Notes on Gabaix (2011): "The Granular Origins of Aggregate Fluctuations"

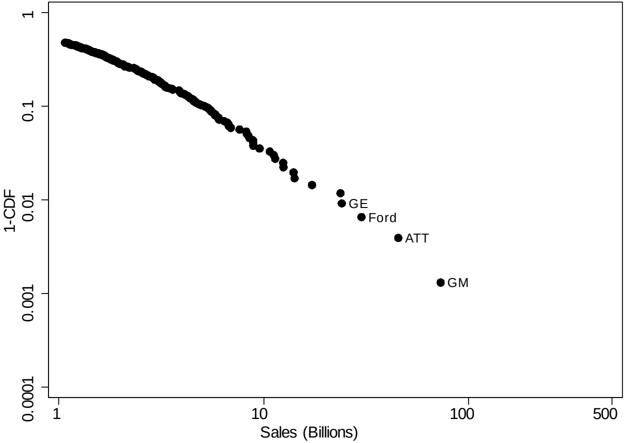
Research question and motivation

- Majority of dynamic general equilibrium models: Firm (scale) heterogeneity does not matter.
- Because some firms are so large, decisions of individual firms can have aggregate implications
  - 2004Q4: Microsoft issues $24 billion one-time dividend. Accounts for 2.1% boost in personal income growth.
  - 2000: Nokia accounts for *half* of Finish private R&D, 1.6 percentage points of GDP growth.
  - Are these anecdotes exceptional or common?
- Question: To what extent are firm-level shocks responsible for aggregate fluctuations?

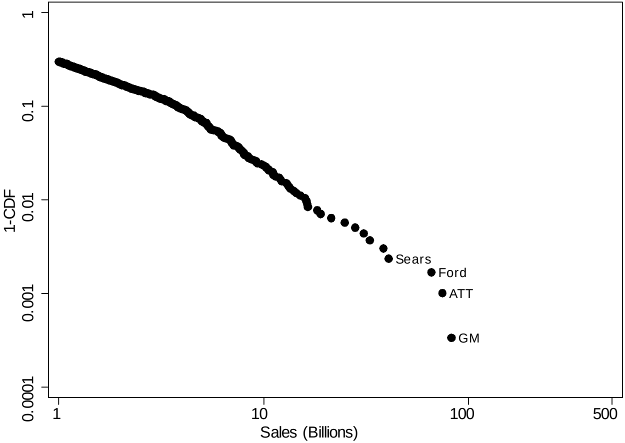
Outline

- Some data
  - Compustat: 1960 to present.
- Theoretical results and calibration
  - The Central Limit Theorem is irrelevant when firm sizes are fat-tailed
  - The herfindahl index is a summary statistic for the importance of firm-specific shocks.
- The granular residual

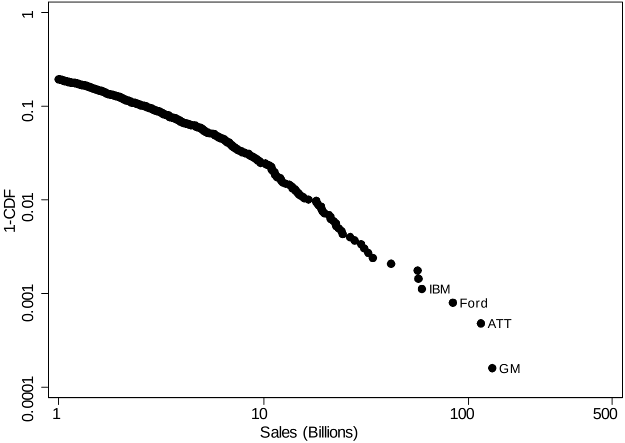
The firm size distribution in 1960



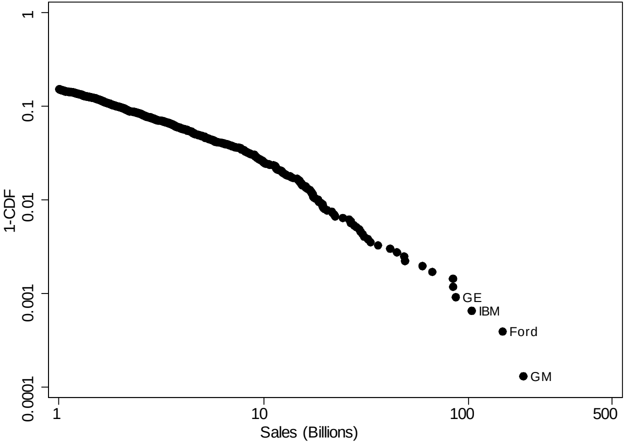
The firm size distribution in 1970



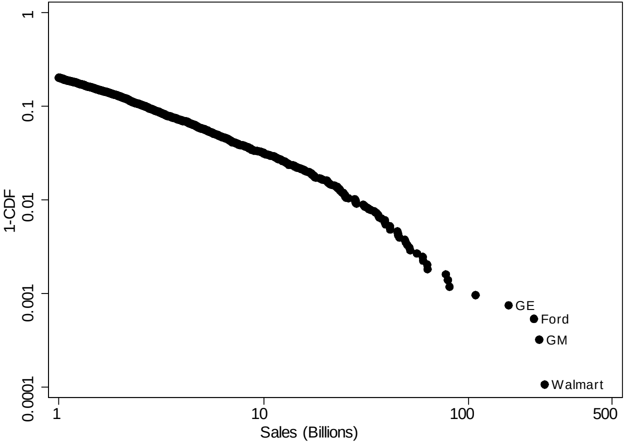
The firm size distribution in 1980



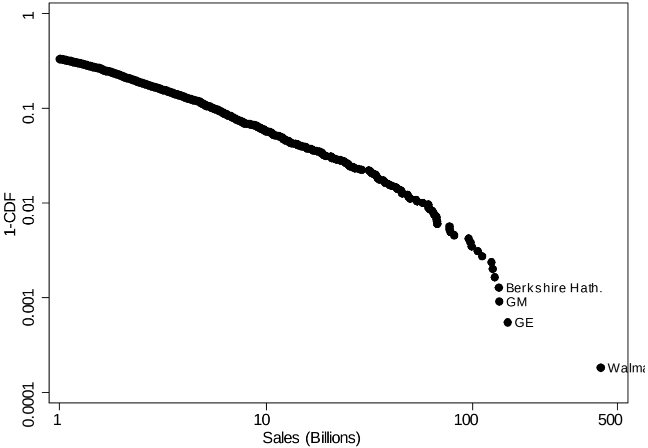
The firm size distribution in 1990



The firm size distribution in 2000



The firm size distribution in 2010



Sales Herfindahl of firms in Compustat

$$h = \left[\sum_i \left(\frac{S_i}{S} \right)^2 \right]^{1/2}$$

Overview of the theoretical results

- If the firm size distribution is Pareto, we can show how the dispersion of GDP growth decreases in economies with more and more firms.
- Even if the firm size distribution is not Pareto, we can relate the dispersion of GDP to:
  - σ: the standard deviation of firm productivity growth rates.
  - h: the HHI of firm sales
  - a combination of other model parameters.

Hulten (1978)

What is the relationship between micro productivity growth and aggregate output growth?

- Economy is made up of n units (firms or industries)
- Utility$= C - \frac{\phi}{\phi+1} L^{\frac{\phi+1}{\phi}}$, where $C \equiv \prod_i \left(\frac{C_i}{\xi_i}\right)^{\xi_i}$
- Production : $Q_i = A_i \left(\frac{L_i}{\alpha}\right)^{\alpha} \left(\frac{M_i}{1-\alpha}\right)^{1-\alpha}$
- Intermediate input bundle: $M_i = \prod_j \left(\frac{M_{j\to i}}{\gamma_{ji}}\right)^{\gamma_{ji}}$
- Market clearing: $Q_i = C_i + \sum_j M_{i\to j}$
- Write
  - P_i as the Lagrange multiplier for the good i market-clearing condition, and $S_i \equiv P_i Q_i$.
  - W as the Lagrange multiplier for the labor market clearing condition.
  - Set C as the numeraire good: $P \equiv \prod_i (P_i)^{\xi_i} = 1$

Hulten (1978)

Step 1: Solve for Total Labor Supply

Consider the problem of the representative consumer who is trying to maximize:

$$C - \frac{\phi}{\phi+1} L^{\frac{\phi+1}{\phi}} \text{ s.t. } C = WL$$

Equilibrium C and L satisfy:

$$L = W^{\phi}$$

$$C = W^{\phi+1} = L^{\frac{\phi+1}{\phi}} \quad (1)$$

Hulten (1978)

Step 2: Solve for Prices

Consider the cost-minimization problem of firm/industry i.

$$\log Q_i = \log A_i + \alpha \log \frac{L_i}{\alpha} + (1-\alpha) \sum_j \gamma_{ji} \log \left(\frac{M_{ji}}{(1-\alpha)\gamma_{ji}} \right)$$

$$= \log A_i + \alpha \log \left(\frac{Q_i P_i}{W} \right) + (1-\alpha) \sum_j \gamma_{ji} \log \left(\frac{Q_i P_i}{P_j} \right)$$

Thus:

$$\log P_i = -\log A_i + \alpha \log W + (1-\alpha) \sum_j \gamma_{ji} \log P_j$$

$$\overrightarrow{\log P} = \left(I - ((1-\alpha)\Gamma)' \right)^{-1} \left(-\overrightarrow{\log A} + \alpha \log W \right) \qquad (2)$$

Hulten (1978)

Step 3: Write out sales in each industry

Using the market clearing conditions

$$S_i = P_i Q_i = P_i C_i + \sum_j P_i M_{i \to j}$$

Plugging in customers' factor demand curves and re-arranging:

$$S_i - (1-\alpha) \sum_j \gamma_{ij} S_j = \xi_i C$$

$$\frac{\overrightarrow{S}}{C} = (I - ((1-\alpha)\Gamma))^{-1} \overrightarrow{\xi}$$

Hulten (1978)

Step 4: Write out total consumption and labor in terms of productivity

Plug Equation (2) into Equation (1)

$$\overrightarrow{\log P} = \left(I - \left(\left(1-\alpha\right)\Gamma\right)'\right)^{-1}\left(-\overrightarrow{\log A} + \alpha \log W\right)$$
$$= \left(I - \left(\left(1-\alpha\right)\Gamma\right)'\right)^{-1}\left(-\overrightarrow{\log A} + \frac{\alpha}{\phi+1}\log C\right)$$

Use the fact that $\xi'\overrightarrow{\log P} = 0$ and $\left(I - \left(\left(1-\alpha\right)\Gamma\right)'\right)^{-1}\alpha\mathbf{1} = \mathbf{1}$:

$$\left(\phi+1\right)\xi'\left(I - \left(\left(1-\alpha\right)\Gamma\right)'\right)^{-1}\overrightarrow{\log A} = \log C$$

Remember the equation for sales

$$\frac{\overrightarrow{S}'}{C} = \overrightarrow{\xi}'\left(I - \left(\left(1-\alpha\right)\Gamma'\right)\right)^{-1}$$

Thus

$$\log C = \left(\phi+1\right)\frac{\overrightarrow{S}'}{C}\overrightarrow{\log A} \qquad \text{and} \qquad \log L = \phi\frac{\overrightarrow{S}'}{C}\overrightarrow{\log A}$$

Hulten (1978)

The Main Results

1. Aggregate productivity is a weighted average of productivity of the individual units:

$$A^{agg} \equiv \log \frac{C}{L} = \frac{\vec{S}\,'}{C} \overrightarrow{\log A}$$

   The sum of the weights is bigger than 1.

2. Total output and labor inputs each depend on aggregate productivity and the labor supply elasticity

$$\log C = (\phi + 1) A^{agg} \text{ and } \log L = \phi A^{agg}$$

3. Combining (1) and (2)

$$\sigma_{\log C} = (\phi + 1) \cdot \frac{\sum S_i}{C} \left[\sum_i \left(\frac{S_i}{S} \right)^2 \right]^{1/2} \sigma = \mu \cdot h \cdot \sigma,$$

   where $\mu \equiv (\phi + 1) \cdot \sum \frac{S_i}{C}$

- Calibration: $h = 6\%$, $\sigma = 12\%$, $\mu = 6 \Rightarrow \sigma_{\log C} = 4.3\%$

The Pareto Distribution

Let $S_i \equiv P_i C_i$ be a Pareto(ζ,x_0) random variable.
$\text{P}(S > x) = \left(\frac{x}{x_0}\right)^{-\zeta}$.

Some useful facts about the Pareto distribution:

- $\mathbb{E}[S] = x_0 \frac{\zeta}{\zeta - 1}$ if $\zeta > 1$, ∞ otherwise
- $\mathbb{E}\left[S^2\right] = (x_0)^2 \frac{\zeta}{\zeta - 2}$ if $\zeta > 2$, ∞ otherwise
- S^α is Pareto $\left(\frac{\zeta}{\alpha},\ (x_0)^\alpha\right)$ distributed.
- αS is Pareto $(\zeta,\ \alpha x_0)$ distributed.
- r^{th} moment of the k^{th} largest value in a sample of $N \equiv \mathbb{E}[S^r_{k:N}] = (x_0)^r \frac{\Gamma\left[k - \frac{r}{\zeta}\right]}{\Gamma[k]} \frac{\Gamma[N+1]}{\Gamma\left(N+1-\frac{r}{\zeta}\right)}$, if $r > \zeta$.
- Many other facts in Gabaix (2009, Section 2)

Classic Central Limit Theorem

Suppose S_1, S_2, ... S_N is a sequence of i.i.d. random variables with $\mathbb{E}\left[S_i\right] = \mu$ and $\text{Var}[S_i] = \sigma^2 < \infty$. Then, as N approaches ∞,

$$\frac{\sqrt{N}}{\sigma}\left(\frac{\sum S_i}{N} - \mu\right) \rightarrow_{\text{d}} \mathcal{N}\left(0, 1\right)$$

"Compared to an economy with 10 firms, an economy with 10 million firms will have GDP growth with a standard deviation 0.1% as large".

What if $\text{Var}[S_i] = \infty$?

Central Limit Theorem with infinite variances

Suppose S_1, S_2, ... S_N is a sequence of i.i.d. nonnegative random variables with $\mathrm{P}(S_i > x) = x^{-\zeta} L(x)$ (where $L(x)$ is a *slowly-varying function*, and $\zeta < 2$). Then

$$\left(\frac{\sum_i S_i - b_N}{a_N} \right) \to \mathcal{L}(\zeta), \text{ where}$$

$$a_N = \inf \left\{ x : P(S_i > x) \leq \frac{1}{N} \right\}$$

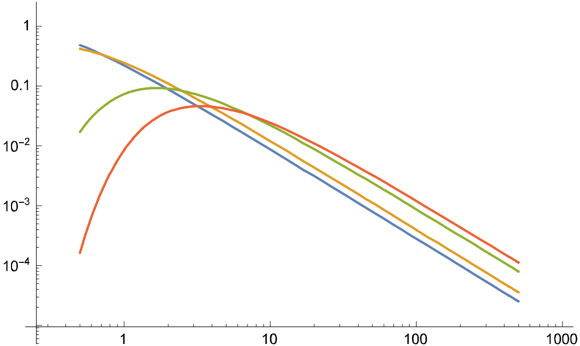
$$\text{and } b_N = N\mathbb{E}\left[S_i \cdot 1_{(X_i \leq a_N)} \right]$$

and $\mathcal{L}(\zeta)$ is a *Levy distribution* with exponent ζ.

- A slowly-varying function, $L(x)$ is one that satisfies $\lim_{x\to\infty} \frac{L(tx)}{L(x)} = 1 \ \forall \ t > 0$.
- If $\mathrm{P}(S_i > x) = \left(\frac{x}{x_0}\right)^{-\zeta}$, then
  $$a_N = \inf \left\{ x : \left(\frac{x}{x_0}\right)^{-\zeta} \leq \frac{1}{N} \right\} = x_0 N^{1/\zeta}, \ b_N = 0$$
- Thus $\frac{N^{1-1/\zeta}}{x_0} \frac{\sum S_i}{N} \to \mathcal{L}(\zeta)$

Levy distribution

PDF of Levy distribution: $\sqrt{\frac{\zeta}{2\pi}} \exp\left\{-\frac{\zeta}{2x}\right\} x^{-3/2}$



Proposition 2

"Consider a series of island economies indexed by N. Economy N has N firms whose growth rate volatility is σ and whose sizes $S_1,..S_N$ are independently drawn from a power law distribution."

$$P(S > x) = ax^{-\zeta}, \text{ with } \zeta \geq 1.$$

As $N \rightarrow \infty$, GDP volatility follows

$$\begin{aligned}
\sigma_{GDP} &\sim \frac{v_\zeta}{\log N}\sigma \text{ for } \zeta = 1 \\
\sigma_{GDP} &\sim \frac{v_\zeta}{N^{1-1/\zeta}}\sigma \text{ for } \zeta \in (1,2) \\
\sigma_{GDP} &\sim \frac{v_\zeta}{N^{1/2}}\sigma \text{ for } \zeta \geq 2
\end{aligned}$$

When $\zeta \geq 2$, v_ζ is a constant; when $\zeta < 2$, v_ζ is the square root of a Levy distributed (with exponent $\zeta/2$) random variable.

Intuition for Proposition 2

In our islands economy, $\sigma_{GDP} = \sigma h$. Looking across economies with different numbers of firms, how does h change as N changes?

Take $P(S > x) = ax^{-\zeta}$, and consider the case in which $\zeta \in (1,2)$, and $a = 1$.

$$\frac{\mathbb{E}[X_{k:N}]}{N\mathbb{E}[X]} = \frac{\Gamma\left[k - \frac{1}{\zeta}\right](\zeta - 1)}{\Gamma[k]\,\zeta} \frac{\Gamma[N]}{\Gamma\left(N + 1 - \frac{1}{\zeta}\right)}$$

$$\to_{N\to\infty} \frac{\Gamma\left[k - \frac{1}{\zeta}\right](\zeta - 1)}{\Gamma[k]\,\zeta} N^{-(1-1/\zeta)}$$

Share of top K firms is proportional to $N^{-(1-1/\zeta)} \Rightarrow h$ is proportional to $N^{-(1-1/\zeta)}$.

Proof of Proposition 2, Part 1

If $\zeta \geq 2$, the variance of S_i is finite. Can apply the formula $\sigma_{GDP} = \sigma h$

$$h = \frac{1}{N^{1/2}} \frac{\left[N^{-1} \sum (S_i)^2\right]^{1/2}}{N^{-1} \sum S_i}$$

$$\sigma_{GDP} \to \frac{\sigma}{N^{1/2}} \cdot \frac{\left(\mathbb{E}\left[S^2\right]\right)^{1/2}}{\mathbb{E}\left[S\right]}$$

Proof of Proposition 2, Part 2

When $\zeta > 1$, $N^{-1} \sum S_i \to \mathbb{E}[S]$

S_i^2 has a power law exponent $\zeta/2$

$$P\left((S_i)^2 > x\right) = ax^{-\zeta/2}$$

Use the CLT with infinite variances, if $\zeta > 1$

$$N^{-2/\zeta} \sum S_i^2 \to_d \mathcal{L}(\zeta/2)$$

$$N^{1-1/\zeta} h = N^{1-1/\zeta} \frac{\left[N^{-2/\zeta}\left(\sum S_i^2\right)\right]^{1/2}}{N^{-1} \sum S_i} \to_d \frac{\left(\mathcal{L}(\zeta/2)\right)^{1/2}}{\mathbb{E}[S]}$$

Putting the pieces together

$$\sigma_{GDP} N^{1-1/\zeta} = \sigma h N^{1-1/\zeta} \to_d \sigma \frac{\left(\mathcal{L}(\zeta/2)\right)^{1/2}}{\mathbb{E}[S]}$$

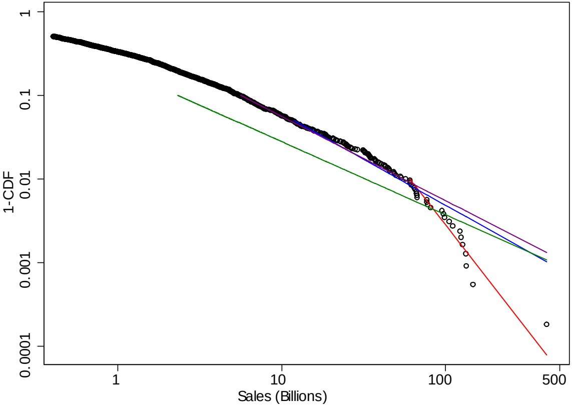
If $\zeta \approx 1.05 \Rightarrow N^{1-1/\zeta} \approx N^{0.05} \Rightarrow$ Compared to an economy with 10 firms, an economy with 10 million firms will have GDP growth with a standard deviation about half as large.

Digression: Is the firm size distribution Pareto?

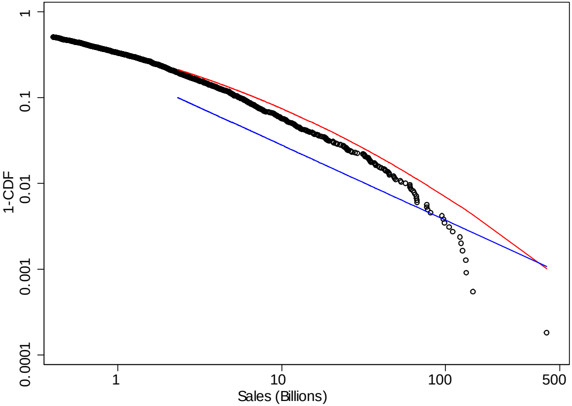
- With moderate sample size it's difficult to distinguish between Pareto distribution (which has infinite variance if $\zeta < 2$) and something like a lognormal distribution (for which regular CLT applies).
- Find best fit, assuming firm sizes are distributed either Pareto or lognormal.
- $f(x) = \frac{\zeta (x_0)^{\zeta}}{x^{\zeta+1}} \Rightarrow \log f(x) = \log \zeta + \zeta \log x_0 - (\zeta + 1) \log x$
- $\frac{\partial \log \mathcal{L}}{\partial \hat{\zeta}} = \sum_{i=1}^{n} \frac{1}{\zeta} + \log\left(\frac{x_0}{x}\right) = 0 \Rightarrow \hat{\zeta} = \left[\frac{1}{N} \sum \log\left(\frac{x}{x_0}\right)\right]^{-1}$

Sample	$\hat{x}_0$	$\hat{\zeta}$
80	2.32	0.87
90	5.62	1.00
95	11.96	1.10
99	60.75	2.52

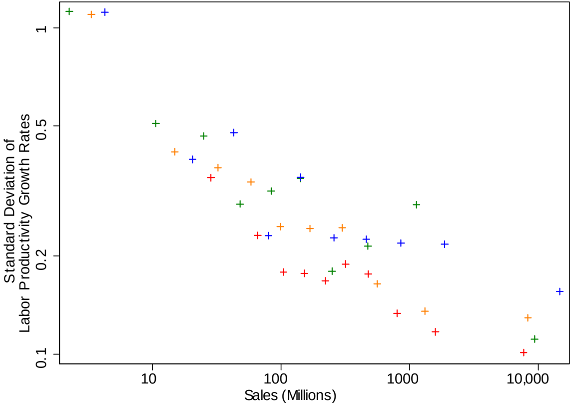
Digression: Is the firm size distribution Pareto?



Digression: Is the firm size distribution Pareto?



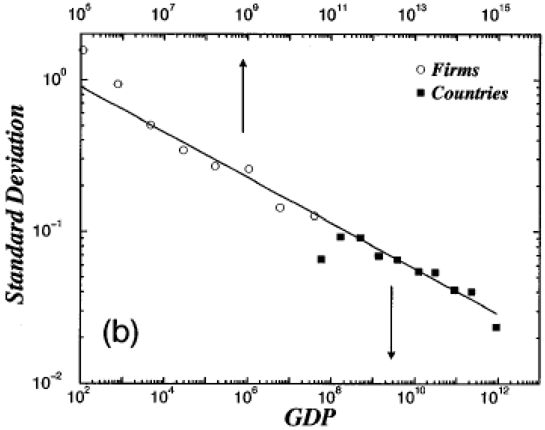
The dispersion of growth rates decreases with size



- $\log\left(\sigma^{\text{Grow}}\right) = \kappa_0 - \kappa_1 \log(\text{size})$;
- $\kappa_1 \in [0.15, 0.25]$, compare to benchmark of perfect correlations of shocks within firms ($\kappa_1 = 0$) or no correlation ($\kappa_1 = \frac{1}{2}$)

The dispersion of growth rates decreases with size

Lee et al. (1998)



We can extend Proposition 2 to allow for firm size and firm volatility to be related.

Consider a series of island economies indexed by N. Economy N has N firms whose growth rate volatility is $\sigma^{\text{firm}}(S) = \sigma\left(\frac{S}{x_0}\right)^{-\alpha}$ and whose sizes $S_1,..S_N$ are independently drawn from a power law distribution.

$$P(S > x) = x^{-\zeta}, \text{ with } \zeta \geq 1.$$

If $\zeta > 1$, the volatility of GDP, $\sigma(Y)$, is proportional to $N^{-\min\left\{\frac{1}{2}, 1-\frac{1-\alpha}{\zeta}\right\}}$.

If $\zeta \approx 1.05$ and $\alpha \approx \frac{1}{6} \Rightarrow N^{1-\frac{1-\alpha}{\zeta}} \approx N^{0.21} \Rightarrow$ Compared to an economy with 10 firms, an economy with 10 million firms will have GDP growth with a standard deviation about 5% as large.

Partial summary

- $h = 6\%$ and $\sigma = 12\%$ $\Rightarrow$ A calibration of a simple "islands" model implies that independent firm shocks can potentially meaningfully contribute to GDP volatility
- Rest of the paper:
  - Construct a measure of productivity shocks to individual firms
  - Regress GDP growth against productivity shocks of the largest firms.

Defining the granular residual

From before

$$\log \frac{Y_t}{Y_{t-1}} \propto \sum_i \frac{S_{i,t-1}}{Y_{it-1}} \log\left(\frac{A_{it}}{A_{i,t-1}}\right)$$

Define

$$\Gamma_t \equiv \sum_{i=1}^{100} \frac{S_{i,t-1}}{Y_{t-1}} \hat{\varepsilon}_{it},$$

$$\hat{\varepsilon}_{it} \equiv z_{it} - z_{i,t-1} - (\bar{z}_{It} - \bar{z}_{I,t-1})$$

where $z_{it} = \log\left(\frac{\text{sales of } i \text{ in year } t}{\text{employees of } i \text{ in year } t}\right)$, and $\bar{z}_{It}$ is the corresponding average labor productivity in firm i's industry, I.

On the granular residual

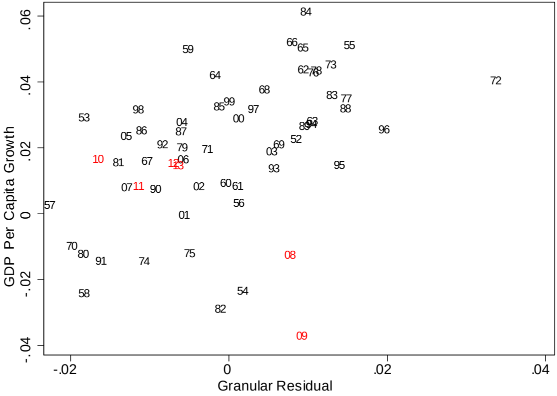
$\log\left(\frac{A_{it}}{A_{i,t-1}}\right)$ measures changes in TFP, not in labor productivity.

- For plants in the manufacturing sector these productivity measures have pretty different patterns (Syverson 2004):
  - 90-10 (75-25) difference of log labor productivity is roughly 1.4 (0.66)
  - 90-10 (75-25) difference of log TFP is 0.7 (0.29)

GDP growth and the granular residual

Sample	1952-2008			1952-2014		
Γ_t	2.8	2.9	3.7	2.9	2.9	3.9
Γ_{t-1}		3.1	3.4		3.1	3.4
Γ_{t-2}			2.1			2.3
Intercept	0.02	0.02	0.02	0.02	0.02	0.02
N	57	56	55	63	62	61
R^2	0.14	0.32	0.40	0.12	0.27	0.36
$\tilde{R}^2$	0.12	0.29	0.36	0.10	0.24	0.32

GDP growth and the granular residual



Granular events

- 1970 Strike at GM (labor productivity down 18%)
- 1972 Ford and Chrsyler have a rush of subcompact sales
- 1983 Launch of IBM PC (labor productivity up 10%)

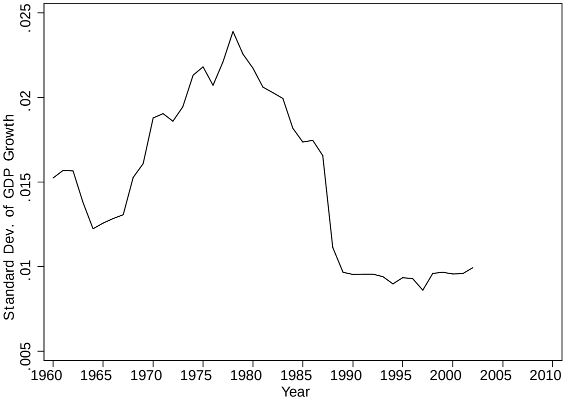
Predictive power of the granular residual

Intercept	0.015	0.019*	0.021*
Γ_{t-1}	3.5**		3.3**
Γ_{t-2}	1.2		2.3*
Monetary_{t-1}		-0.04	-0.05
Monetary_{t-1}		-0.02	0.04
Oil_{t-1}		$-8.7 \cdot 10^{-5}$	$-1.7 \cdot 10^{-4}$
Oil_{t-2}		$-6.9 \cdot 10^{-5}$	$-1.2 \cdot 10^{-4}$
3-month t-bill_{t-1}		-0.45	-0.41
3-month t-bill_{t-2}		0.43	0.39
Term Spread_{t-1}		0.38	0.40
Term Spread_{t-2}		0.27	-0.38
$\tilde{R}^2$	0.19	0.19	0.34

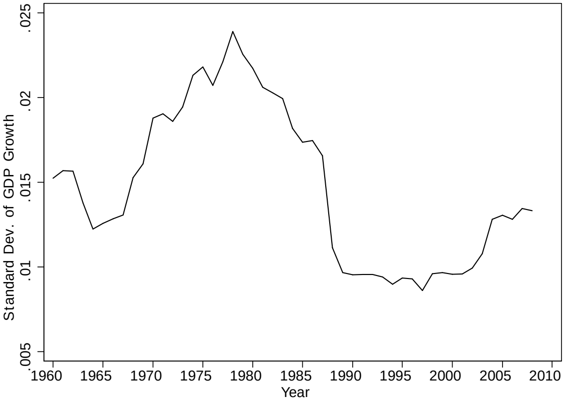
- Oil: (Hamilton 2003): current vs. last year's max oil price.
- Monetary policy shock: Residuals from FOMC decisions vs. FOMC forecasts (Romer and Romer 2004)
- Term spread: 5 year bond yield - 3 month bond yield.

Notes on Carvalho and Gabaix (2013): "The Great Diversification and its Undoing"

The Great Moderation



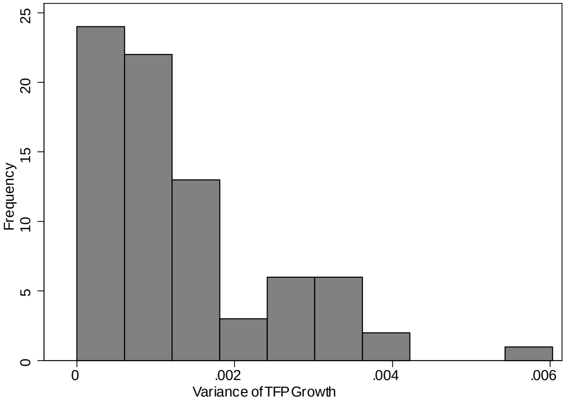
The Great Moderation and its Undoing



Motivation and question

- Why did volatility of GDP growth decrease beginning around 1980? And why did volatility increase beginning around 2005?
- Previous answers to the first question:
  - Stock and Watson (2003): It *doesn't* seem to have to do with better inventory management or more aggressive monetary policy.
  - Arias, Hansen, and Ohanian (2007): Aggregate TFP shocks have become less volatile)
  - Jaimovich and Siu (2009): Fewer young people (those with more elastic labor supply) in the work force
- New answer in this paper: Industry composition affects aggregate volatility.

Industries differ in their volatility



Fundamental Volatility

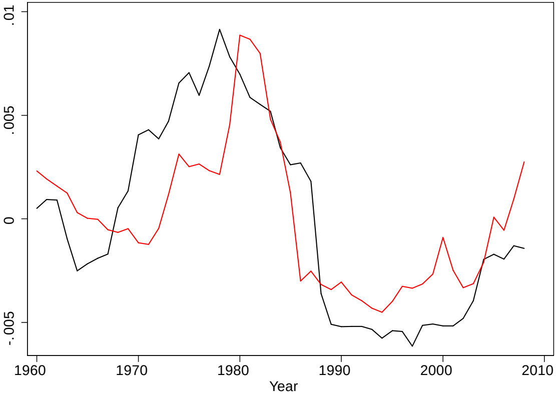
- Reminder from our islands economy
  - Suppose economy is made up of n units (firms or industries)
  - $\log GDP_t = \sum_i \frac{S_{it}}{GDP_t} \log(A_{it})$
  - $\log\left(\frac{GDP_{t+1}}{GDP_t}\right) \approx \sum_i \frac{S_{it}}{GDP_t} \log\left(\frac{A_{i,t+1}}{A_{it}}\right)$
- Suppose $\frac{A_{i,t+1}}{A_{it}}$ are i.i.d. across time and industries, with standard deviation σ_i

$$\text{SD}\left[\log\left(\frac{GDP_{t+1}}{GDP_t}\right)\right] \approx \left[\sum_i \left(\frac{S_{it}}{GDP_t}\right)^{\frac{1}{2}} (\sigma_i)^2\right]^{1/2}$$

- Potentially
  - productivity shocks are correlated, have volatilities that change over time.
  - things besides industries' TFP change from one period to the next

Fundamental Volatility

$$\sigma_{Ft} = \left[\sum_i \left(\frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2}$$



Outline

- Definitions and data sources
  - TFP in each industry
  - σ_{GDP}
- Fundamental volatility accounts for the break in GDP volatility.
- Sources of fundamental volatility
- Fundamental volatility and GDP volatility in other countries

Industry TFP Volatility

- KLEMS data from Dale Jorgenson (http://hdl.handle.net/1902.1/11155)
- For each industry×year define TFP growth as:

$$\begin{aligned}\Delta TFP_{it} = & \log\left(\frac{y_{it+1}}{y_{it}}\right) \\ & - \frac{1}{2}\left(s_{it}^{k} + s_{it+1}^{k}\right)\log\left(\frac{k_{it+1}}{k_{it}}\right) \\ & - \frac{1}{2}\left(s_{it}^{l} + s_{it+1}^{l}\right)\log\left(\frac{l_{it+1}}{l_{it}}\right) \\ & - \frac{1}{2}\left(s_{it}^{m} + s_{it+1}^{m}\right)\log\left(\frac{m_{it+1}}{m_{it}}\right)\end{aligned}$$

  where s_{it}^{l} (s_{it}^{m}, s_{it}^{k}) is industry i's cost share of labor (intermediate inputs, capital) at time t.
- $\sigma_i \equiv \text{SD}(\Delta TFP_{it})$

GDP Volatility

Three measures:

1) Rolling standard deviation

$$\sigma_t^{roll} = \text{SD}\left(y_{t-10}^{HP}, \ldots y_{t+10}^{HP}\right), \text{ where}$$

y_t^{HP} is deviation of log GDP from trend

2) Instantaneous standard deviation

$$\Delta y_s = \psi + \phi \Delta y_{s-1} + \epsilon_s$$

$$\sigma_t^{Inst} \equiv \frac{1}{2}\sqrt{\frac{\pi}{2}} \sum_{q=1}^{4} |\hat{\epsilon}_{t,q}|$$

3) σ_t^{HP} is the HP smoothed version of σ_t^{Inst}

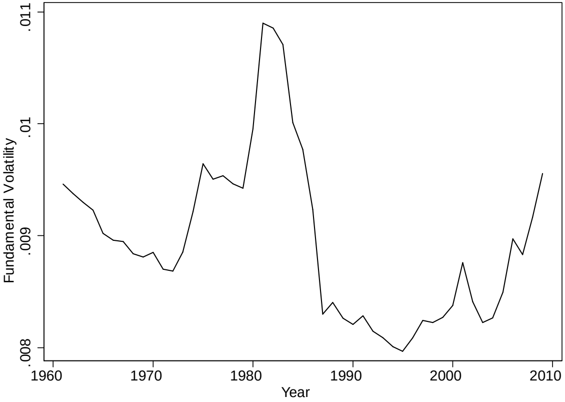
Fundamental Volatility accounts for the break in GDP volatility.

$$LR_T = \frac{\prod_{t=1960}^{T} f_1(\eta_t) \prod_{T+1}^{2008} f_2(\eta_t)}{\prod_{t=1960}^{2008} f_0(\eta_t)}$$

	$\sigma_{Yt}^{inst} = a + \eta_t$		$\sigma_{Yt}^{inst} = a + b\sigma_{Ft} + \eta_t$	
H_0	No break in a		No break in b	No break in a or b
$\max_T LR_T$	26.50	8.32	8.64	8.91
Reject null?	Yes	No	No	No
Estimated break date	1983	NA	NA	NA

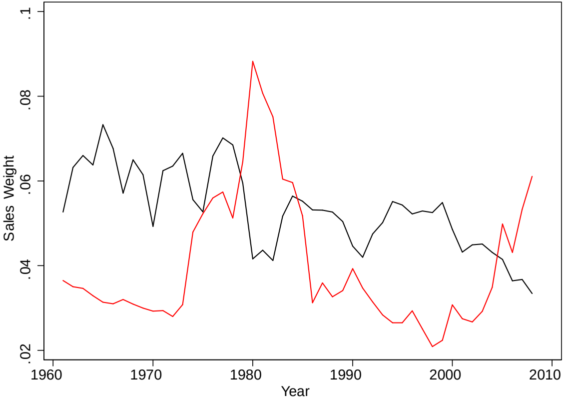
Fundamental Volatility

$$\sigma_{Ft} = \left[\sum_i \left(\frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2}$$



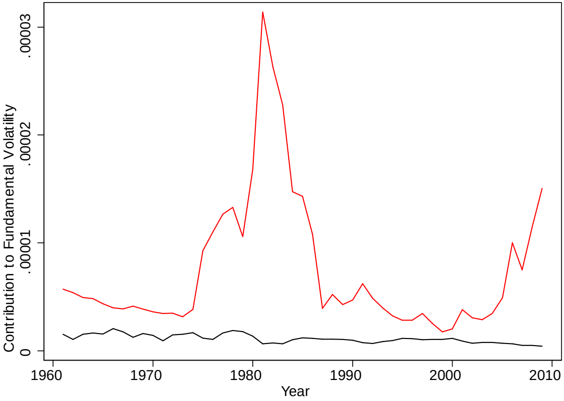
Sales weights: motor vehicles and petroleum

$$\sigma_{Ft} = \left[\sum_i \left(\frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2}$$



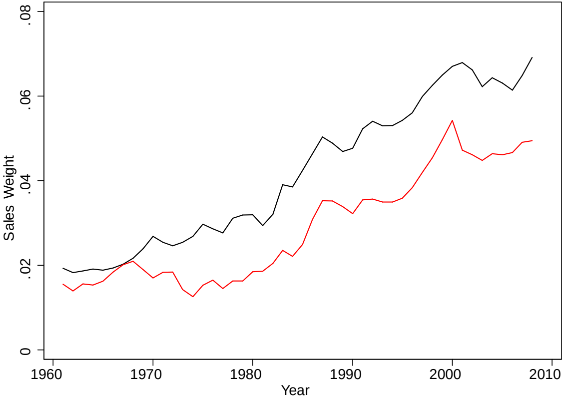
Contribution to fundamental volatility: motor vehicles and petroleum

$$\sigma_{Ft} = \left[\sum_i \left(\frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2}$$



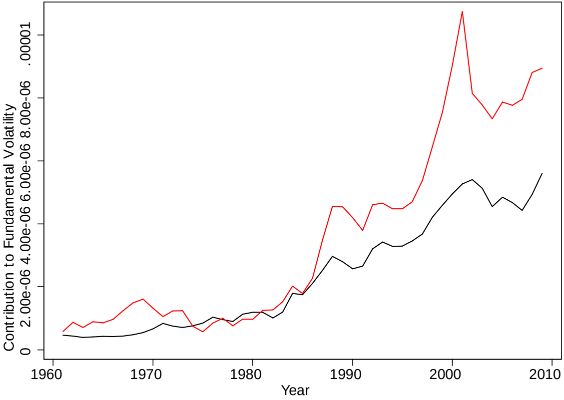
Sales weights: depository and nondepository financial institutions

$$\sigma_{Ft} = \left[\sum_i \left(\frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2}$$

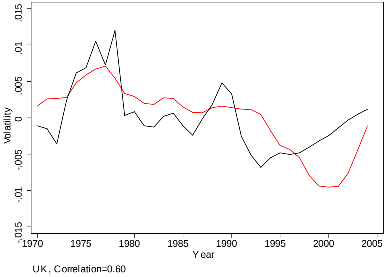


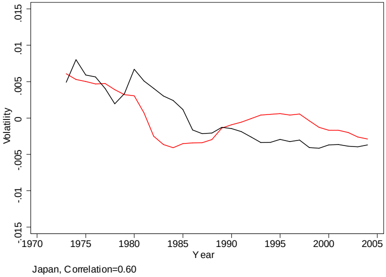
Contribution to fundamental volatility: depository and nondepository financial institutions

$$\sigma_{Ft} = \left[\sum_i \left(\frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2}$$

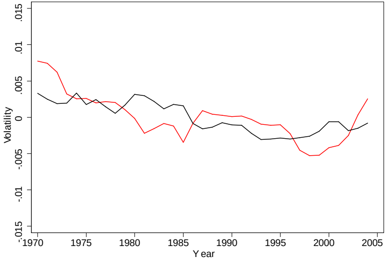


Fundamental volatility tracks GDP volatility in other countries

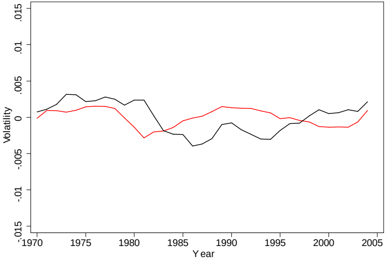




Fundamental volatility tracks GDP volatility in other countries



Germany, Correlation=0.54



France, Correlation=0.03

Conclusion

Summary:

- GDP volatility changes over time.
- Volatility changes reflect changes in the importance of different types of firms in the economy.
  - Implies that firm/industry-level shocks are important for aggregate volatility.

Next steps:

- To what extent are the economy's shocks independent across firms (or industries)?