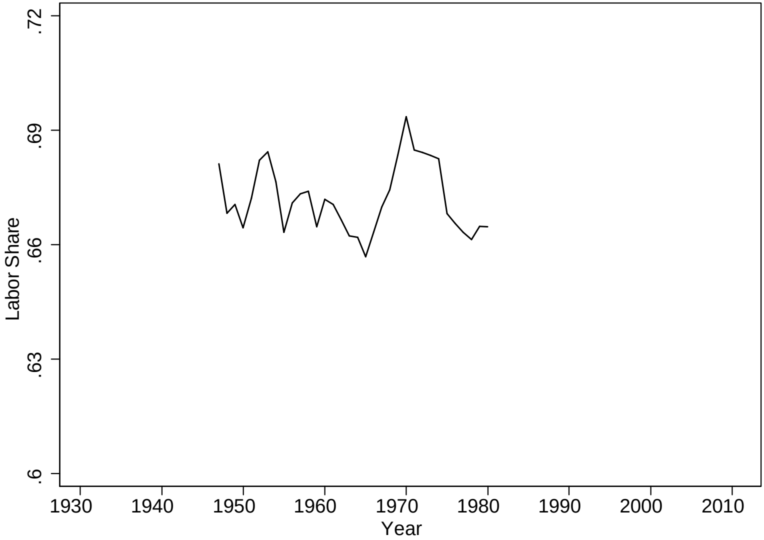
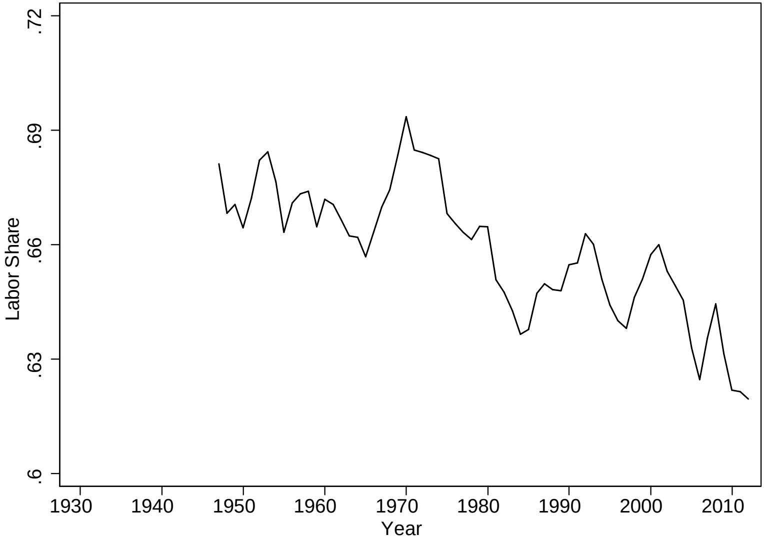
Kaldor Facts & Kuznets Facts

- Kaldor Facts
  1. $\frac{Y}{L}$ grows at a sustained rate
  2. $\frac{K}{L}$ grows at a sustained rate
     $(1) + (2) \Rightarrow \frac{Y}{K}$ is roughly stable.
  3. $r = i - \pi$ is stable
  4. The capital and labor shares of national income are stable (roughly $\frac{1}{3}$ and $\frac{2}{3}$)
  5. Y per capita grows at a stable rate
- Kuznets Facts: As economies grow, the shares of income/consumption in services grow, in agriculture shrink, and in manufacturing are roughly constant (grow and then shrink).

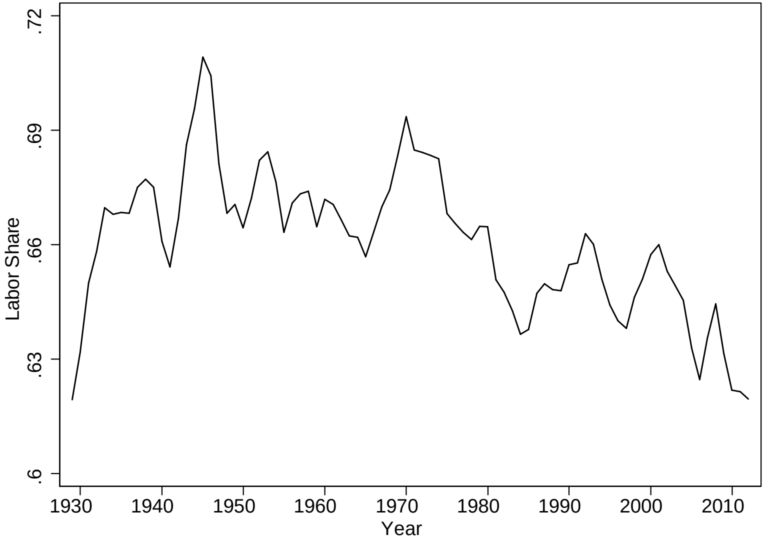
Labor Share of Income



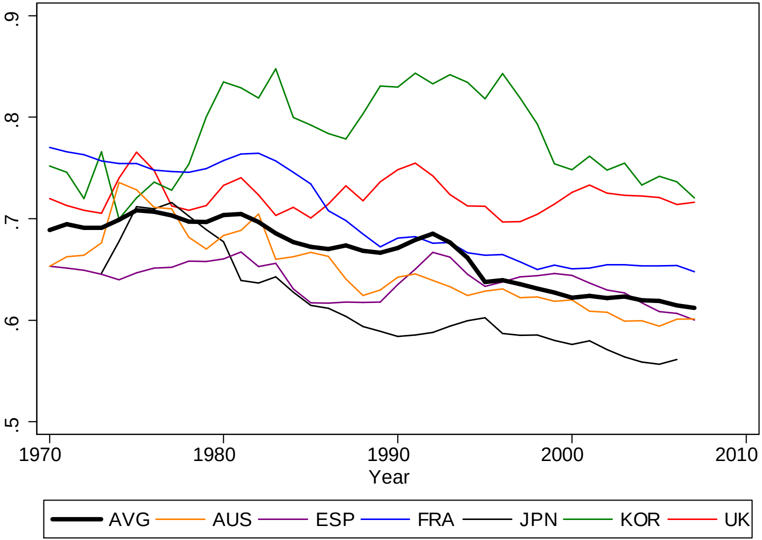
Labor Share of Income



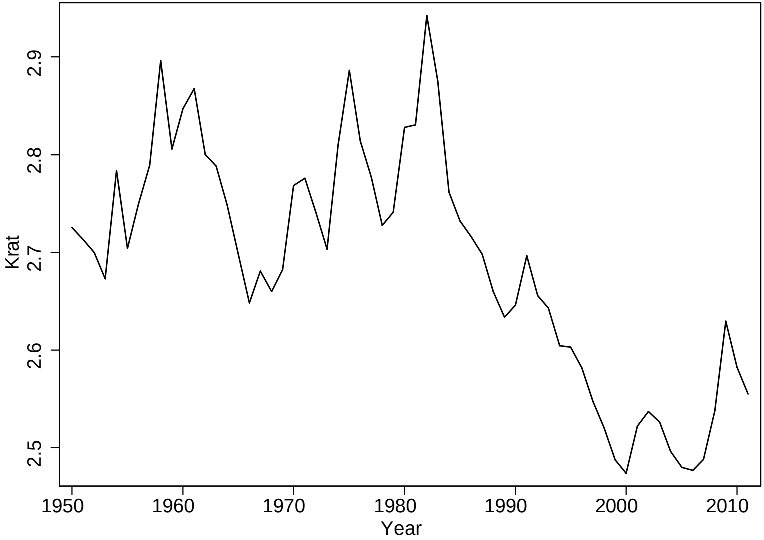
Labor Share of Income



Labor Share of Income: Other Countries



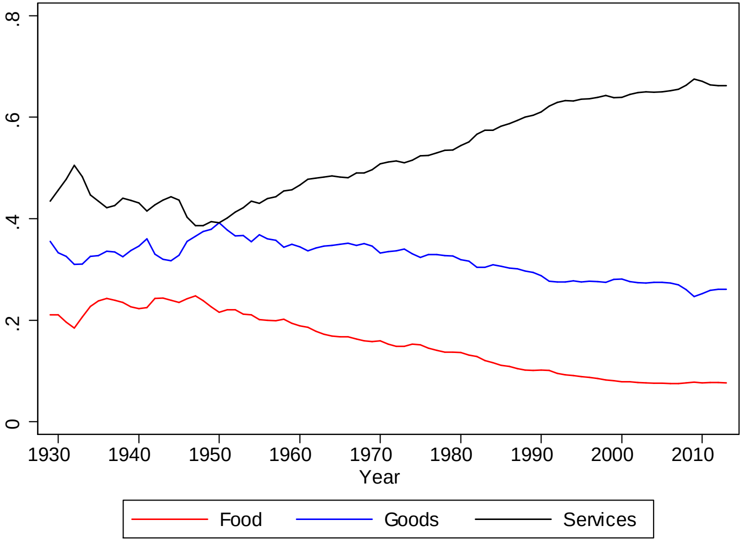
Ratio of K/Y



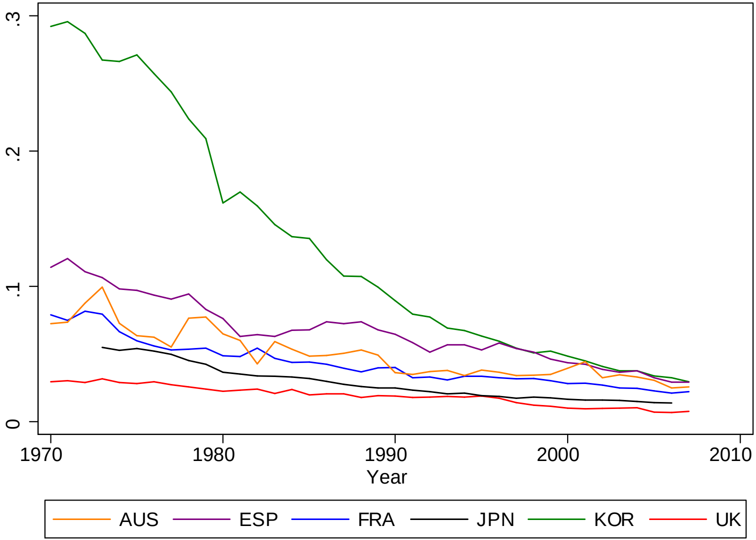
Real Return of S&P 500

Period	Return
1930-1950	4.8%
1950-1970	9.2%
1970-1990	4.7%
1990-2010	6.2%

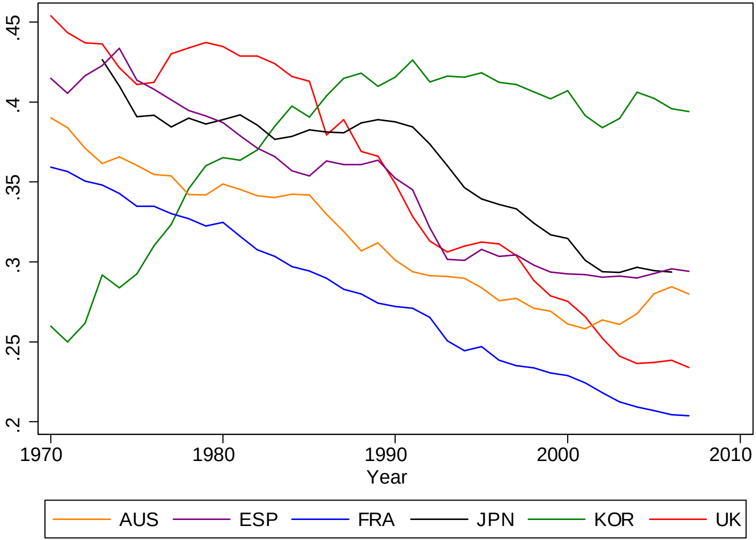
Kuznets Facts for the US



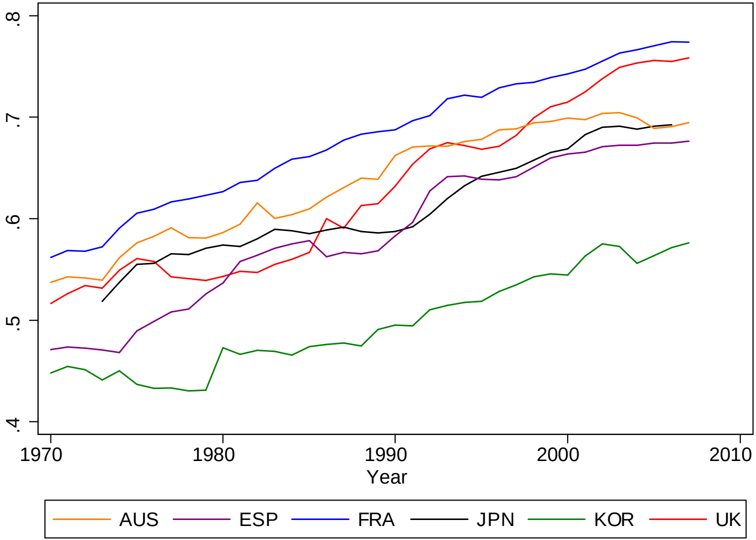
Agriculture Value Added Share of GDP



Manufacturing Value Added Share of GDP



Services Value Added Share of GDP



Note: We'll go over the following papers on the board

- Kongsamut, Rebelo, Xie (2001), "Beyond Balanced Growth"
- Ngai and Pissarides (2007), "Structural Change in a Multisector Model of Growth"

Notes on Kongsamut et al. (2001): "Beyond Balanced Growth"

Overview

- Goods differ in their income elasticities: agriculture has a relatively low income elasticity, services has a high income elasticity.
  - Technological progress
  - $\rightarrow$ Higher income
  - $\rightarrow$ Greater fraction of consumption expenditures spent on services
  - $\rightarrow$ Labor reallocation to services, away from agriculture.
- What are the implications for long-run productivity growth in this model, compared to Ngai and Pissarides?
- Why do we care about models that have balanced growth paths?

Evidence on income elasticities: Aguiar and Bils (2015)

- Assume log-linear Engel curves:

$$\underbrace{\log x^*_{hjt} - \log \bar{x}^*_{jt}}_{\text{cons of j by h, relative to peers}} = \alpha^*_{jt} + \beta_j \log X^*_{ht} + \Gamma_j \underbrace{Z_h}_{\text{hh characteristics}} + \underbrace{\varphi_{hjt}}_{\text{taste shock}}$$

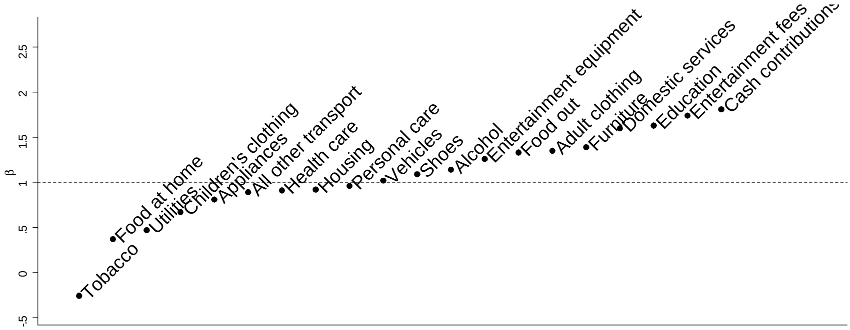
- Measurement equation:

$$\log x^{\text{measured}}_{hjt} - \log \bar{x}^{\text{measured}}_{jt} = \alpha_{jt} + \beta_j \log X^{\text{measured}}_{ht} + \Gamma_j Z_h + u_{hjt}$$

- A key challenge
  - Our observed value of expenditures on good j by household h are measured with error $x^{\text{measured}}_{hjt} \neq x^*_{hjt}$.
  - Measurement error in individual goods, component of u_{hjt}, will be correlated with $\log X_{ht}$ term.
- Solution: Instrument $\log X_{ht}$ with $\log I_{ht}$
  - Idea behind $\log I_{ht}$ instrument: Consumption reflects permanent income, which will be correlated with current income, but uncorrelated with measurement error.

Evidence on income elasticities

Results from the CEX



Stone-Geary Preferences

$$U = \left(A - \bar{A}\right)^{\beta} M^{\gamma} \left(S + \bar{S}\right)^{\theta}, \text{ where } \beta + \gamma + \theta = 1$$

Consider maximizing U s.t. $I = A \cdot P_A + M + S \cdot P_S$

First order conditions:

$$S + \bar{S} = \frac{\theta}{P_S}\left(I + P_S\bar{S} - P_A\bar{A}\right)$$
$$A - \bar{A} = \frac{\beta}{P_A}\left(I + P_S\bar{S} - P_A\bar{A}\right)$$
$$M = \gamma\left(I + P_S\bar{S} - P_A\bar{A}\right)$$

Just like Cobb-Douglas, except income shifted by $P_S\bar{S} - P_A\bar{A}$, consumption shifted by $\bar{A}$ and $\bar{S}$, resp.

Stone-Geary Preferences

From last slide

$$S + \bar{S} = \frac{\theta}{P_S}\left(I + P_S\bar{S} - P_A\bar{A}\right)$$
$$A - \bar{A} = \frac{\beta}{P_A}\left(I + P_S\bar{S} - P_A\bar{A}\right)$$
$$M = \gamma\left(I + P_S\bar{S} - P_A\bar{A}\right)$$

Compute income elasticity

$$\frac{\partial S}{\partial I}\frac{I}{S} = \frac{\theta}{P_S} \cdot \frac{I}{\frac{\theta}{P_S}\left(I + P_S\bar{S} - P_A\bar{A}\right) - \bar{S}}$$
$$= \frac{\theta I}{\theta\left(I + P_S\bar{S} - P_A\bar{A}\right) - P_S\bar{S}}$$
$$= \frac{\theta I}{\theta I - \theta P_A\bar{A} - (1-\theta)P_S\bar{S}} > 1$$

Model setup

1. Representative consumer with Stone-Geary preferences.
2. Services and agriculture are consumed, manufacturing either consumed or invested.
   Identical homogeneous of degree 1 production functions:

$$
\begin{aligned}
A_t &= B_A F(\phi_t^A K_t, N_t^A X_t) \\
S_t &= B_S F(\phi_t^S K_t, N_t^S X_t) \\
M_t + \delta K + \dot{K} &= B_M F(\phi_t^M K_t, N_t^M X_t)
\end{aligned}
$$

   Productivity growth: $\dot{X} = X \cdot g$
3. Market clearing conditions

$$
\begin{aligned}
N_t^A + N_t^M + N_t^S &= 1 \\
\phi_t^A + \phi_t^M + \phi_t^S &= 1
\end{aligned}
$$

Implications of static optimization

- Marginal rate of transformation in each sector is the same:

$$\frac{P_A \cdot B_A \cdot F_K^A}{P_A \cdot B_A \cdot F_L^A} = \frac{B_M \cdot F_K^M}{B_M \cdot F_L^M} = \frac{P_S \cdot B_S \cdot F_K^S}{P_S \cdot B_S \cdot F_L^S} \Rightarrow \frac{\phi_t^A}{N_t^A} = \frac{\phi_t^M}{N_t^M} = \frac{\phi_t^S}{N_t^S}$$

- Relative prices equals relative productivities:

$$P_A = \frac{B_M}{B_A} \; ; \; P_S = \frac{B_M}{B_S}$$

- Combining these conditions with the production functions (HW), and using homogeneity of F

$$M_t + \delta K_t + \dot{K}_t + P_A A_t + P_S S_t = B_M F\left(\frac{K_t}{X_t}, 1\right) X_t$$

(Generalized) balanced growth path

- Definition: In a GBGP, the real interest rate, r, is constant.
- Rough argument for deriving r
  - Consumers can buy unit of capital
    - Rent it to firms for $B_M F_1(k, 1)$
    - Sell the undepreciated portion the next period, for $(1 - \delta)$
  - Or receive a gross interest rate of $(1 + r)$
- $r = B_M F_1(k, 1) - \delta$
- r is constant if and only if $k = \frac{K}{X}$ is constant; K must grow at rate g.

Conditions for a generalized balanced growth path

- Revisit budget constraint:

$$\delta K_t + \dot{K}_t + M_t + P_A A_t + P_S S_t = B_M F(k_t, 1) X_t$$

Right-hand side grows at rate g. Need $M_t + P_A A_t + P_S S_t$ to grow at the same rate.

- From before:

$$\begin{aligned} M_t &= \gamma \left(I + P_S \bar{S} - P_A \bar{A}\right) \\ P_S S_t &= -P_S \bar{S} + \theta \left(I + P_S \bar{S} - P_A \bar{A}\right) \\ P_A A_t &= P_A \bar{A} + \beta \left(I + P_S \bar{S} - P_A \bar{A}\right) \end{aligned}$$

$\Rightarrow -P_S \bar{S} + P_A \bar{A} + \left(I + P_S \bar{S} - P_A \bar{A}\right)$ needs to grow at rate g.
$\Rightarrow$ Can happen iff $P_S \bar{S} = P_A \bar{A}$

Consumption growth rates

$$
\begin{aligned}
S_t + \bar{S} &= \frac{\theta}{P_S}\left(I + P_S\bar{S} - P_A\bar{A}\right) \quad \text{(last two terms =0 along BGP)} \\
&= \left(S_0 + \bar{S}\right)e^{gt} \\
\dot{S}_t &= \frac{\partial S_t}{\partial t} = \frac{\partial\left(S_0 e^{gt} + \bar{S}\left(e^{gt} - 1\right)\right)}{\partial t} = g\left(S_0 + \bar{S}\right)e^{gt} \\
\frac{\dot{S}_t}{S_t} &= g\frac{S_t + \bar{S}}{S_t}
\end{aligned}
$$

Similarly:

$$
\begin{aligned}
\frac{\dot{M}_t}{M_t} &= g \\
\frac{\dot{A}_t}{A_t} &= g\frac{A_t - \bar{A}}{A_t}
\end{aligned}
$$

Output growth rates of each sector approach g as $t \to \infty$.

Labor growth rates

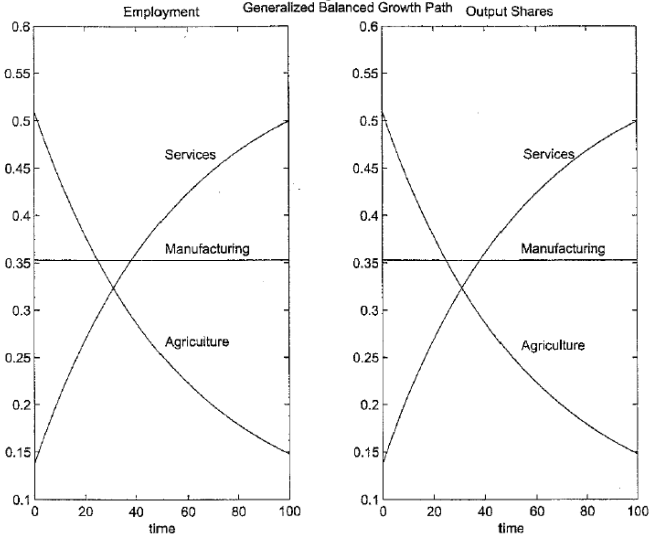
$$
\begin{aligned}
N_t^A &= \frac{A_t}{B_A F(k,1) X_t} \\
\dot{N}_t^A &= \frac{\dot{A}_t}{B_A F(k,1) X_t} - \frac{A_t \cdot \dot{X}_t}{B_A F(k,1) \left(X_t\right)^2} \\
&= \frac{g\left(A_t - \bar{A}\right)}{B_A F(k,1) X_t} - \frac{A_t \cdot g}{B_A F(k,1) X_t} \\
&= -\frac{g\bar{A}}{B_A F(k,1) X_t} \text{ ; (goes to 0 as } t \to \infty)
\end{aligned}
$$

Similarly:

$$
\begin{aligned}
\dot{N}_t^M &= 0 \\
\dot{N}_t^S &= \frac{g\bar{S}}{B_S F(k,1) X_t}
\end{aligned}
$$

Employment and Output Growth Rates

Figure 3
Generalized Balanced Growth Path



A GBGP transforms the problem to that of a one-sector model

Original formulation

$$\max U = \int_0^\infty e^{-\rho t} \frac{\left[\left(A(t) - \bar{A}\right)^\beta M(t)^\gamma \left(S(t) + \bar{S}\right)^\theta \right]^{1-\sigma}}{1-\sigma} dt$$

$$\text{s.t.} \quad \dot{K}(t) = B_M F(K(t), X(t)) - \delta K(t) - M(t) - P_A A(t) - P_S S(t)$$

With $P_S \bar{S} = P_A \bar{A}$, this problem is equivalent to

$$\max U = \int_0^\infty e^{-\rho t} \frac{M(t)^{1-\sigma}}{1-\sigma} dt$$

$$\text{s.t.} \quad \dot{K}(t) = B_M F(K(t), X(t)) - \delta K(t) - \frac{M(t)}{\gamma}$$

Why? Note that, when $P_S \bar{S} = P_A \bar{A}$:

$$S + \bar{S} = \frac{\theta}{P_S} \frac{M}{\gamma} \text{ and } A - \bar{A} = \frac{\beta}{P_A} \frac{M}{\gamma}$$

What happens when the condition doesn't hold?

- Suppose $P_S \bar{S} - P_A \bar{A} \equiv \varepsilon \neq 0$. Then the transformed planner's problem is to

$$\max U = \int_0^\infty e^{-\rho t} \frac{M(t)^{1-\sigma}}{1-\sigma} dt$$

$$\text{s.t. } \dot{K}(t) = B_M F(K(t), X(t)) - \delta K(t) - \frac{M(t)}{\gamma} + \varepsilon$$

- Can't employ standard solutions (phase diagrams) to study the transition $\frac{K}{X}$.

- Eventually the quantitative impact of the ε term will diminish.

Notes on Herrendorf et al. (2013): "Two Perspectives on Preferences and Structural Transformation"

Review: two views of structural transformation

- Facts:
  - Agriculture shrinks, manufacturing first grows and then shrinks, services grow.
  - These shifts are more pronounced in nominal rather than real terms.
- Ngai and Pissarides
  - Differential growth rates in sectors' productivity.
  - Nonunitary elasticity of substitution across goods.
  - Low-growth sector (services) has larger relative prices; draws more resources into the economy.
- Kongsamut et al.
  - Identical productivity growths.
  - Nonunitary income elasticity for different goods.
  - Agriculture has subunitary elasticity of substitution; services has income elasticity > 1.

Review: two (or three) views of structural transformation

- Facts:
  - Agriculture shrinks, manufacturing first grows and then shrinks, services grow.
  - These shifts are more pronounced in nominal rather than real terms.
- Ngai and Pissarides
- Kongsamut et al.
- Acemoglu and Guerrieri (2008)
  - Similar to Ngai and Pissarides, except:
  - Capital deepening, rather than differential productivity growth, is responsible for changes in industries' relative output prices.

- In these papers, there was little distinction between commodities and the industries that produced them.

Contribution of Herrendorf, Rogerson, and Valentinyi

- Construct and estimate a model that nests Ngai and Pissarides and Kongsamut et al.
- Show that the attribution of transformation to income/price effects depends on how we view what consumers value:
  1. "Final Consumption Expenditures": $u(c_a, c_m, c_s)$
     - c_a: food and beverages purchases or off-premises consumption
     - c_m: goods, excluding food and beverages...
     - c_s: services; government consumption expenditure
  2. "Consumption Value Added": $u(c_a, c_m, c_s)$
     - c_a: farms; forestry, fishing
     - c_m: construction; manufacturing; mining
     - c_s: all other industries
- Provide a link between the two perspectives.

Outline

1. Model
2. Data
3. Estimation using the "Final Consumption Expenditures" perspective
4. Estimation using the "Consumption Value Added" perspective
5. Linking the two perspectives.

Model (1)

Consider the problem of a consumer who is trying to maximize:

$$u(c_{at}, c_{mt}, c_{st}) = \left(\sum_{i \in \{a,m,s\}} \omega_i^{\frac{1}{\sigma}} \left(c_{it} + \bar{c}_i\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to $\sum_{i \in \{a,m,s\}} p_{it} c_{it} = C_t$

Note:

- If $\bar{c}_i = 0 \Rightarrow$ Preferences as in Ngai and Pissarides.
- If $\sigma = 1$ and $\bar{c}_m = 0 \Rightarrow$ Preferences as in Kongsamut et al.
- Nothing about the technology side of the economy is explicitly specified.
- Intertemporal decisions play little/no role.

Model (2)

- Solving the static problem from the previous slide:

$$\frac{p_{mt}c_{mt}}{C_t} = -\frac{p_{mt}\bar{c}_m}{C_t} + \frac{\omega_m p_{mt}^{1-\sigma}}{\sum_{i\in\{a,m,s\}}\omega_i p_{it}^{1-\sigma}}\left(1 + \sum_{i\in\{a,m,s\}}\frac{p_{it}\bar{c}_i}{C_t}\right) \quad (1)$$

$$\frac{p_{st}c_{st}}{C_t} = -\frac{p_{st}\bar{c}_s}{C_t} + \frac{\omega_s p_{st}^{1-\sigma}}{\sum_{i\in\{a,m,s\}}\omega_i p_{it}^{1-\sigma}}\left(1 + \sum_{i\in\{a,m,s\}}\frac{p_{it}\bar{c}_i}{C_t}\right) \quad (2)$$

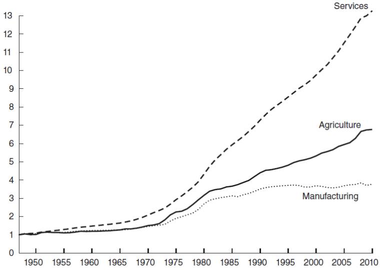
- The equation for $p_{at}c_{at}/C_t$ is redundant.
- Taking the model to the data
  - Parameters: ω_a, ω_m, σ, $\bar{c}_a$, $\bar{c}_s$
  - Data: Time series on $p_{mt}c_{mt}$, $p_{st}c_{st}$, p_{at}, p_{mt} and p_{st},
  - Fit Equations (1) and (2) as best as possible.

Data Sources

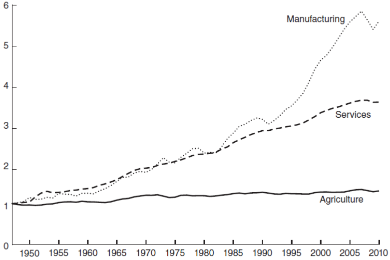
- Consumption Final Expenditure Data ($p^f_{st}c^f_{st}$ and p^f_{st})
  - National Income Product Accounts: Values and Quantity Indices (see http://www.econstats.com/nipa/)
- Consumption Value Added Data:
  - Bureau of Economic Analysis Industry Accounts: Value Added and Quantity Indices by Industry.
  - Need to subtract off investment from the production value added data. (Investment goods produced by all industries, not just manufacturing)
    - In previous papers $c_m + \dot{k} - \delta k = m$. But, after 2002 $\dot{k} - \delta k > m$!
    - BEA: 2002 Table of service shares for different types of investment goods.
- Bureau of Economic Analysis Input-Output Tables: (Useful in Linking FE and VA perspectives.)

Final Expenditures Data

Price Indices



Quantity Indices



- Quantity goes up most for manufacturing, least for food.
- Prices goes up most for services, least for manufacturing.

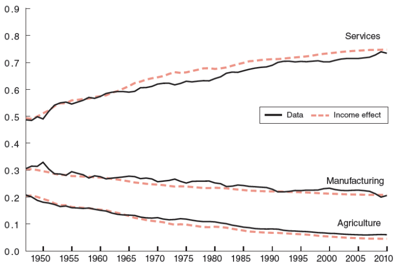
Estimating Final Consumption Expenditure Preferences

	(1)	(2)	(3)
σ	0.85	1	0.89
$\bar{c}_a$	-1350	-1316	
$\bar{c}_s$	11237	19748	
ω_a	0.02	0.02	0.11
ω_m	0.17	0.15	0.24
ω_s	0.81	0.84	0.65
$\chi^2(\bar{c}_a = 0, \bar{c}_s = 0)$	3867	4065	
AIC	-932.55	-931.35	-666.03

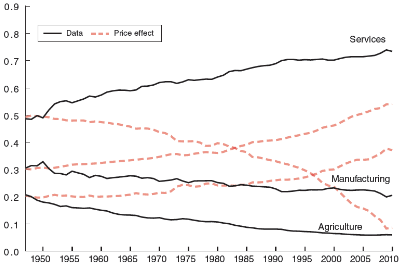
Note: AIC$=2k - 2\log\mathcal{L}$

Income effects are important in fitting expenditure share data

Prices Fixed at 1947 Values



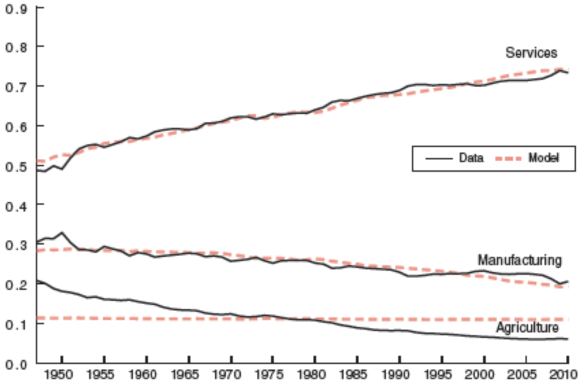
Income Fixed at 1947 Values



Nonhomotheticity terms:

	1947	2010
$p_a \bar{c}_a / C$	-0.17	-0.04
$p_s \bar{c}_s / C$	0.73	0.32

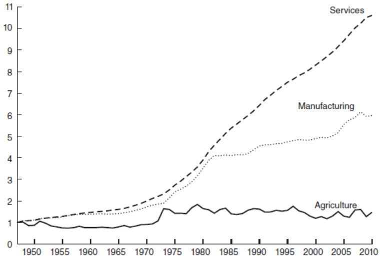
Fit of estimated model, $\bar{c}_a = \bar{c}_s = 0$



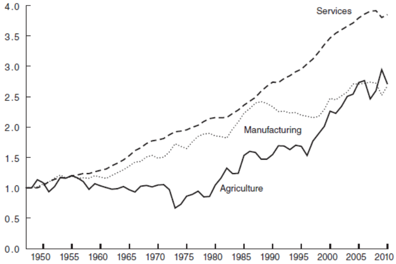
- $\{\hat{\sigma}, \omega_a, \omega_m, \omega_s\} = \{0.89, 0.11, 0.24, 0.65\}$

Value Added Data

Price Indices



Quantity Indices



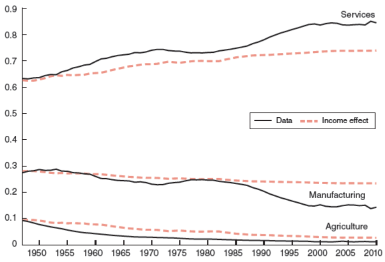
- Correlation between prices indices and quantity indices is much stronger in the value added data (89%) than in the final expenditure data (48%).

Estimating Value Added Preferences

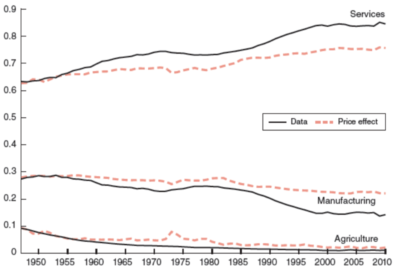
	(1)	(2)	(3)
σ	0.00	0	0
$\bar{c}_a$	-138.7	-138.9	
$\bar{c}_s$	4261.8	4268.1	
ω_a	0.002	0.002	0.01
ω_m	0.15	0.15	0.18
ω_s	0.85	0.85	0.81
$\chi^2(\bar{c}_a = 0, \bar{c}_s = 0)$	1424	216	
AIC	-837.3	-875.4	-739.4

Income and price effects are both important in fitting the value added data

Prices Fixed at 1947 Values



Income Fixed at 1947 Values



Nonhomotheticity terms:

	1947	2010
$p_a \bar{c}_a / C$	-0.08	-0.01
$p_s \bar{c}_s / C$	0.34	0.12

Why are the $\bar{c}_a$, $\bar{c}_s$ terms less important?

- Consumption over Commodities' Final Expenditure
  - Food from supermarkets is an agricultural commodity ($\bar{c}_a < 0$)
  - Meals from restaurants is a service ($\bar{c}_s > 0$)
- Consumption over Industries' Value Added
  - Both food from supermarkets and food from restaurants are produced by the agriculture industry; $\bar{c}_a$ & $\bar{c}_s$ balance out.

Linking the two approaches: theory

- Assume that final added consumption is a CES aggregate of value added from the three sectors:

$$c_{it}^{f} = \left[\sum_{j \in \{a,m,s\}} \left(A_{it}\phi_{j\rightarrow i}\right)^{\frac{1}{\eta_i}} \left(c_{j\rightarrow i,t}^{v}\right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}}$$

- Cost minimization of the "final expenditure bundler" implies that:

$$p_j^v c_{j\rightarrow i,t}^{v} = \frac{\phi_{j\rightarrow i}\left(p_j^v\right)^{1-\eta_i}}{\sum_{k\in\{a,m,s\}} \phi_{k\rightarrow i}\left(p_k^v\right)^{1-\eta_i}} p_i^f c_{it}^f \qquad (3)$$

- Taking the model to the data
  - Parameters: $\eta_i, \phi_{j\rightarrow i}$; $i, j \in \{a, m, s\}$.
  - Fit Equation (3) as best as possible, separately for each $i \in \{a, m, s\}$.

Linking the two perspectives: data

How are the $p_j^v c_{j\to i,t}^v$ constructed?

- Bureau of Economic Analysis "Total Requirements" Tables
  - For firms producing commodity j, what is the total value of purchases from industry i?
  - For each i, what is the gross output $(p_i^g c_i^g)$, value added $(p_i^v c_i^v)$, and final expenditures $(p_i^f c_i^f,)$?
- Define $T_{ij} = \frac{\text{purchases of commodity } j \text{ for firms producing in } i}{\text{value added in } i + \text{total purchases of firms in } i}$
- ji element of $(I-T)^{-1}$: dollar amount of commodity j that industry i uses per dollar of its sales. Note $(I-T)^{-1} = I + T + T^2 + T^3 + ...$
- Using this definition:

$$p_j^g c_{j\to i}^g = \left((I-T)^{-1}\right)_{ji} p_i^f c_i^f$$

An example from the data

How are the $p_j^v c_{j\to i,t}^v$ constructed? BEA "Total Requirements" Tables, from 1963

IO Table:

Agric.	17818	0	326	1112	25641	259	3410
Min'g	128	1138	737	3686	10949	46	2914
Const.	567	416	25	588	814	1556	10906
Durab.	795	1081	27329	97129	8018	3160	6299
N-Dur	6851	588	4234	11582	69029	6683	17745
Trans.	2795	876	9789	11605	12615	7278	11526
Serv.	4774	3529	5814	14041	15974	26717	60931
VA	22702	11050	37022	95905	75063	112320	233569

$(I - T)^{-1} =$

Agric.	1.50	0.02	0.04	0.04	0.26	0.02	0.04
Min'g	0.02	1.07	0.03	0.04	0.08	0.01	0.02
Const.	0.02	0.03	1.01	0.01	0.02	0.02	0.04
Durab.	0.08	0.18	0.57	1.71	0.13	0.06	0.07
N-Dur	0.29	0.08	0.14	0.15	1.52	0.09	0.11
Trans.	0.11	0.07	0.17	0.11	0.12	1.07	0.06
Serv.	0.21	0.26	0.18	0.17	0.21	0.23	1.25

An example from the data

How are the $p_j^v c_{j\to i,t}^v$ constructed? BEA "Total Requirements" Tables, from 1963:

$(I-T)^{-1} =$

Agric.	1.50	0.02	0.04	0.04	0.26	0.02	0.04
Min'g	0.02	1.07	0.03	0.04	0.08	0.01	0.02
Const.	0.02	0.03	1.01	0.01	0.02	0.02	0.04
Durab.	0.08	0.18	0.57	1.71	0.13	0.06	0.07
N-Dur	0.29	0.08	0.14	0.15	1.52	0.09	0.11
Trans.	0.11	0.07	0.17	0.11	0.12	1.07	0.06
Serv.	0.21	0.26	0.18	0.17	0.21	0.23	1.25

In 1963, each dollar of final expenditures in agriculture generates 0.21 dollars of gross output in services, 0.11 in transport.

Since $p_{A,1963}^f c_{A,1963}^f = \$348$ per capita,
we have that $p_{S\to A}^g c_{S\to A}^g = \$348 \cdot (0.21 + 0.11) = \$111$

Estimates of the production commodities

Now use:

$$p_j^v c_{j\to i,t}^v \approx p_j^g c_{j\to i,t}^g \cdot \frac{va_j}{go_j}$$

Reminder:

$$p_j^v c_{j\to i,t}^v = \frac{\phi_{j\to i}\left(p_j^v\right)^{1-\eta_i}}{\sum_{k\in\{a,m,s\}}\phi_{k\to i}\left(p_k^v\right)^{1-\eta_i}} p_i^f c_{it}^f$$

	Food	Goods	Services
η_i	0.19*	0.00	0.00
$\phi_{a\to i}$	0.05*	0.02*	0.01*
$\phi_{m\to i}$	0.33*	0.36*	0.09*
$\phi_{s\to i}$	0.62*	0.62*	0.90*

- Except for agriculture, production of final expenditures is Leontief.
- Services are an important input in all commodities.
- Agriculture is relatively unimportant in the production of the three commodities.

Conclusion (1)

- Summary
  - To fit the growth of service FE, and the decline of food FE $\Rightarrow$ income effects are important.
  - To link FE data and VA added data $\Rightarrow$ complementarity in production of fixed expenditures.
- Next Steps
  - What productivity trajectories will generate the observed relative price movements?
  - Look at within-sector price & quantity paths.
    - Are they similar across industries, within sectors?
    - What are the within-industry productivity paths?

Conclusion (2)

What are the underlying productivity paths?

Herrendorf, Herrington, and Valentinyi (2014)

- Production functions of the form:

$$G_{it} = [F_{it}(K_{it}, L_{it})]^{\eta_i} [X_{it}(Z_{it})]^{1-\eta_i}, \text{ where}$$

$$F_{it} = \left[\alpha_i [\exp(\gamma_{ik} t) K_{it}]^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i) [\exp(\gamma_{il} t) L_{it}]^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}}$$

- Main result: $\gamma_{Al} > \gamma_{Ml} > \gamma_{Sl}$; $\sigma \approx 1$ fit the price data well.

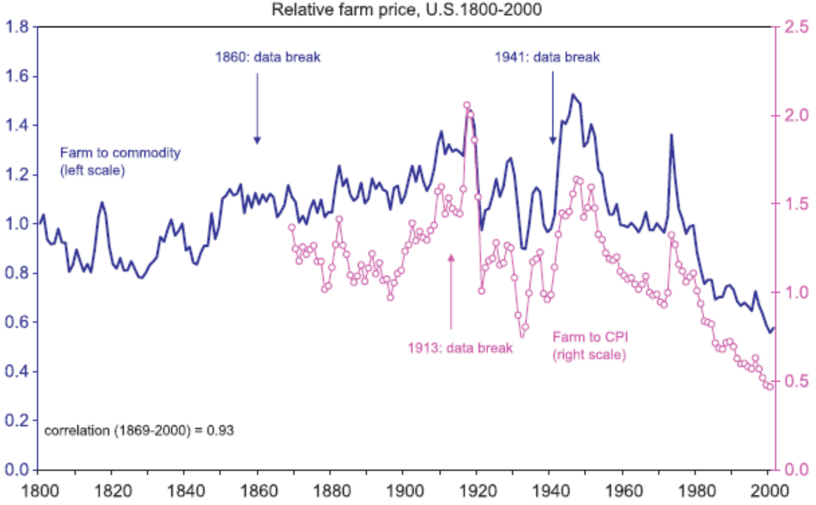
Conclusion (3)

Substantial differences within Services

	Prices: % Ann. Growth	Quantity: % Ann. Growth
GDP	3.5%	3.4%
Wholesale	1.9%	4.8%
Retail	2.7%	3.6%
Transportation	2.9%	2.9%
Information	2.5%	5.3%
Finance & Insurance	5.0%	4.0%
Real Estate	3.7%	4.0%
Professional Services	5.3%	4.5%
Management	4.2%	3.0%
Administration	4.6%	5.3%
Education	5.8%	3.0%
Health	5.3%	4.2%
Arts & Entertainment	4.2%	3.4%
Accommodation	4.0%	3.1%
Other Services	4.9%	1.6%

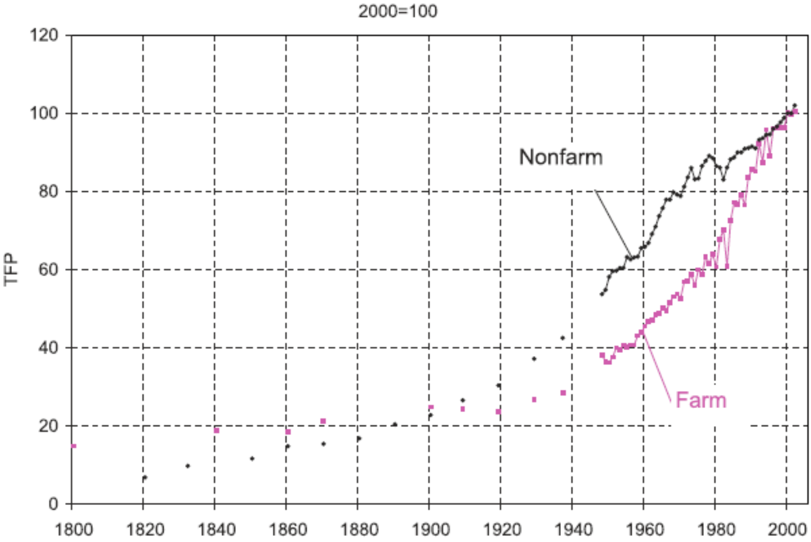
Conclusion (4)

Longer time horizons: Dennis & İşcan (2009)



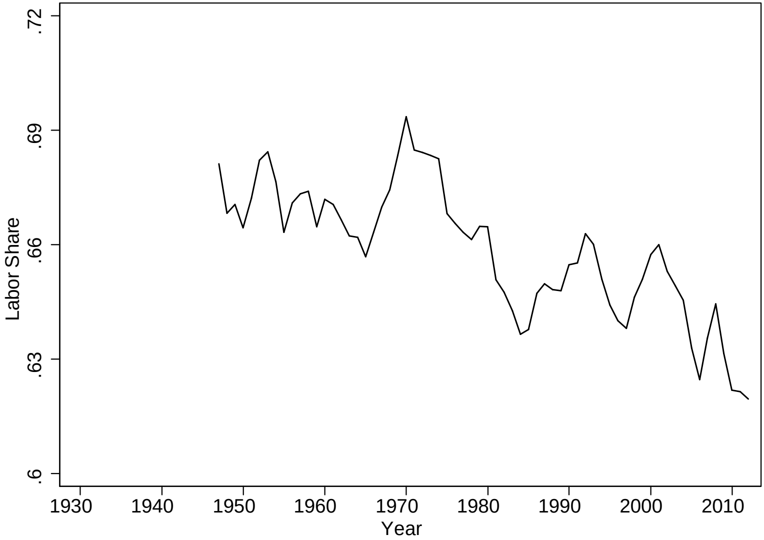
Conclusion (4)

Longer time horizons: Dennis & İşcan (2009)



Notes on Oberfield and Raval (2014) "Micro Data and Macro Technology"

Review: The labor share of income is declining



Summary

- Question 1: What is σ^{agg}, the (aggregate) elasticity of substitution between capital and labor?
- Question 2: Given σ^{agg}, how much of the observed decline in the labor share is due to changes in the price of capital vs. labor?
- Contribution:
  - A new estimate of a parameter that we care about.
  - A new method to apply micro data to "build up" to estimates of aggregate elasticities of substitution.

Why do we care about σ^{agg}?

- Does an increase in $\frac{K}{L}$ increase incentive to innovate in labor- or capital-intensive technologies? (Acemoglu, 2002, 2003)
- How much of the GDP per capita differences between poor and rich countries is explained by differences in $\frac{K}{L}$? (Caselli, 2005)
- How fast does an economy converge to its steady state.

Estimating factor-augmenting technical growth using time series data

- Assume:

$$Y_t = \left[(A_t K_t)^{\frac{\sigma_t - 1}{\sigma_t}} + (B_t L_t)^{\frac{\sigma_t - 1}{\sigma_t}} \right]^{\frac{\sigma_t}{\sigma_t - 1}}$$

- FOC (normalizing price of output to 1)

$$\pi_t = \frac{r_t K_t}{Y_t} = \frac{(A_t K_t)^{\frac{\sigma_t - 1}{\sigma_t}}}{Y_t}$$

$$1 - \pi_t = \frac{w_t L_t}{Y_t} = \frac{(B_t L_t)^{\frac{\sigma_t - 1}{\sigma_t}}}{Y_t}$$

- So

$$\log\left(\frac{\pi_t}{1 - \pi_t}\right) = \frac{\sigma_t - 1}{\sigma_t} \log\left(\frac{A_t}{B_t}\right) + \frac{\sigma_t - 1}{\sigma_t} \log\left(\frac{K_t}{L_t}\right)$$

- One approach (Klump, McAdam, William): Assume σ_t is constant, and parameterize the growth rate of A_t and B_t.

Set-up

Monopolistically competitive plants produce using caital and labor

$$Y_i = \left[(A_i K_i)^{\frac{\sigma-1}{\sigma}} + (B_i L_i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

... taking the wage and capital rental rate as given; same for all plants

Consumers have Dixit-Stiglitz preferences

$$Y = \left[\sum (D_i)^{\frac{1}{\varepsilon}} (Y_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

The goal is to derive an expression for

$$\sigma_n^N = \frac{d \log \frac{K}{L}}{d \log \frac{w}{r}}$$

Notation:

- θ_{ni}: sales share of plant i.
- α_{ni}: capital cost share of plant i

Two additions

From our homework:

$$\sigma_n^N = (1 - \chi_n)\,\sigma + \chi_n \varepsilon, \text{ where}$$

$$\chi_n = \sum \theta_{ni} \frac{(\alpha_{ni} - \alpha_n)^2}{\alpha_n (1 - \alpha_n)}$$

1. Include materials in plants' production functions:

$$F(K_{ni}, L_{ni}, \quad) = \left[\left[(A_{ni} K_{ni})^{\frac{\sigma_n - 1}{\sigma_n}} + (B_{ni} L_{ni})^{\frac{\sigma_n - 1}{\sigma_n}} \right]^{\frac{\sigma_n}{\sigma_n - 1}} \right]$$

2. Write out the aggregate elasticity in terms of industry-level terms.

Two additions

From our homework:

$$\sigma_n^N = (1-\chi_n)\,\sigma + \chi_n \varepsilon \text{ where}$$
$$\chi_n = \sum \theta_{ni} \frac{(\alpha_{ni}-\alpha_n)^2}{\alpha_n (1-\alpha_n)}$$

1. Include materials in plants' production functions:

$$F(K_{ni}, L_{ni}, M_{ni}) = \left[\left[(A_{ni}K_{ni})^{\frac{\sigma_n-1}{\sigma_n}} + (B_{ni}L_{ni})^{\frac{\sigma_n-1}{\sigma_n}} \right]^{\frac{\sigma_n}{\sigma_n-1}\frac{\zeta_n-1}{\zeta_n}} \right.$$
$$\left. + C_{ni} M_{ni}^{\frac{\zeta_n-1}{\zeta_n}} \right]^{\frac{\zeta_n}{\zeta_n-1}}$$

2. Write out the aggregate elasticity in terms of industry-level terms.

Building up to the aggregate EoS

- The industry-level elasticity of substitution equals:

$$\sigma_n^N = (1 - \chi_n)\,\sigma_n + \chi_n \left[\left(1 - \bar{s}_n^M\right) \varepsilon_n + \bar{s}_n^M \zeta_n \right]$$

where $\chi_n = \sum_i \frac{(\alpha_{ni} - \alpha_n)^2}{(1 - \alpha_n)\,\alpha_n} \theta_{ni}$, and

$\bar{s}_n^M$ is a weighted average of plants' intermediate input shares.

- The aggregate elasticity of substitution equals:

$$\sigma^{agg} = (1 - \chi^{agg})\,\bar{\sigma}^N + \chi^{agg} \left[\left(1 - \bar{s}^M\right) \eta + \bar{s}^M \bar{\zeta}_n \right]$$

where $\chi^{agg} = \sum_i \frac{(\alpha_n - \alpha)^2}{(1 - \alpha)\,\alpha} \theta_n$, and

  - $\bar{\sigma}^N$ ($\bar{\zeta}_n$) is a weighted average of the industry capital-labor (materials) EoS.
  - $\bar{s}^M$ is a weighted average of industries' intermediate input shares.

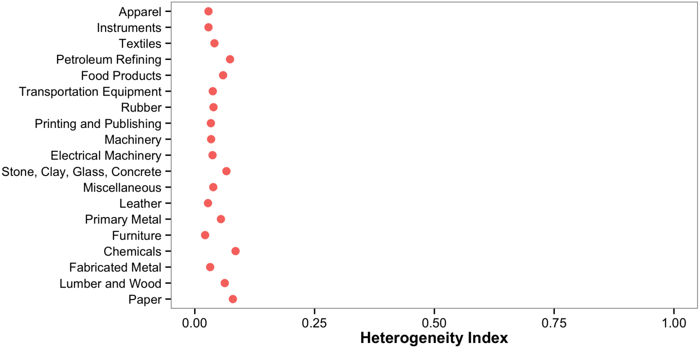
The Census of Manufacturers & Annual Survey of Manufacturers

- Census of Manufacturers (CM)
  - All plants within the US with $\geq$ 5 employees (180,000 out of 350,000)
  - Every five years (1972, 1977,... 2012)
  - Book value of capital is imputed for non ASM plants (except for 1987, 1997)
  - Materials expenditures, labor expenditures, output.
- Annual Survey of Manufacturers
  - A subset of plants (50,000), oversampling of larger plants
  - Materials expenditures, labor expenditures, output.

Building blocks of σ_n^N

- χ: variation in plant-level capital shares (within value added)
- $\bar{s}_n^M$: average materials cost share
- σ_n: plant-level elasticity of substitution, between capital and labor
- ε_n: elasticity of demand
- ζ_n: elasticity of substitution between materials and value added

Building blocks of $\sigma_n^N = (1-\chi_n)\sigma_n + \chi_n[(1-\bar{s}_n^M)\varepsilon_n + \zeta_n \bar{s}_n^M]$: χ_N



$$\chi_n \approx 0 \Rightarrow \sigma_n^N \approx \sigma_n$$

Building blocks of $\sigma_n^N = (1-\chi_n)\sigma_n + \chi_n[(1-\bar{s}_n^M)\varepsilon_n + \zeta_n \bar{s}_n^M]$: σ_n

From the plants' cost-minimization condition:

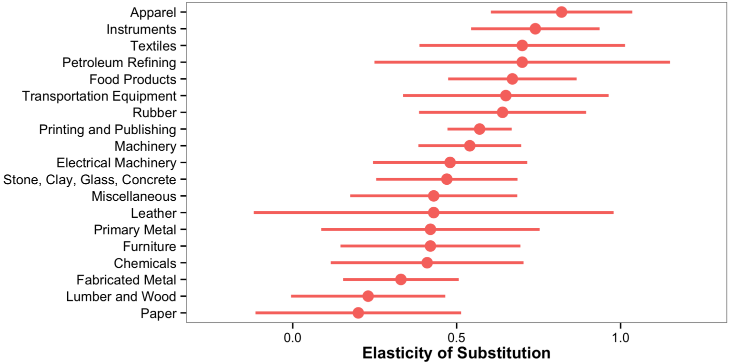
$$\log\left(\frac{rK}{wN}\right)_{ni} = \kappa + (\sigma_n - 1)\left(\frac{w}{R}\right)_{ni}$$

Specification from Raval (2014):

$$\log\left(\frac{rK}{wN}\right)_{ni} = \kappa + (\sigma_n - 1)\log w_{ni}^{MSA} + \text{Controls} + \epsilon_{ni}$$

- w_{ni}^{MSA}: hourly wage in the MSA of plant i, after controlling for worker education, experience, industry, occupation, demographics.
- Controls: age of the plant, indicator for whether it is part of a multi-unit firm.
- Key Assumptions: $R_{ni} \perp w_{ni}^{MSA}$ (or more generally, $w_{ni}^{MSA} \perp \epsilon_{ni}$)

Building blocks of $\sigma_n^N = (1-\chi_n)\sigma_n + \chi_n[(1-\bar{s}_n^M)\varepsilon_n + \zeta_n \bar{s}_n^M]$: σ_n



Average of $\sigma_n \approx 0.5$.

Building blocks of $\sigma_n^N = (1-\chi_n)\sigma_n + \chi_n[(1-\bar{s}_n^M)\varepsilon_n + \zeta_n \bar{s}_n^M]$: ζ_n

From the plants' cost-minimization condition:
Similar specification to identify ζ:

$$\log\left(\frac{qM}{wN+rK}\right)_{ni} = (\zeta-1)(1-\alpha_i)\log w_{ni}^{MSA} + \text{Controls} + \epsilon_{ni}$$

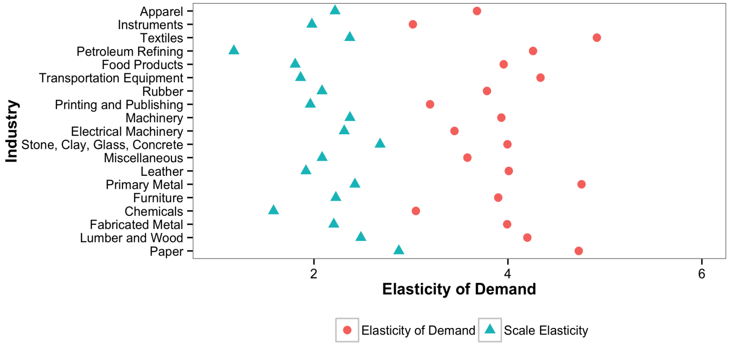
Results from pooled regression

	$\hat{\zeta}$
1987	0.90
1997	0.67
N	140,000

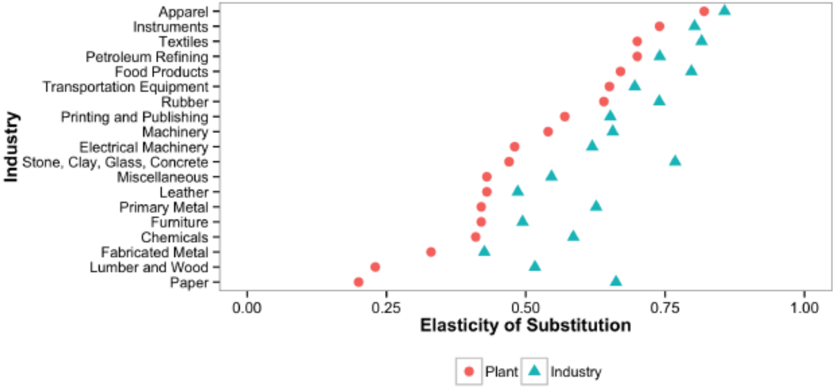
Building blocks of $\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n[(1 - \bar{s}_n^M)\varepsilon_n + \zeta_n \bar{s}_n^M]$: $\bar{s}_n^M$ and ε_N

- $\bar{s}_n^M$, average materials cost share: average=0.59.
- ε_n: Demand elasticity.
  - According to the model, the markup equals revenues divided by total costs $\Rightarrow \frac{\varepsilon_n}{\varepsilon_n - 1} = \frac{P_{ni} Y_{ni}}{wL_{ni} + rK_{ni} + qM_{ni}}$
  - $\varepsilon_n \in [3, 5]$

Building blocks of $\sigma_n^N = (1-\chi_n)\sigma_n + \chi_n[(1-\bar{s}_n^M)\varepsilon_n + \zeta_n\bar{s}_n^M]$: $[(1-\bar{s}_n^M)\varepsilon_n + \bar{\zeta}_n\bar{s}_n^M]$

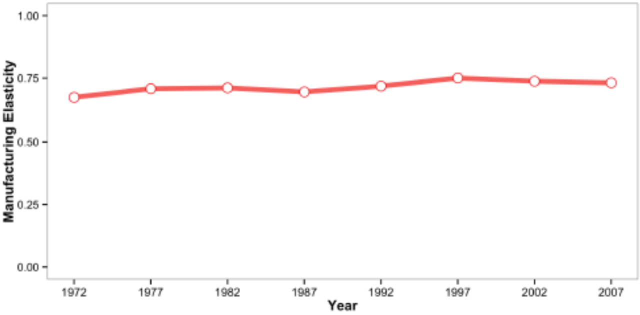


Building blocks of $\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n[(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M]$

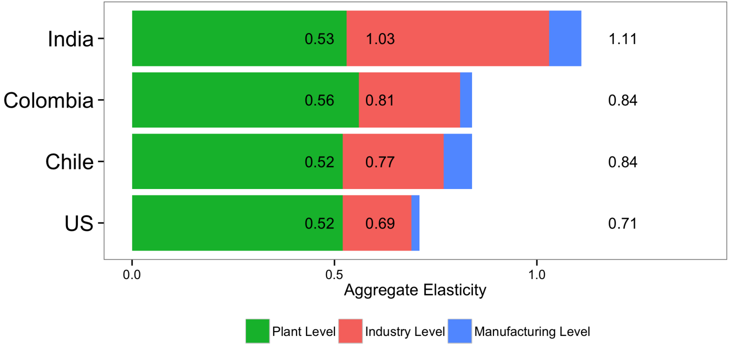


Building blocks of $\sigma^{agg} = (1-\chi^{agg})\bar{\sigma}_n^N + \chi^{agg}[(1-\bar{s}^M)\eta + \bar{s}^M\bar{\zeta}_n]$

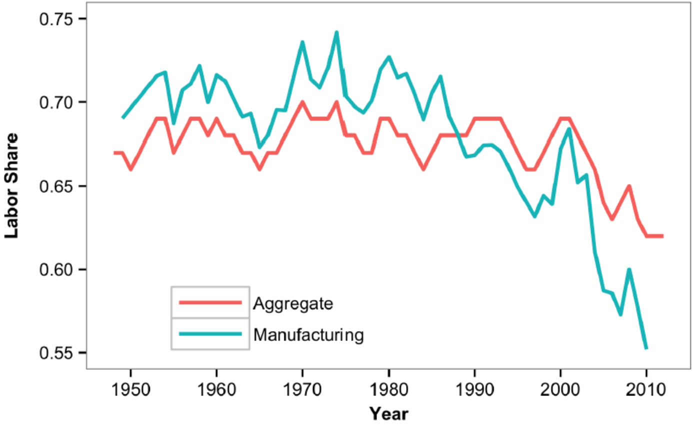
- η, elasticity of demand across industries: 1
- $\bar{\sigma}_n^N$, χ^{agg}, $\bar{s}^M$, and $\bar{\zeta}_n$ all come from industry-level data.
- Estimate in 1987: 0.70
- Allowing the χs, $\bar{s}$s to vary across years:



σ^{agg} ranges from 0.80 to 1.15 for other countries



Reminder: The labor share has fallen



Why has the labor share fallen? A decomposition

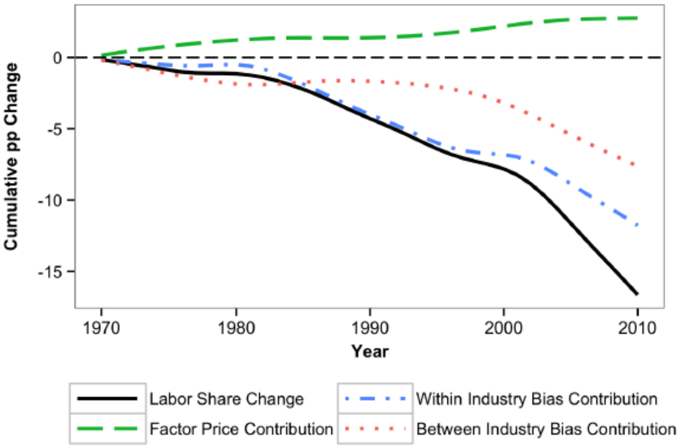
$$ds^{v,L} = \frac{\partial s^{v,L}}{\partial \log w/r} d \log w/r + \left[ds^{v,L} - \frac{\partial s^{v,L}}{\partial \log w/r} d \log w/r \right]$$
$$= \left(1 - \sigma^{agg}\right) d \log w/r + \left[ds^{v,L} - \frac{\partial s^{v,L}}{\partial \log w/r} d \log w/r \right]$$

Data on w, r:

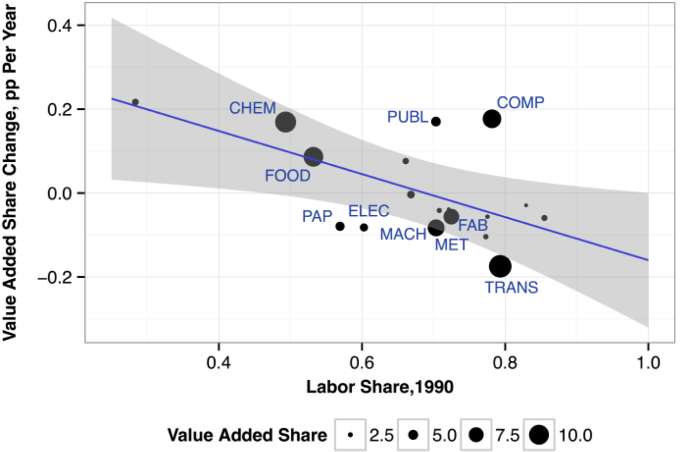
- For w: NIPA. $w = \frac{\text{Labor compensation}}{\text{Employees}}$, adjust for changes in skills.
- For r:
  - Capital prices from NIPA
  - Real rental rate of capital 3.5%
  - Tax rates and depreciation allowances from Jorgenson

w/r has gone up & $1 - \sigma^{agg} > 0 \Rightarrow$ Contribution of factor prices is positive.

Almost none of the change in the labor share is from w/r increasing.



Within manufacturing, industries with high labor shares have declined in importance



Why has the labor share fallen? A decomposition

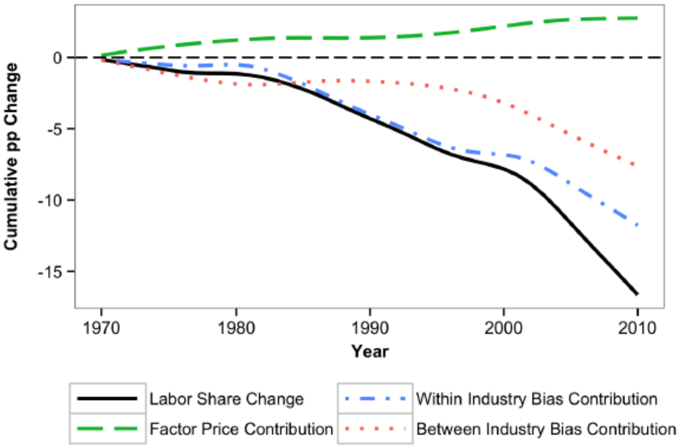
$$ds = (1 - \sigma^{agg})\, d\log w/r + \left[ds^{v,L} - \frac{\partial s^{v,L}}{\partial \log w/r} d\log w/r \right]$$

$$= (1 - \sigma^{agg})\, d\log w/r + \underbrace{\sum_n v_n \left(ds_n^{v,L} - \frac{\partial s_n^{v,L}}{\partial \ln w/r} d\log w/r \right)}_{\text{within-industry contribution}}$$

$$+ \sum_n \left(s_n^{v,L} - s^{v,L} \right) \underbrace{\left(dv_n - \frac{\partial v_n}{\partial \ln w/r} d\log w/r \right)}_{\text{between-industry contribution}}$$

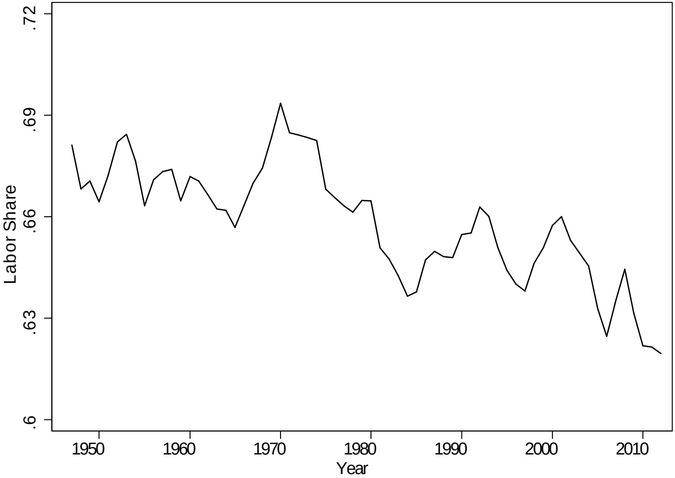
- $s_n^{v,L}$: labor share in industry n
- v_n: share of industry n in overall value added.

Almost none of the change in the labor share is from w/r increasing.

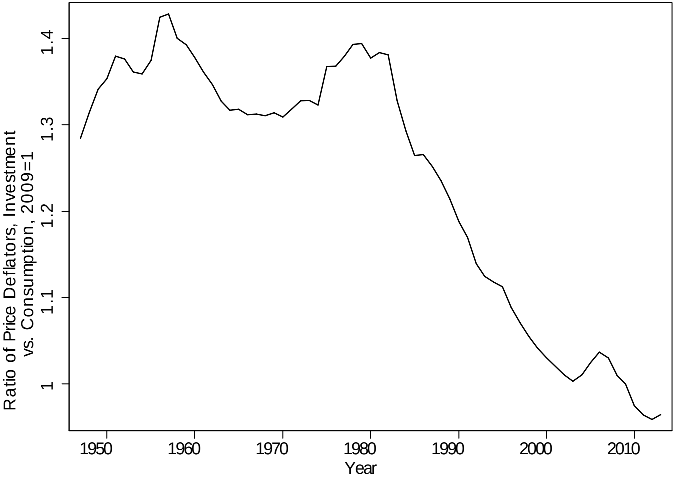


Notes on Karabarbounis and Neiman (2014) "The Global Decline of the Labor Share"

Review: Labor Share of Income



Relative Price of Capital Is Falling, Especially After 1980



A complication when computing the labor share

How do you classify entrepreneurs' income? Taxes?

Line		2012 I	2012 II	2012 III	2012 IV	2013 I	2013 II	2013 III	2013 IV	2014 I
1	**Gross domestic income**	**16,104.6**	**16,150.3**	**16,269.6**	**16,522.0**	**16,690.9**	**16,847.8**	**17,004.6**	**17,181.4**	**17,121.3**
2	**Compensation of employees, paid**	**8,522.3**	**8,562.6**	**8,599.5**	**8,795.5**	**8,756.1**	**8,844.0**	**8,896.8**	**8,973.8**	**9,049.5**
3	Wages and salaries	6,850.3	6,882.3	6,913.2	7,094.6	7,048.2	7,126.1	7,171.3	7,237.7	7,301.2
4	To persons	6,836.1	6,867.3	6,898.4	7,080.0	7,033.8	7,111.0	7,156.2	7,222.5	7,286.5
5	To the rest of the world	14.1	15.0	14.8	14.6	14.4	15.1	15.1	15.2	14.8
6	Supplements to wages and salaries	1,672.1	1,680.3	1,686.2	1,700.9	1,707.9	1,717.8	1,725.5	1,736.2	1,748.3
7	**Taxes on production and imports**	**1,124.4**	**1,122.2**	**1,118.8**	**1,126.3**	**1,140.7**	**1,138.8**	**1,149.0**	**1,158.3**	**1,166.7**
8	**Less: Subsidies 1**	**57.8**	**57.6**	**56.0**	**57.7**	**58.0**	**58.9**	**59.1**	**58.7**	**56.8**
9	**Net operating surplus**	**4,008.1**	**3,989.4**	**4,052.2**	**4,083.0**	**4,248.2**	**4,292.0**	**4,358.2**	**4,416.9**	**4,240.1**
10	Private enterprises	4,032.5	4,015.5	4,080.7	4,114.8	4,283.7	4,331.0	4,399.6	4,461.3	4,285.5
11	Net interest and miscellaneous payments, domestic industries	613.6	580.8	611.7	583.3	630.3	591.7	615.5	638.8	633.4
12	Business current transfer payments (net)	115.7	110.0	102.6	99.5	121.9	125.8	120.1	129.9	122.5
13	Proprietors' income with inventory valuation and capital consumption adjustments	1,214.4	1,217.8	1,220.0	1,247.5	1,334.6	1,341.5	1,360.7	1,358.5	1,359.5
14	Rental income of persons with capital consumption adjustment	524.8	537.8	546.7	555.4	574.9	587.7	596.6	603.2	611.9
15	Corporate profits with inventory valuation and capital consumption adjustments, domestic industries	1,564.0	1,569.1	1,599.8	1,629.1	1,622.1	1,684.3	1,706.8	1,730.9	1,558.4
16	Taxes on corporate income	437.2	429.7	439.1	433.2	408.2	418.2	417.8	431.1	458.9
17	Profits after tax with inventory valuation and capital consumption adjustments	1,126.8	1,139.4	1,160.7	1,196.0	1,213.8	1,266.1	1,289.0	1,299.8	1,099.5
18	Net dividends	569.1	572.5	577.3	735.3	616.6	874.7	769.4	787.8	674.7
19	Undistributed corporate profits with inventory valuation and capital consumption adjustments	557.8	566.9	583.4	460.7	597.3	391.4	519.5	512.0	424.8
20	Current surplus of government enterprises 1	-24.5	-26.1	-28.5	-31.8	-35.5	-39.0	-41.4	-44.3	-45.5
21	**Consumption of fixed capital**	**2,507.6**	**2,533.7**	**2,555.1**	**2,575.0**	**2,603.8**	**2,631.9**	**2,659.6**	**2,691.0**	**2,721.9**
22	Private	2,018.7	2,041.0	2,059.8	2,077.6	2,103.3	2,128.5	2,153.5	2,180.5	2,208.6
23	Government	488.9	492.7	495.3	497.4	500.5	503.4	506.1	510.5	513.3
	Addendum:									
24	Statistical discrepancy	-63.0	10.1	86.4	-101.7	-155.6	-186.8	-91.7	-91.8	-105.3

Two main contributions of Karabarbounis and Neiman (2014)

- Measurement: Compiling data for corporate labor shares for ~ 60 countries.
- Estimation: New method (using cross-sectional data) of estimating capital-labor substitutability ($\sigma \equiv \frac{d \log(K/N)}{d \log(w/r)}$).

Outline

- Data sources
- Stylized facts
  - Labor share
  - Relative price of capital
- Theory: Linking the labor share to the relative price of capital
- Estimating σ and sources of the decline in the labor share

Labor Share Data

- Decomposition of GDP

$$Y = \underbrace{Q_C}_{\text{Corporate VA}} + Q_H + Q_G + \text{Tax}_{\text{products}}$$

$$Q_C = W_C N_C + \text{Tax}_{\text{production,C}} + \text{ Operating Surplus}_C$$

- Total labor share$= \frac{WN}{Y}$
- Corporate labor share$=\frac{W_C N_C}{Q_C}$
- Major data sources
  - Country-specific web pages, UN + OECD websites, books
  - EUKLEMS : Includes data by industry. No seperation into corporate vs. household/government.

Investment Price Data

1. Penn World Tables

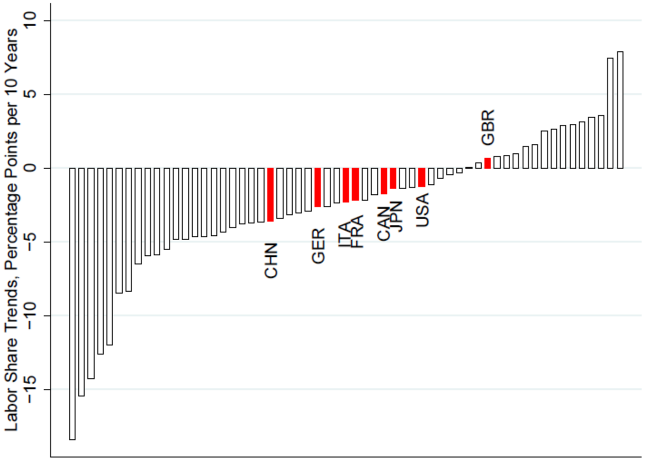
$$\xi_{it} = \frac{P^{PPP}_{I,i,t} / P^{PPP}_{I,US,t}}{P^{PPP}_{C,i,t} / P^{PPP}_{C,US,t}} \times \frac{P^{BEA}_{I,US,t}}{P^{BEA}_{C,US,t}}$$

   From the second term: incorporate adjustments that the BEA makes for relative improvements the quality of investment/consumption goods.

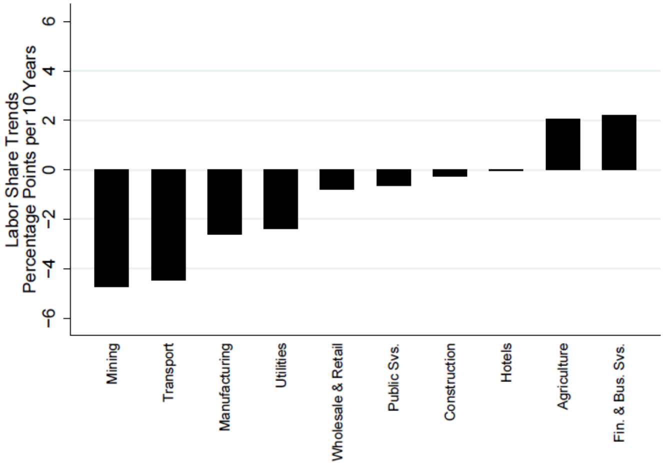
2. World Bank: World Development Indicators (Fixed Investment Deflator, CPI)
3. EUKLEMS

Both the overall and corporate labor share are declining

The labor share is declining for most countries

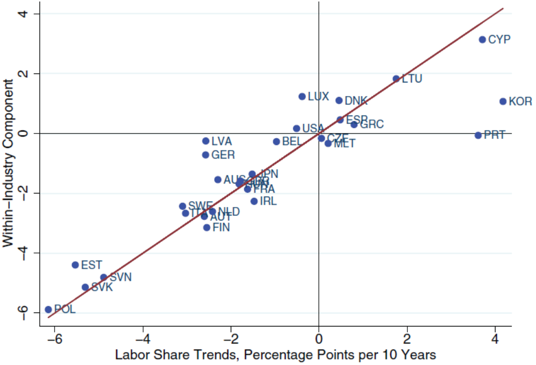


The labor share is declining for most industries

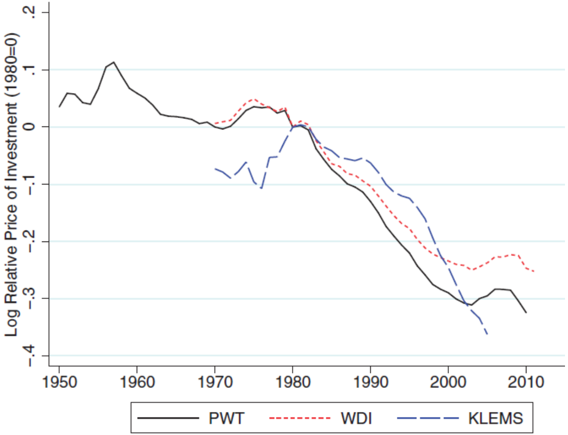


Changes in the labor share come from "within industry" changes

$$\Delta s_{Li} = \underbrace{\sum_k \bar{\omega}_{i,k} \Delta s_{Li,k}}_{\text{Within-industry}} + \underbrace{\sum_k \bar{s}_{Li,k} \Delta \omega_{i,k}}_{\text{Between-industry}}$$



Investment Price Decline, Across Data Sources



Model: Overview

1. Goal: Account for the decline of the labor share.
2. Two sectors: Producing consumption goods and investment goods.
   2.1 Produce using capital & labor with identical production (CES) technologies.
   2.2 Relative price of the two goods dictated by technology differences (ξ).
   2.3 Inputs are supplied by monopolistically competitive (with markup μ) continuum of firms.
3. Household side straightforward.
4. Key parameter of interest : σ, elasticity of substitution between capital/labor

Model: Household Problem

- Maximize

$$\max_{\{C_t, L_t, X_t, K_{t+1}, B_{t+1}\}} \sum \beta^t V\left(C_t, N_t; \chi_t\right) \text{ subject to}$$

$$W_t L_t + R_t K_t + \Pi_t = C_t + \xi X_t + B_{t+1} - \left(1 + r_t\right) B_t$$

$$K_{t+1} = \left(1 - \delta\right) K_t + X_t$$

- FOC for capital:

$$R_{t+1} = \xi_t \left(1 + r_{t+1}\right) - \xi_{t+1} \left(1 - \delta\right)$$

$\xi_t =$price of the investment good at time t (more details on the next slide).

- Euler Equation:

$$\beta \left(1 + r_{t+1}\right) = \frac{V_C\left(C_t, N_t; \chi_t\right)}{V_C\left(C_{t+1}, N_{t+1}; \chi_{t+1}\right)}$$

Model: Production

- Three products: intermediate inputs $z \in \{0,1\}$, final investment good X, final consumption good C.

$$C_t = \left[\int_0^1 c_t(z)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dz\right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}} \quad ; X_t = \frac{1}{\xi_t}\left[\int_0^1 x_t(z)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dz\right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$$

- Intermediate input supplier:

$$y_t(z) = \left(\alpha^{\frac{1}{\sigma}} \left(A_{K,t} k_t(z)\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} \left(A_{L,t} n_t(z)\right)^{\frac{\sigma-1}{\sigma}}\right)^{\sigma/(\sigma-1)}$$

  $A_{K,t}$ and $A_{L,t}$ are capital- and labor-augmenting productivity.

- Market-clearing conditions:

$$y_t(z) = c_t(z) + x_t(z)$$

$$K_t = \int_0^1 k_t(z) dz$$

$$L_t = \int_0^1 n_t(z) dz$$

Model: Input choices of each intermediate input supplier

- Problem of the intermediate input supplier:

$$\max p_t(z)y_t(z) - k_t(z)R_t - n_t(z)W_t$$

- First order conditions (For each z):

$$R_t = \frac{\partial (p_t y_t)}{\partial k_t} = \frac{\partial \left(\left(\frac{y_t}{Y_t} \right)^{-\frac{1}{\varepsilon}} y_t \right)}{\partial k_t}$$
$$= \frac{(Y_t)^{\frac{1}{\varepsilon}} \partial \left((y_t)^{1-\frac{1}{\varepsilon}} \right)}{\partial k_t}$$

- Define

$$\mu_t = \frac{\varepsilon - 1}{\varepsilon}$$

Model: Input choices of each intermediate input supplier

- Remember from last slide, definition of y_t

$$y_t(z) = \left(\alpha^{\frac{1}{\sigma}} \left(A_{K,t} k_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} \left(A_{L,t} n_t\right)^{\frac{\sigma-1}{\sigma}}\right)^{\sigma/(\sigma-1)}$$

- Then

$$R_t = \frac{(Y_t)^{\frac{1}{\varepsilon}} \partial \left((y_t)^{1-\frac{1}{\varepsilon}}\right)}{\partial k_t}$$

$$\mu_t R_t = \alpha^{\frac{1}{\sigma}} \left(A_{Kt}\right)^{\frac{\sigma-1}{\sigma}} p_t \left(\frac{k_t}{y_t}\right)^{-\frac{1}{\sigma}} \Rightarrow \mu_t \underbrace{\frac{k_t R_t}{y_t p_t}}_{s_{K,t}(z)} = \alpha \left(\frac{A_{Kt}}{\mu_t R_t}\right)^{\sigma-1}$$

- Similarly:

$$\mu_t \underbrace{\frac{l_t W_t}{y_t p_t}}_{s_{L,t}(z)} = (1-\alpha) \left(\frac{A_{Lt}}{\mu_t W_t}\right)^{\sigma-1}$$

Model: Input choices of each intermediate input supplier

From the last slide:

$$\mu_t(z)s_{K,t} = \alpha\left(\frac{A_{Kt}}{\mu_t(z)R_t}\right)^{\sigma-1}$$

But also:

$$s_{\Pi t}(z) \equiv \frac{\Pi_t(z)}{p_t(z)\cdot y_t(z)} = \frac{\mu_t - 1}{\mu_t}$$

Since

$$s_{\Pi t}(z) + s_{Lt}(z) + s_{Kt}(z) = 1$$
$$\mu_t s_{Lt}(z) + \mu_t s_{Kt}(z) = 1$$

Thus:

$$1 - \mu_t s_{Lt}(z) = \alpha\left(\frac{A_{Kt}}{\mu_t R_t}\right)^{\sigma-1}$$

Comparing two periods:

$$\left(\frac{1}{1-s_L\mu}\right)\left(1 - s_L\left(1+\hat{s}_L\right)\mu\left(1+\hat{\mu}\right)\right) = \left(\frac{1+\hat{A}_K}{1+\hat{R}}\right)^{\sigma-1}\left(1+\hat{\mu}\right)$$

Model: Estimating Equation

From the last slide:

$$\left(\frac{1}{1-s_L\mu}\right)\left(1-s_L\left(1+\hat{s}_L\right)\mu\left(1+\hat{\mu}\right)\right)=\left(\frac{1+\hat{A}_K}{1+\hat{R}}\right)^{\sigma-1}\left(1+\hat{\mu}\right)$$

From the FOC for capital:

$$1+\hat{R}=\left(1+\hat{\xi}\right)\cdot\left(1-\hat{\delta}\frac{\beta\delta}{1-\beta+\beta\delta}\right)$$

Plugging this equation

$$\left(\frac{1}{1-s_L\mu}\right)\left(1-s_L\left(1+\hat{s}_L\right)\mu\left(1+\hat{\mu}\right)\right)$$
$$=\left(\frac{1+\hat{A}_K}{1+\hat{\xi}}\right)^{\sigma-1}\left(1+\hat{\mu}\right)^{\sigma-1}\left(1-\hat{\delta}\frac{\beta\delta}{1-\beta+\beta\delta}\right)^{1-\sigma}$$

So, the labor share can change if ξ, A_K, μ or δ change.

Estimation

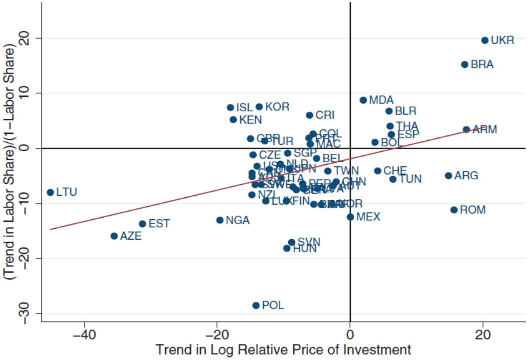
For now, set $\mu - 1 = \hat{\mu} = \hat{\delta} = 0$. Take logs:

$$\frac{s_L}{1 - s_L}\hat{s}_L = (\sigma - 1)\,\hat{\xi} + \underbrace{(1 - \sigma)\,\hat{A}_K}_{\gamma + u}$$

In the benchmark regressions, assume $\hat{\xi} \perp \hat{A}_K$.

Estimation

$$\frac{s_L}{1 - s_L}\hat{s}_L = \gamma + (\sigma - 1)\,\hat{\xi} + u$$



- Slope: 0.28 $\Rightarrow$ $\hat{\sigma} \approx 1.28$.

Estimation

Investment Price	Labor Share	$\hat{\sigma}$	Obs
PWT	KN Merged	1.25 (0.08)	58
WDI	KN Merged	1.29 (0.07)	54
PWT	OECD & UN	1.20 (0.08)	50
WDI	OECD & UN	1.31 (0.06)	47

Markup Shocks?

What if $\hat{\mu}_j \neq 0$ or $\mu_j \neq 1$?

$$\left(\frac{s_{Lj}\mu_j}{1 - s_{Lj}\mu_j}\right)\left(\hat{s}_{Lj} + \hat{\mu}_j + \hat{s}_{Lj}\hat{\mu}_j\right) = \gamma + (\sigma - 1)\left(\hat{\xi}_j + \hat{\mu}_j\right) + u_j$$

- Assuming β, δ are constant over time, same for all countries:

$$s_{Kj} = \frac{R_j K_j}{Y_j} = \frac{\xi_j X_j}{Y_j}\left(\frac{1/\beta - 1 + \delta}{\delta}\right)$$
$$\hat{s}_{Kj} = \widehat{\xi_j X_j / Y_j}$$

- From before $\mu s_{Lj} + \mu s_{Kj} = 1$. And:

$$\hat{\mu}_j = \frac{1}{\mu_j\left(s_{Lj}\hat{s}_{Lj} + s_{Kj}\hat{s}_{Kj}\right)}$$

Markup Shocks?

⇒ Countries with declining labor shares had (on average) declines in capital shares and increases in markups.

Markup Shocks?

Investment Price	Investment Rate	$\hat{\sigma}$	Obs
PWT	Corporate	1.03 (0.09)	55
WDI	Corporate	1.29 (0.08)	52
PWT	Total	1.11 (0.11)	54
WDI	Total	1.35 (0.08)	52

Capital-Augmenting Technical Change?

Again, when $\mu = \hat{\mu} - 1 = \hat{\delta} = 0$:

$$\frac{s_L}{1 - s_L}\hat{s}_L = \gamma + (\sigma - 1)\,\hat{\xi} + (1 - \sigma)\,\hat{A}_K + u$$

Up to know, we had assumed $\text{corr}\left(\hat{A}_k, \hat{\xi}\right) = 0$. If not:

$$\underbrace{\tilde{\sigma} - \sigma}_{\text{Bias}} = (1 - \sigma)\,\text{corr}\left(\hat{A}_k, \hat{\xi}\right)\frac{\text{sd}\left(\hat{A}_k\right)}{\text{sd}\left(\hat{\xi}\right)}$$

- If $\text{corr}\left(\hat{A}_k, \hat{\xi}\right) < 0$, then
    - $\tilde{\sigma} > \sigma$ iff $\sigma > 1$
    - $\tilde{\sigma} \to \sigma$ if $\sigma \to 1$.

Capital-Augmenting Technical Change?

- From the last slide:

$$\underbrace{\tilde{\sigma} - \sigma}_{\text{Bias}} = (1 - \sigma)\,\text{corr}\left(\hat{A}_k, \hat{\xi}\right) \frac{\text{sd}\left(\hat{A}_k\right)}{\text{sd}\left(\hat{\xi}\right)}$$

- If
  - $\text{corr}\left(\hat{A}_k, \hat{\xi}\right) = -0.28$
  - $\text{sd}\left(\hat{A}_k\right) = 0.10$
  - $\text{sd}\left(\tilde{\xi}\right) = 0.11$
- then if $\sigma = 1.25 \Rightarrow \tilde{\sigma} = 1.20$

Effect of the markup and investment price shocks

σ	1	1.25	1	1.25	1	1.25
	$\hat{\xi}$		$\hat{\mu}$		$(\hat{\xi},\ \hat{\mu})$	
Labor share (% points)	0.0	-2.6	-3.1	-2.6	-3.1	-4.9
Capital share (% points)	0.0	2.6	-1.9	-2.4	-1.9	-0.1
Profit share (% points)	0.0	0.0	5.0	5.0	5.0	5.0
Rental rate	-22.1	-22.1	0.0	0.0	-22.1	-22.1
Capital-to-output	28.4	36.6	-5.2	-6.4	21.8	27.9
Welfare-equiv. consumption	18.1	22.1	-3.0	-3.4	13.2	15.8

The discrepancy between Oberfield and Raval and Karabarbounis and Neiman?

- Sample: Manufacturing (OR) vs the whole economy (KN)

	Primary	Construction	Manuf.	Transport
θ	0.03	0.05	0.20	0.61
α	0.55	0.19	0.35	0.45
	Electricity/ Gas Serv.	Wholesale/ Retail	FIRE	Other Services
θ	0.05	0.15	0.17	0.28
α	0.58	0.29	0.67	0.18

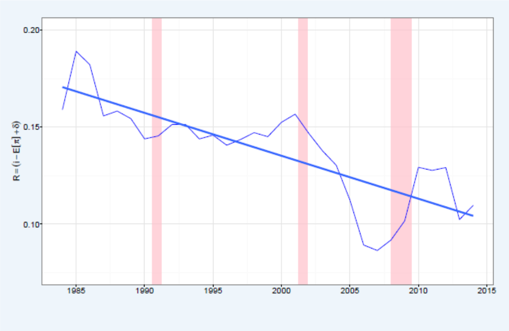
$\chi^{\text{full}} = 0.14$.

- Omitted variable bias? See Loukas' discussion of OR on his webpage.

Barkai (2016): "Declining Labor and Capital Shares"

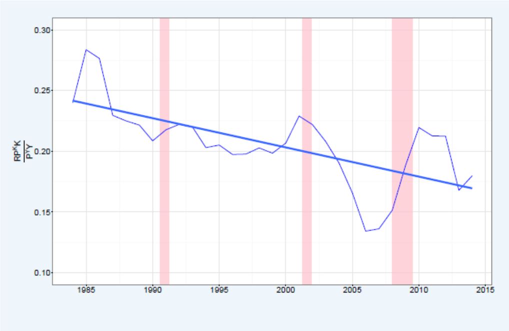
- Karabarbounis and Neiman: Whether declining labor shares are due to $\hat{\xi}$ and $\hat{\mu}$ have dramatically different welfare implications.
- Compute, for different types of capital, the required rate of return on capital

$$\tilde{R}_s = \underbrace{i - \mathbb{E}\left[\pi_s\right]}_{\text{real interest rate}} + \delta_s$$



Barkai (2016): "Declining Labor and Capital Shares"

As a result, capital shares declines more than when $\tilde{R}$ is fixed.



... and the profit share increased considerably, from 2 percent to 16 percent.