Problem Set 4: Due Wednesday, November 12

Note: For this problem set, you only need to complete problem 1 plus *either* problem 2 or problem 3.

Problem 1

Consider a competitive industry wherein firms sell output at a constant price $p = 1$ and produce using a Cobb-Douglas production function of the form:

$$\begin{aligned} Y_{it} &= A_{it} K_{it}^{\alpha} L_{it}^{1-\alpha}, \text{ where} \\ \log A_{it} &= \gamma_i + \omega_{it}, \text{ and} \\ \omega_{it} &= \rho \omega_{i,t-1} + \varepsilon_{it} \end{aligned}$$

1) Construct a simulated sample of 200 firms by 50 periods. In your simulation, suppose that

- $\rho = 0.8$ and $\alpha = 0.3$
- γ_i is $\mathcal{N}(0,1)$, independently distributed across firms.
- ε_{it} is $\mathcal{N}(0,0.1)$, independently distributed across firms and periods. Initialize $\omega_{i,0}$ to be equal to 0.
- $\log K_{it}$ and $\log w_{it}$ are each $\mathcal{N}(0,0.1)$, independently distributed across firms and periods.
- The γs, εs, $\log K_{it}$, and $\log w_{it}$ are each uncorrelated to one another.

2) Consider the problem of a firm who is choosing its labor demand in period t. Suppose the information that it has available is γ_i, K_{it}, $\omega_{i,t-1}$, and w_{it} (but not ε_{it}). In other words, the plant has to choose L_{it} before ε_{it} is realized. The expected profits of the firm in period t are equal to

$$\begin{aligned} & \mathbb{E}\left[A_{it} K_{it}^{\alpha} L_{it}^{1-\alpha} - w_{it} L_{it}\right] \\ = \; & \mathbb{E}\left[\exp\left\{\gamma_i + \rho\omega_{i,t-1} + \varepsilon_{it} + \alpha \log K_{it} + (1-\alpha)\log L_{it}\right\} - \exp\left\{\log w_{it} + \log L_{it}\right\}\right] \\ = \; & e^{\gamma_i + \rho\omega_{i,t-1} + \alpha \log K_{it} + (1-\alpha)\log L_{it}} \mathbb{E}\left[\exp\left\{\varepsilon_{it}\right\}\right] - e^{\log w_{it} + \log L_{it}} \end{aligned}$$

Write out for the profit-maximizing choice of $\log L_{it}$ as a function of γ_i, $\log K_{it}$, $\omega_{i,t-1}$, and $\log w_{it}$. Then compute $\log Y_{it}$ for each firm-year.

3) Using your answer to part 2, you should now have a panel of observations of log sales, log labor and log capital inputs. Suppose you ran the OLS regressions of the form

$$\log Y_{it} = \beta_0 + \beta_1 \log K_{it} + \beta_2 \log L_{it} \tag{1}$$

How do your coefficient estimates (β_1 and β_2) compare to the true production function parameters, α and $(1-\alpha)$?

4) Re-run the regression of Equation (1), but now include firm-level fixed effects. Are your estimates any closer to the true production function parameters?

5) Note that, by assumption, w_{it} are uncorrelated to ε_{it} and γ_i. Using this fact, describe how you can construct an consistent two-step estimator of α. Run the two-stage regression that you have formulated. Are your estimates any closer to the true production function parameters?

Problem 2

Reproduce, as closely as you can, Figure 2 of Caballero and Hammour (1994). The authors describe their algorithm to solve for $\dot{\phi}(t)$ and $\dot{\bar{a}}(t)$ for a given path of $\dot{\bar{D}}(t)$ on pages 1365-1366. From here, use the definitions $CC(t) \equiv \frac{\dot{\phi}(t)}{N(t)}$ and $DD(t) \equiv \delta + \frac{f(\bar{a}(t),t)\left[1-\dot{\bar{a}}(t)\right]}{N(t)}$. Plot $CC(t)$ and $DD(t)$.

Include, in your answer to this problem, your code (in whatever programming language you prefer).

Problem 3

In this problem, we will explore the investment dynamics in the model discussed in Caballero and Engel (1999).

The course webpage contains code that computes the value function defined by Equations 5 and 6 of the paper (integrating over the entire distribution of ω).

1. In the code what does the gamcdf function do? What about the circshift function?

2. Draw a sequence of 50 normally distributed random variables with mean 0 and standard deviation 0.1. Using ...

- the realization of these random variables as ν_t

- the invariant distribution of x as $f(x,0)$, and
- Equations 15, 16, and 18 of the paper

  ... compute the path of aggregate $\frac{I_t^A}{K_t^A}$ for $t \in \{1, ..., 50\}$.

3. Repeat problem 2 for different combinations of μ_ω, cv_ω: {0.1, 0.16}, {0.3, 0.16}, {0.1, 0.3}, {0.3, 0.3}. Use the *same realization* of the aggregate ν_t shocks (a random seed may be helpful). For each μ_ω, cv_ω combination plot $\frac{I_t^A}{K_t^A}$ for $t \in \{1, ..., 50\}$. In addition, plot the v_t shocks.

Include, in your answer to this problem, your MATLAB code.