambda) \widehat{L}(\lambda) \pi_t(\lambda, \kappa, \omega)}{\sum_{\lambda, \kappa} w(\lambda) \widehat{L}_t(\lambda) \pi_t(\lambda, \kappa, \omega_1)} \cdot \left(\frac{\hat{q}(\omega)}{\hat{q}(\omega_1)} \right)^{(1-\alpha)(\rho-1)}$$

- ▶ And finally, to back out labor productivity:

$$\frac{\hat{T}(\lambda)}{\hat{T}(\lambda_1)} = \frac{\hat{w}(\lambda)}{\hat{w}(\lambda_1)} \cdot \left[\frac{\hat{s}(\lambda)}{\hat{s}(\lambda_1)} \right]^{1/\theta}$$

where $\hat{s}(\lambda)$ is an ugly expression we have already solved for, describing λ -specific average equipment productivity changes.

Results

Period	Data	Labor Composition	Occupation Shifters
84-89	0.057	-0.031	0.026
89-93	0.064	-0.017	-0.009
93-97	0.037	-0.023	0.044
97-03	-0.007	-0.043	-0.011
84-03	0.151	-0.114	0.049

Equipment Productivity	Labor Productivity
0.052	0.009
0.045	0.046
0.021	-0.005
0.042	0.006
0.159	0.056

Conclusion

- ▶ Declining prices of computer equipment account for a 16 percentage point increase in the college/high-school wage gap.
- ▶ Increased relative supply of college-educated workers has a countervailing, 11 percentage point effect on the college wage premia.
- ▶ Final sections in the paper endogenize decline in computer equipment prices, change in the demand for occupation services, through declining trade costs with other countries.
- ▶ But what about residual wage inequality?
- ▶ And what about non-technological, non-supply factors, such as declining unionization rates?

Notes on Firpo, Fortin, Lemieux (2014):
"Occupational Tasks and Changes in
the Wage Structure"

Changes in Attributes

	1983-85	2003-05
Unionization	26.2	15.2
Experience		
<10 years	34.4	25.2
10-20 years	29.0	25.3
20-30 years	17.3	26.2
30-40 years	12.7	18.5
>40 years	6.6	4.7
Education		
< High School	12.5	8.0
High School	38.5	31.1
Some College	19.2	27.6
College	12.9	19.5
Post-graduate	9.9	10.1

Occupational Characteristics

- ▶ Top occupations per O*NET Element:
 - ▶ Information: Life Scientist, Physicist, Engineer
 - ▶ Automation: Production Supervisor, Prepress Technicians, Power Plant Operator
 - ▶ Face-to-Face: Supervisor of Security Personnel, Clergy, Doctor
 - ▶ On-Site Job: Mining Operator, Material Moving Workers, Pilot
 - ▶ Decision-Making: Chief Executive, Supervisor of Security Personnel, Farming Supervisor

- ▶ Differences across the sample period:

	1983-85	2003-05
Information	-0.02	0.06
Automation	0.01	-0.03
Face-to-Face	-0.04	0.10
On-Site Job	-0.02	0.06
Decision-Making	-0.03	0.07

Research Question

- ▶ What is the effect of changes in
 - ▶ occupational characteristics
 - ▶ unionization
 - ▶ education and experience... on inequality in the wage distribution?

Review of the Oaxaca-Blinder Decomposition

Assume that log wages obey the linear model

$$\log w_{i,t} = X_i \cdot \beta_t + \epsilon_{i,t}$$

Then, the mean wage, between periods t_0 and t_1 is given by

$$\begin{aligned}\Delta_O^\mu &= \underbrace{(\bar{X}_{t_1} - \bar{X}_{t_0}) \cdot \beta_{t_0}}_{\equiv \Delta_X} + \underbrace{\bar{X}_{t_1} \cdot (\beta_{t_1} - \beta_{t_0})}_{\equiv \Delta_S} \\ &= \sum_k (\bar{X}_{t_1 k} - \bar{X}_{t_0 k}) \beta_{t_0 k} \\ &\quad + \beta_{t_1,0} - \beta_{t_0,0} + \sum_k \bar{X}_{t_1 k} \cdot (\beta_{t_1 k} - \beta_{t_0 k})\end{aligned}$$

where k is an individual covariate

Can't extend this idea exactly to other distributional statistics

A key step of this decomposition was

$$\begin{aligned}\Delta_O^\mu &\equiv \mathbb{E} [\log w_{i,t_1} - \log w_{i,t_0}] \\ &= \mathbb{E}_{t_1} [\mathbb{E} [\log w_{i,t_1} | X]] - \mathbb{E}_{t_0} [\mathbb{E} [\log w_{i,t_0} | X]] \\ &= \mathbb{E}_{t_1} [X] \cdot \beta_{t_1} - \mathbb{E}_{t_1} [X] \cdot \beta_{t_0} + \mathbb{E}_{t_1} [X] \cdot \beta_{t_0} - \mathbb{E}_{t_0} [X] \cdot \beta_{t_0}\end{aligned}$$

In the second line, we use the law of iterated expectations.

Could we have done the same for the median (or 75th percentile, etc..)? No, because:

$$Q_{50}(\log w_{i,t}) \neq \mathbb{E}_t [Q_{50}[\log w_{i,t} | X]]$$

- ▶ Problem: Analogue of law of iterated expectations does not hold.

Quoting Thomas Lemieux:

"Decomposing proportions is easier than decomposing quantiles"

- ▶ "Example: 10 percent of men earn more than 80K a year, but only 5 percent of women do so."
- ▶ "Easy to do a decomposition by running [linear probability] model for the probability of earning less (or more) than 80K, and perform a Oaxaca decomposition on the proportions."
- ▶ "By contrast, much less obvious how to decompose the difference between the 90th quantile for men (80K) and women (say 65K)"
- ▶ A way to write out the proportion of individuals making more than some quantity will be given by the recentered influence function.

Recentered Influence Function (RIF)

Definition (for a particular quantile):

$$RIF(\log w, Q_{50}) = Q_{50}(\log w) + \frac{0.5 - \mathbf{1}\{\log w < Q_{50}\}}{f_{\log w}(Q_{50})}$$

Why is this useful? (Now plugging in a general τ)

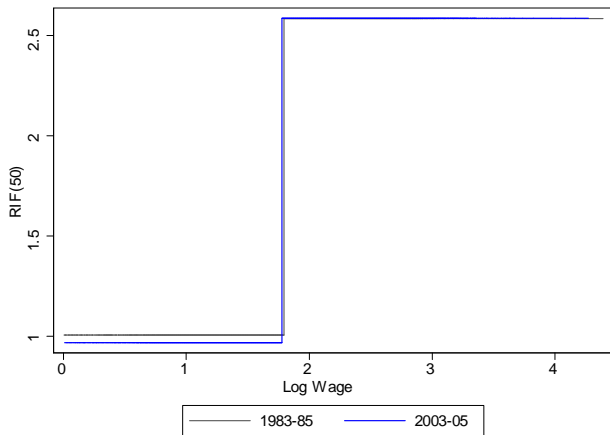
$$Q_{\tau}(\log w) = \int \mathbb{E}[RIF(\log w, Q_{\tau}) | X = x] dF_X(X)$$

⇒ We've re-written the problem in a way so that we can use the law-of-iterated expectations.

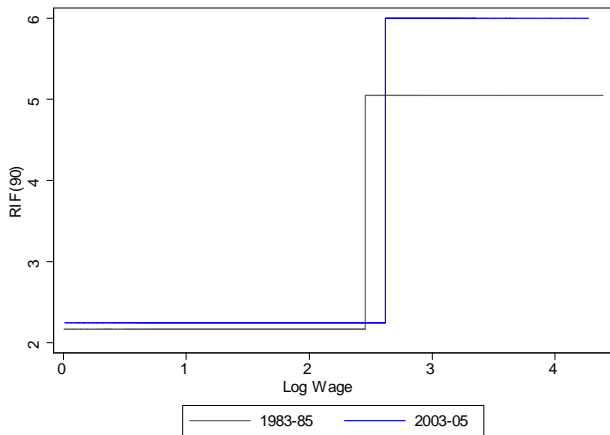
- ▶ If we assume that $\mathbb{E}[RIF(\log w, Q_{\tau}) | X = x]$ is a linear function of X , we can use the Oaxaca-Blinder decomposition on $RIF(\log w, Q_{\tau})$
- ▶ Note:

$$\begin{aligned}\mathbb{E}[RIF(\log w, Q_{\tau}) | X] &= c_{1\tau} \mathbb{E}[\mathbf{1}\{\log w < Q_{\tau}\} | X] + c_{2\tau} \\ &= c_{1\tau} \cdot \Pr\{\log w < Q_{\tau}\} + c_{2\tau}\end{aligned}$$

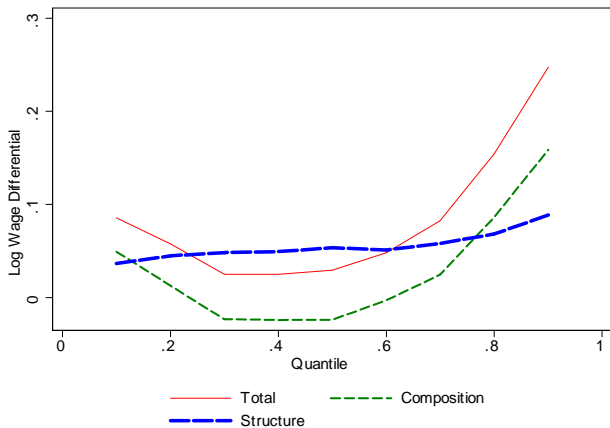
RIF (50)



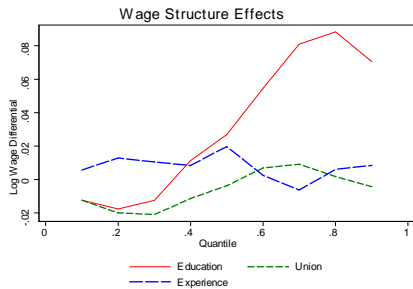
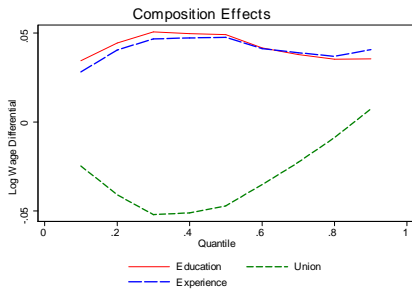
RIF (90)



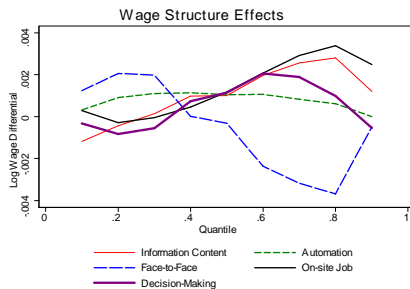
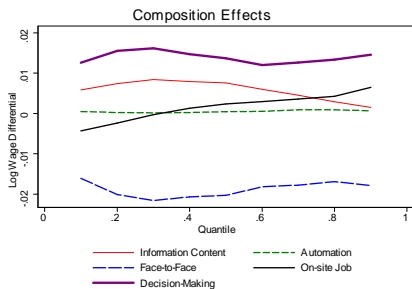
Overall Decomposition



Education/Union/Experience



Occupation



Summary

- ▶ These decompositions are a useful way to summarize changes in wages for different types of workers.
- ▶ But, interpretation of what these decompositions are telling us is difficult:
 - ▶ Are declining unionization rates or declines in the real minimum wage exogenous?
 - ▶ What are the general equilibrium effects of e.g. declining unionization rates?