Notes on Burstein, Morales, Vogel (2016): "Changes in between-group inequality: computers, occupations, and international trade"

## Research Question

- To what extent can skill-biased technical change account for a higher skill premium? (Or more generally the changes in between-group inequality which we have observed)
- Others (including Krusell et al. 2000) have addressed this in the past.


## Approach

- Overall goal: Use individual-level data from different points in time to recover sources of between-group premia changes.
- Workers of different types (college vs. high-school educated; women vs. men; experienced vs. inexperienced)
- tend to use computer capital at different intensities;
- tend to be employed in different occupations (which may be growing or shrinking for non-computer related reasons);
- may be experiencing different trends in unobserved (quality); and
- may have growing/shrinking observed labor supply
- Construct a general equilibrium model in which these four forces can lead to changes in
- which occupations workers work in.
- the wages of different types of workers
- ... to back out the importance of supply; occupational demand; unobserved labor quality; increased computer capital in generating wage premia.


## October CPS has a supplement on Computer/Internet Usage

- Ask things like:
- Does the worker "directly use computer at work?"
- Is a "computer used at work for electronic mail?"
- Is a "computer used at work for programming?"
- Is a "Computer used at home for household record keeping, taxes, etc.?"
- Every few years between 1989 (in the paper, 1984) and 2003. More frequently after.

Occupations differ considerably in their workers, computer usage

|  |  | Computer | Female |
| ---: | :--- | :---: | :---: |
| 1 | Executive, Admin. | 57 | 34 |
| 2 | Mgmt. Related | 80 | 49 |
| 3 | Architect | 46 | 12 |
| 4 | Engineer | 79 | 8 |
| 5 | Life, Physical Science | 76 | 29 |
| 6 | Computer/Math | 96 | 36 |
| 7 | Social Services | 38 | 47 |
| 8 | Lawyer | 47 | 23 |
| 9 | Education | 47 | 65 |
| 10 | Arts | 49 | 41 |
| 11 | Health diagnosing | 33 | 17 |
| 12 | Health treatment | 51 | 82 |
| 13 | Technician | 67 | 43 |
| 14 | Financial Sales | 50 | 35 |
| 15 | Retail Sales | 36 | 45 |

Occupations differ considerably in their workers, computer usage

|  | SHS | Some <br> College | Collge+ |  |
| ---: | :--- | :---: | :---: | :---: |
| 1 | Executive, Admin. | 44 | 28 | 28 |
| 2 | Mgmt. Related | 32 | 36 | 32 |
| 3 | Architect | 7 | 42 | 52 |
| 4 | Engineer | 24 | 42 | 34 |
| 5 | Life, Physical Science | 10 | 19 | 71 |
| 6 | Computer/Math | 28 | 32 | 40 |
| 7 | Social Services | 20 | 19 | 61 |
| 8 | Lawyer | 3 | 3 | 94 |
| 9 | Education | 7 | 12 | 81 |
| 10 | Arts | 40 | 26 | 34 |
| 11 | Health diagnosing | 2 | 2 | 95 |
| 12 | Health treatment | 14 | 33 | 53 |
| 13 | Technician | 48 | 31 | 20 |
| 14 | Financial Sales | 54 | 27 | 19 |
| 15 | Retail Sales | 62 | 24 | 14 |

## College+ Share and Computer Share Are Moderately Positively Correlated



## College+ occupations (as of 1989) grew between 1989 and 2003



## Computer-heavy occupations (as of 1989) grew between 1989 and 2003



## Reasons why college premium could increase/decrease:

1. Demand for occupations (those with many college workers) has been increasing
2. Price of computers has been decreasing.
3. Labor quality of college workers has been increasing relative to other workers.
4. Supply of college workers has been increasing

## Background on the Frechet Distribution

- $G(\varepsilon)=\exp \left\{-\varepsilon^{-\theta}\right\}$
- Pdf of the Frechet distribution for $\theta \in\{2,3,5\}$ :

- Less dispersion when $\theta$ is large.


## Background on the Frechet Distribution

- Suppose we have a sample of $N$ Frechet independently distributed random variables, $\phi_{1}, \ldots \phi_{N} \ldots$


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- What is $\pi_{n} \equiv \operatorname{Pr}\left\{T_{n} \phi_{n}>\max _{m \neq n} T_{m} \phi_{m}\right\}$ ?


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- What is $\pi_{n} \equiv \operatorname{Pr}\left\{T_{n} \phi_{n}>\max _{m \neq n} T_{m} \phi_{m}\right\}$ ?
- Let's start with an easier problem.

$$
\begin{aligned}
\operatorname{Pr}\left\{\max _{m \neq n} T_{m} \phi_{m}<x\right\} & =\prod_{m \neq n} \operatorname{Pr}\left\{\max _{m \neq n} \phi_{m}<\frac{x}{T_{m}}\right\} \\
& =\prod_{m \neq n} \exp \left\{-\left(\frac{x}{T_{m}}\right)^{-\theta}\right\} \\
& =\exp \left[-\sum x^{-\theta} T_{m}^{\theta}\right] \exp \left\{x^{-\theta} T_{n}^{\theta}\right\}
\end{aligned}
$$

## Background on the Frechet Distribution

- From last slide:

$$
\operatorname{Pr}\left\{\max _{m \neq n} T_{m} \phi_{m}<x\right\}=\exp \left[-\sum x^{-\theta} T_{m}^{\theta}\right] \exp \left\{x^{-\theta} T_{n}^{\theta}\right\}
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$$

- Back to our original problem:

$$
\begin{aligned}
\pi_{n} & =\int_{0}^{\infty} \operatorname{Pr}\left\{\max _{m \neq n} T_{m} \phi_{m}<x\right\} d G(x) \\
& =\int_{0}^{\infty} \exp \left[-x^{-\theta} \sum T_{m}^{\theta}\right] \exp \left\{x^{-\theta} T_{n}^{\theta}\right\} \cdot d G(x) \\
& =\theta T_{n}^{\theta} \int_{0}^{\infty} \exp \left[-x^{-\theta} \sum T_{m}^{\theta}\right] x^{-1-\theta} d x \\
& =\left.\theta T_{n}^{\theta} \frac{\exp \left\{-\sum T_{m}^{\theta} x^{-\theta}\right\}}{\sum T_{m}^{\theta} \cdot \theta}\right|_{x=0} ^{\infty} \\
& =\frac{T_{n}^{\theta}}{\sum T_{m}^{\theta}}
\end{aligned}
$$

## Background on the Frechet Distribution

- Summary

$$
\pi_{n}=\frac{T_{n}^{\theta}}{\sum_{m} T_{m}^{\theta}}
$$

- Heterogeneity in idiosyncratic draws ( $\theta$ is small) means that $\pi$ is less sensitive to the $T$ 's.
- Another formula. Back in the Frechet case:

$$
\mathbb{E}\left[\max _{m} T_{m} \phi_{m}\right]=\left[\sum T_{m}^{\theta}\right]^{1 / \theta} \cdot \Gamma\left[\frac{\theta-1}{\theta}\right]
$$

- Formula for the choice probability resembles that in the Gumbel (type 1 extreme value) distribution.
- $G(x)=\exp \{-\exp \{-x\}\}$.
- In this case the probabilities would look like:

$$
\pi_{n}=\frac{\exp \left\{\theta T_{n}\right\}}{\sum_{m} \exp \left\{\theta T_{m}\right\}}
$$

## Model Overview

- Aggregate output is a CES composite of occupation ( $\omega$ ) output, can be used for consumption or capital investment

$$
\left.\begin{array}{rl}
Y_{t} & =[\sum_{\omega} \underbrace{\mu_{t}^{1 / \rho}(\omega)}_{\text {demand shifter }} Y_{t}(\omega)^{\frac{\rho-1}{\rho}}
\end{array}\right]^{\rho /(\rho-1)}
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- Workers z come of different types, $\lambda$ (education; gender; experience).
- Workers have idiosyncratic productivity in different occupations, capital types, $\varepsilon(z, \kappa, \omega)$ is distributed according to a Frechet distribution.


## Model Overview

- Occupation output is the combination of capital and labor of different types. A worker of type $\lambda$ with I units of efficiency labor, using capital $k$ of type $\kappa$, produces in occupation $\omega$ :

$$
k(\kappa)^{\alpha} \cdot I(\lambda)^{1-\alpha} \cdot T(\lambda, \kappa, \omega)^{1-\alpha}
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$$

- $T_{t}(\lambda, \kappa, \omega)$ is the productivity in producing occupation output when combining capital $\kappa$ and labor type $\lambda$


## Model Overview

- Things that are exogeneous
- supply of workers of type $\lambda$
- $T(\lambda, \kappa, \omega)$
- demand for occupations $\mu(\omega)$
- price of capital $p(\kappa)$
- What we want to solve for
- Occupational choice and wages of different types of workers
- Choice on computer vs. non-computer capital.


## Occupational Choice

- Suppose the price of $\omega$ 's output is $p(\omega)$ and of capital $\kappa$ is $p(\kappa)$. A profit maximizing firm is maximizing

$$
p(\omega) \cdot k(\kappa)^{\alpha} \cdot /(\lambda)^{1-\alpha} \cdot T(\lambda, \kappa, \omega)^{1-\alpha}-\underbrace{v(\lambda, \kappa, \omega)}_{\text {efficiency wage, tbd }} \cdot /(\lambda)-p(\kappa) k
$$

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- Plug in profit maximizing choice of capital to get indirect profit function as a function of a firm. Then compute marginal productivity of labor to get

$$
v(\lambda, \kappa, \omega)=(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} \cdot p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} T(\lambda, \kappa, \omega)
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$$

- Important point. Efficiency wage is just a function of its type.
- Workers choose occupations to maximize

$$
v(\lambda, \kappa, \omega) \cdot \varepsilon(z, \kappa, \omega)
$$

## Occupational Choice

- From last slide, the value of a worker of type $\lambda$ in occupation $\omega$ with capital type $\kappa$ :

$$
v(\lambda, \kappa, \omega)=(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} \cdot p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} T(\lambda, \kappa, \omega)
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- The probability that a type $\lambda$ worker chooses $\kappa, \omega$ is

$$
\begin{equation*}
\pi(\lambda, \kappa, \omega)=\frac{\left[p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} \cdot T(\lambda, \kappa, \omega)\right]^{\theta}}{\left[\sum_{\kappa^{\prime}, \omega^{\prime}} p\left(\kappa^{\prime}\right)^{\frac{-\alpha}{1-\alpha}} \cdot p\left(\omega^{\prime}\right)^{\frac{1}{1-\alpha}} T\left(\lambda^{\prime}, \kappa^{\prime}, \omega\right)\right]^{\theta}} \tag{1}
\end{equation*}
$$

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- From last slide, the value of a worker of type $\lambda$ in occupation $\omega$ with capital type $\kappa$ :

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v(\lambda, \kappa, \omega)=(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} \cdot p(\kappa)^{\frac{-\alpha}{1-\alpha}} \cdot p(\omega)^{\frac{1}{1-\alpha}} T(\lambda, \kappa, \omega)
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\end{equation*}
$$

- And the wage of a worker of type $\lambda$

$$
\begin{equation*}
w(\lambda)=\Gamma\left[\frac{\theta-1}{\theta}\right] \cdot\left[\sum_{\kappa, \omega} v(\lambda, \kappa, \omega)^{\theta}\right]^{1 / \theta} \tag{2}
\end{equation*}
$$

## Goods market clearing

For each occupation, expenditures equals payments to factors in that occupation

$$
\begin{equation*}
\mu_{t}(\omega) \cdot p_{t}(\omega)^{1-\rho} \cdot E_{t}=\frac{1}{1-\alpha} \cdot \sum_{\lambda, \kappa} w_{t}(\lambda) \cdot L_{t}(\lambda) \cdot \pi_{t}(\lambda, \kappa, \omega) \tag{3}
\end{equation*}
$$

- LHS: Total expenditure $\left(E_{t}\right)$ multiplied by expenditure share of occupation $\omega$.
- RHS: The sum equals payments just to labor. The $\frac{1}{1-\alpha}$ indicates that a fraction $\alpha$ of payments go to capital, so to get revenues scale up labor costs by $(1-\alpha)^{-1}$.

Equations (1)-(3) characterize and equilibrium.

## Effect of...

- Decrease in the price of computers (or equivalently an increase in the productivity of working with computers, through $T(\lambda, \kappa, \omega)$ ) :
- Raises the wage of workers who use computers intensively.
- Reduces the price of occupations which use computers.
- Lowers the wage of workers in these occupations who don't use computers.
- Increase in the demand for an occupation (or equivalently an increase in the productivity of workers in an occupation):


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- Increase in the supply of workers of type $\lambda$


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- Increase in the supply of workers of type $\lambda$
- Reduces the wages of $\lambda$


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- Increase in the demand for an occupation (or equivalently an increase in the productivity of workers in an occupation):
- Raises the wage of individuals disproportionately in this occupation.
- Increase in the supply of workers of type $\lambda$
- Reduces the wages of $\lambda$
- Reduces the prices of occupations $\omega$ which use $\lambda$ intensively, wages of workers who work in same occupations as $\lambda$.


## Backing out the exogeneous shifters

- We are interested in the changes of wages, worker allocation as a function of changes in labor supply, the price of capital, occupational demand, and productivity
- Assume a particular functional form for how $T$ changes over time

$$
T_{t}(\lambda, \kappa, \omega)=T_{t}(\lambda) \cdot T_{t}(\kappa) \cdot T_{t}(\omega) \cdot T(\lambda, \kappa, \omega)
$$

$T(\lambda, \kappa, \omega)$ will be unobserved. Use a hat to refer to a change:
$\hat{X}_{t} \equiv \frac{X_{t+\tau}}{X_{t}}$. So:

$$
\hat{T}_{t}(\lambda, \kappa, \omega)=\hat{T}_{t}(\lambda) \cdot \hat{T}_{t}(\kappa) \cdot \hat{T}_{t}(\omega)
$$

## Backing out the exogeneous shifters

- Can't tell apart differences in capital productivity and exogeneous capital prices. Define a variable which combines their effect:

$$
\hat{q}_{t}(\kappa)=\left[\hat{p}_{t}(\kappa)\right]^{-\frac{\alpha}{1-\alpha}} \cdot \hat{T}_{t}(\kappa)
$$

- Similarly for occupation prices:

$$
\hat{q}_{t}(\omega)=\hat{p}_{t}(\omega)^{\frac{1}{1-\alpha}} \cdot \hat{T}_{t}(\omega)
$$

- And occupational output:

$$
\hat{a}_{t}(\omega)=\hat{\mu}_{t}(\omega)^{\frac{1}{1-\alpha}} \cdot \hat{T}_{t}(\omega)^{(1-\alpha)(\rho-1)}
$$

## Backing out the exogeneous shifters

- Changes in $\hat{L}(\lambda) / \hat{L}\left(\lambda_{1}\right)$ can be read off of data.
- With the variable re-definitions from the previous slide, Equations (1) to (3) imply that when comparing two periods:

$$
\left(\frac{\hat{q}(\kappa)}{\hat{q}\left(\kappa_{1}\right)}\right)^{\theta}=\frac{\hat{\pi}(\lambda, \kappa, \omega)}{\hat{\pi}\left(\lambda, \kappa_{1}, \omega\right)}
$$

The right hand side is observable, which (if $\theta$ where known) would allow us to back out changes in capital productivity.

- Similarly one can compute changes in occupational productivity

$$
\left(\frac{\hat{q}(\omega)}{\hat{q}\left(\omega_{1}\right)}\right)^{\theta}=\frac{\hat{\pi}(\lambda, \kappa, \omega)}{\hat{\pi}\left(\lambda, \kappa, \omega_{1}\right)}
$$

## Backing out the exogeneous shifters

- Finally, for a given value of $\rho$ one can back out shocks to occupational output

$$
\frac{\hat{a}(\omega)}{\hat{a}\left(\omega_{1}\right)}=\frac{\sum_{\lambda, \kappa} w(\lambda) \widehat{L(\lambda)} \pi_{t}(\lambda, \kappa, \omega)}{\sum_{\lambda, \kappa} w(\lambda) \widehat{L_{t}(\lambda)} \pi_{t}\left(\lambda, \kappa, \omega_{1}\right)} \cdot\left(\frac{\hat{q}(\omega)}{\hat{q}\left(\omega_{1}\right)}\right)^{(1-\alpha)(\rho-1)}
$$

- And finally, to back out labor productivity:

$$
\frac{\hat{T}(\lambda)}{\hat{T}\left(\lambda_{1}\right)}=\frac{\hat{w}(\lambda)}{\hat{w}\left(\lambda_{1}\right)} \cdot\left[\frac{\hat{s}(\lambda)}{\hat{s}\left(\lambda_{1}\right)}\right]^{1 / \theta}
$$

where $\hat{s}(\lambda)$ is an ugly expression we have already solved for, describing $\lambda$-specific average equipment productivity changes.

## Results

| Period | Data | Labor <br> Composition | Occupation <br> Shifters |
| :---: | :---: | :---: | :---: |
| $84-89$ | 0.057 | -0.031 | 0.026 |
| $89-93$ | 0.064 | -0.017 | -0.009 |
| $93-97$ | 0.037 | -0.023 | 0.044 |
| $97-03$ | -0.007 | -0.043 | -0.011 |
| $84-03$ | 0.151 | -0.114 | 0.049 |


| Equipment <br> Productivity | Labor <br> Productivity |
| :---: | :---: |
| 0.052 | 0.009 |
| 0.045 | 0.046 |
| 0.021 | -0.005 |
| 0.042 | 0.006 |
| 0.159 | 0.056 |

## Conclusion

- Declining prices of computer equipment account for a 16 percentage point increase in the college/high-school wage gap.
- Increased relative supply of college-educated workers has a countervailing, 11 percentage point effect on the college wage premia.
- Final sections in the paper endogenize decline in computer equipment prices, change in the demand for occupation services, through declining trade costs with other countries.
- But what about residual wage inequality?
- And what about non-technological, non-supply factors, such as declining unionization rates?

Notes on Firpo, Fortin, Lemieux (2014):
"Occupational Tasks and Changes in the Wage Structure"

## Changes in Attributes

|  | $1983-85$ | $2003-05$ |
| :--- | :---: | :---: |
| Unionization | 26.2 | 15.2 |
| Experience |  |  |
| $<10$ years | 34.4 | 25.2 |
| 10-20 years | 29.0 | 25.3 |
| $20-30$ years | 17.3 | 26.2 |
| $30-40$ years | 12.7 | 18.5 |
| $>40$ years | 6.6 | 4.7 |
| Education |  |  |
| $<$ High School | 12.5 | 8.0 |
| High School | 38.5 | 31.1 |
| Some College | 19.2 | 27.6 |
| College | 12.9 | 19.5 |
| Post-graduate | 9.9 | 10.1 |

## Occupational Characteristics

- Top occupations per O*NET Element:
- Information: Life Scientist, Physicist, Engineer
- Automation: Production Supervisor, Prepress Technicians, Power Plant Operator
- Face-to-Face: Supervisor of Security Personnel, Clergy, Doctor
- On-Site Job: Mining Operator, Material Moving Workers, Pilot
- Decision-Making: Chief Executive, Supervisor of Security Personnel, Farming Supervisor
- Differences across the sample period:

|  | $1983-85$ | $2003-05$ |
| :--- | :---: | :---: |
| Information | -0.02 | 0.06 |
| Automation | 0.01 | -0.03 |
| Face-to-Face | -0.04 | 0.10 |
| On-Site Job | -0.02 | 0.06 |
| Decision-Making | -0.03 | 0.07 |

## Research Question

- What is the effect of changes in
- occupational characteristics
- unionization
- education and experience
... on inequality in the wage distribution?


## Review of the Oaxaca-Blinder Decomposition

Assume that log wages obey the linear model

$$
\log w_{i, t}=X_{i} \cdot \beta_{t}+\epsilon_{i, t}
$$

Then, the mean wage, between periods $t_{0}$ and $t_{1}$ is given by

$$
\begin{aligned}
\Delta_{O}^{\mu} & =\underbrace{\left(\bar{X}_{t_{1}}-\bar{X}_{t_{0}}\right) \cdot \beta_{t_{0}}}_{\equiv \Delta_{x}}+\underbrace{\bar{X}_{t_{1}} \cdot\left(\beta_{t_{1}}-\beta_{t_{0}}\right)}_{\equiv \Delta_{s}} \\
& =\sum_{k}\left(\bar{X}_{t_{1} k}-\bar{X}_{t_{0} k}\right) \beta_{t_{0} k} \\
& +\beta_{t_{1}, 0}-\beta_{t_{0}, 0}+\sum_{k} \bar{X}_{t_{1} k} \cdot\left(\beta_{t_{1} k}-\beta_{t_{0} k}\right)
\end{aligned}
$$

where $k$ is an individual covariate

## Can't extend this idea exactly to other distributional statistics

A key step of this decomposition was

$$
\begin{aligned}
\Delta_{O}^{\mu} & \equiv \mathbb{E}\left[\log w_{i, t_{1}}-\log w_{i, t_{0}}\right] \\
& \left.=\mathbb{E}_{t_{1}} \mathbb{E}\left[\log w_{i, t_{1}} \mid X\right]\right]-\mathbb{E}_{t_{0}}\left[\mathbb{E}\left[\log w_{i, t_{0}} \mid X\right]\right] \\
& =\mathbb{E}_{t_{1}}[X] \cdot \beta_{t_{1}}-\mathbb{E}_{t_{1}}[X] \cdot \beta_{t_{0}}+\mathbb{E}_{t_{1}}[X] \cdot \beta_{t_{0}}-\mathbb{E}_{t_{0}}[X] \cdot \beta_{t_{0}}
\end{aligned}
$$

In the second line, we use the law of iterated expectations.
Could we have done the same for the median (or 75 th percentile, etc..)? No, because:

$$
Q_{50}\left(\log w_{i, t}\right) \neq \mathbb{E}_{t}\left[Q_{50}\left[\log w_{i, t} \mid X\right]\right]
$$

- Problem: Analogue of law of iterated expectations does not hold.


## Quoting Thomas Lemieux: <br> "Decomposing proportions is easier than decomposing quantiles"

- "Example: 10 percent of men earn more than 80 K a year, but only 5 percent of women do so."
- "Easy to do a decomposition by running [linear probability] model for the probability of earning less (or more) than 80 K , and perform a Oaxaca decomposition on the proportions."
- "By contrast, much less obvious how to decompose the difference between the 90th quantile for men ( 80 K ) and women (say 65K)"
- A way to write out the proportion of individuals making more than some quantity will be given by the recentered influence function.

FFL decomposition is a generalization of Oaxaca-Blinder

Definition (for a particular quantile):

$$
R I F\left(\log w, Q_{50}\right)=\log Q_{50}+\frac{0.5-1\left\{\log w<Q_{50}\right\}}{f_{\log w}\left(Q_{50}\right)}
$$

This satisfies

$$
\mathbb{E}\left[R I F\left(\log w, Q_{50}\right)\right]=Q_{50}
$$

Note

$$
\begin{aligned}
\mathbb{E}\left[R I F\left(\log w, Q_{\tau}\right) \mid X\right] & =c_{1 \tau} \cdot \mathbb{E}\left[\mathbf{1}\left\{\log w<Q_{\tau}\right\} \mid X\right]+c_{2 \tau} \\
& =c_{1 \tau} \cdot \operatorname{Pr}\left[\log w<Q_{\tau} \mid X\right]+c_{2 \tau}
\end{aligned}
$$

Assume

$$
\mathbb{E}\left[R I F\left(\log w, Q_{\tau}\right) \mid X\right]=X \gamma^{\tau}
$$

## FFL decomposition is a generalization of Oaxaca-Blinder

Proceed in steps for each quantile $\tau$

1. Construct the RIF for $\tau$
2. Obtain $\hat{\gamma}^{\tau}$ from

$$
\mathbb{E}\left[R I F\left(\log w, Q_{\tau}\right) \mid X\right]=X \gamma^{\tau}
$$

- Regression coefficient of $\mathbf{1}\left(\log w<Q_{\tau}\right)$ on $X$ : change in the earnings quantile from a one unit change in $X$
- $f_{\log w}\left(Q_{T}\right)$ : change in the earnings quantile from a one unit change in earnings
- $\hat{\gamma}^{\tau}$ : change in (the unconditional) earnings, at the $\tau^{\text {th }}$ quantile, from a unit increase in $X$

3. Decompose the change in $\tau$ between $t=1$ and $t=0$ as

$$
\Delta_{O}^{\mu}=Q_{\tau 1}-Q_{\tau 0}=\sum_{k=1}^{K} \bar{X}_{1 k}\left(\hat{\gamma}_{1 k}^{\tau}-\hat{\gamma}_{0 k}^{\tau}\right)+\sum_{k=1}^{K}\left(\bar{X}_{1 k}-\bar{X}_{0 k}\right) \hat{\gamma}_{0 k}^{\tau}
$$

RIF (50)


## RIF (90)



## Overall Decomposition



## Education/Union/Experience



W age Structure Effects


## Occupation




## Summary

- These decompositions are a useful way to summarize changes in wages for different types of workers.
- But, interpretation of what these decompositions are telling us is difficult:
- Are declining unionization rates or declines in the real minimum wage exogenous?
- What are the general equilibrium effects of e.g. declining unionization rates?

