Kaldor Facts & Kuznets Facts

Kaldor Facts

- 1. $\frac{Y}{L}$ grows at a sustained rate 2. $\frac{K}{L}$ grows at a sustained rate

$$(1) + (2) \Rightarrow \frac{Y}{K}$$
 is roughly stable.

3.
$$r = i - \pi$$
 is stable

- 4. The capital and labor shares of national income are stable (roughly $\frac{1}{3}$ and $\frac{2}{3}$)
- 5. Y per capita grows at a stable rate
- Kuznets Facts: As economies grow, the shares of income/consumption in services grow, in agriculture shrink, and in manufacturing are roughly constant (grow and then shrink).

Labor Share of Income



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Labor Share of Income



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Labor Share of Income



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Labor Share of Income: Other Countries



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Ratio of K/Y



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Real Return of S&P 500

Period	Return
1930-1950	4.8%
1950-1970	9.2%
1970-1990	4.7%
1990-2010	6.2%

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Kuznets Facts for the US



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Agriculture Value Added Share of GDP



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Manufacturing Value Added Share of GDP



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Services Value Added Share of GDP



Note: We'll go over the following papers on the board

▶ Kongsamut, Rebelo, Xie (2001), "Beyond Balanced Growth"

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 Ngai and Pissarides (2007), "Structural Change in a Multisector Model of Growth" Notes on Herrendorf et al. (2013): "Two Perspectives on Preferences and Structural Transformation"

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Review: two views of structural transformation

- Facts:
 - Agriculture shrinks, manufacturing first grows and then shrinks, services grow.
 - These shifts are more pronounced in nominal rather than real terms.
- Ngai and Pissarides
 - Differential growth rates in sectors' productivity.
 - Nonunitary elasticity of substitution across goods.
 - Low-growth sector (services) has larger relative prices; draws more resources into the economy.
- Kongsamut et al.
 - Identical productivity growths.
 - Nonunitary income elasticity for different goods.
 - Agriculture has subunitary elasticity of substitution; services has income elasticity > 1.
- In these papers, there was little distinction between commodities and the industries that produced them.

Contribution of Herrendorf, Rogerson, and Valentinyi

- Construct and estimate a model that nests Ngai and Pissarides and Kongsamut et al.
- Show that the attribution of transformation to income/price effects depends on how we view what consumers value:
 - 1. "Final Consumption Expenditures": $u(c_a, c_m, c_s)$
 - c_a: food and beverages purchases or off-premises consumption
 - c_m: goods, excluding food and beverages...
 - cs: services; government consumption expenditure
 - 2. "Consumption Value Added": $u(c_a, c_m, c_s)$
 - c_a: farms; forestry, fishing
 - c_m: construction; manufacturing; mining
 - c_s: all other industries
- Provide a link between the two perspectives.

Outline

- 1. Model
- 2. Data
- 3. Estimation using the "Final Consumption Expenditures" perspective
- 4. Estimation using the "Consumption Value Added" perspective

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5. Linking the two perspectives.

Model (1)

Consider the problem of a consumer who is trying to maximize:

$$u(c_{at}, c_{mt}, c_{st}) = \left(\sum_{i \in \{a, m, s\}} \omega_i^{\frac{1}{\sigma}} (c_{it} + \bar{c}_i)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$
subject to
$$\sum_{i \in \{a, m, s\}} p_{it} c_{it} = C_t$$

Note:

- If $\bar{c}_i = 0 \Rightarrow$ Preferences as in Ngai and Pissarides.
- If $\sigma = 1$ and $\bar{c}_m = 0 \Rightarrow$ Preferences as in Kongsamut et al.
- Nothing about the technology side of the economy is explicitly specified.
- Intertemporal decisions play little/no role.

Model (2)

Solving the static problem from the previous slide:

$$\frac{p_{mt}c_{mt}}{C_{t}} = -\frac{p_{mt}\bar{c}_{m}}{C_{t}} + \frac{\omega_{m}p_{mt}^{1-\sigma}}{\sum_{i\in\{a,m,s\}}\omega_{i}p_{it}^{1-\sigma}} \left(1 + \sum_{i\in\{a,m,s\}}\frac{p_{it}\bar{c}_{i}}{C_{t}}\right)$$
(1)
$$\frac{p_{st}c_{st}}{C_{t}} = -\frac{p_{st}\bar{c}_{s}}{C_{t}} + \frac{\omega_{s}p_{st}^{1-\sigma}}{\sum_{i\in\{a,m,s\}}\omega_{i}p_{it}^{1-\sigma}} \left(1 + \sum_{i\in\{a,m,s\}}\frac{p_{it}\bar{c}_{i}}{C_{t}}\right)$$
(2)

- The equation for $p_{at}c_{at}/C_t$ is redundant.
- Taking the model to the data
 - Parameters: ω_a , ω_m , σ , \bar{c}_a , \bar{c}_s
 - Data: Time series on p_{mt}c_{mt}, p_{st}c_{st}, p_{at}, p_{mt} and p_{st},
 - Fit Equations (1) and (2) as best as possible.

Data Sources

- Consumption Final Expenditure Data $(p_{st}^f c_{st}^f \text{ and } p_{st}^f)$
 - National Income Product Accounts: Values and Quantity Indices (see http://www.econstats.com/nipa/)
- Consumption Value Added Data:
 - Bureau of Economic Analysis Industry Accounts: Value Added and Quantity Indices by Industry.
 - Need to subtract off investment from the production value added data. (Investment goods produced by all industries, not just manufacturing)
 - ▶ In previous papers $c_m + \dot{k} \delta k = m$. But, after 2002 $\dot{k} - \delta k > m!$
 - BEA: 2002 Table of service shares for different types of investment goods.
- Bureau of Economic Analayis Input-Output Tables: (Useful in Linking FE and VA perspectives.)

Final Expenditures Data



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Quantity goes up most for manufacturing, least for food.

Prices goes up most for services, least for manufacturing.

Estimating Final Consumption Expenditure Preferences

	(1)	(2)	(3)
σ	0.85	1	0.89
- <i>C</i> _a	-1350	-1316	
īc _s	11237	19748	
ω_a	0.02	0.02	0.11
ω_m	0.17	0.15	0.24
ω_s	0.81	0.84	0.65
$\chi^2(\bar{c}_a=0,\bar{c}_s=0)$	3867	4065	
AIC	-932.55	-931.35	-666.03

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Note: AIC= $2k - 2 \log \mathcal{L}$

Income effects are important in fitting expenditure share data



Income Fixed at 1947 Values



Nonhomotheticity terms:

	1947	2010
$p_a \bar{c}_a / C$	-0.17	-0.04
$p_s \bar{c}_s / C$	0.73	0.32

Fit of estimated model, $\bar{c}_a = \bar{c}_s = 0$



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• $\{\hat{\sigma}, \omega_a, \omega_m, \omega_s\} = \{0.89, 0.11, 0.24, 0.65\}$

Value Added Data



 Correlation between prices indices and quantity indices is much stronger in the value added data (89%) than in the final expenditure data (48%).

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Estimating Value Added Preferences

	(1)	(2)	(3)
σ	0.00	0	0
- <i>C</i> _a	-138.7	-138.9	
\bar{c}_s	4261.8	4268.1	
ω_a	0.002	0.002	0.01
ω_m	0.15	0.15	0.18
ω_s	0.85	0.85	0.81
$\chi^2(\bar{c}_a=0,\bar{c}_s=0)$	1424	216	
AIC	-837.3	-875.4	-739.4

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Income and price effects are both important in fitting the value added data



Nonhomotheticity terms:

	1947	2010
$p_a \bar{c}_a / C$	-0.08	-0.01
$p_s \bar{c}_s / C$	0.34	0.12

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Why are the \bar{c}_a , \bar{c}_s terms less important?

- Consumption over Commodities' Final Expenditure
 - Food from supermarkets is an agricultural commodity ($\bar{c}_a < 0$)
 - Meals from restaurants is a service $(\bar{c}_s > 0)$
- Consumption over Industries' Value Added
 - Both food from supermarkets and food from restaurants are produced by the agriculture industry; c
 _a & c
 _s balance out.

Linking the two approaches: theory

 Assume that final added consumption is a CES aggregate of value added from the three sectors:

$$c_{it}^{f} = \left[\sum_{j \in \{a,m,s\}} \left(A_{it}\phi_{j \to i}\right)^{\frac{1}{\eta_{i}}} \left(c_{j \to i,t}^{\vee}\right)^{\frac{\eta_{i}-1}{\eta_{i}}}\right]^{\frac{\eta_{i}}{\eta_{i}-1}}$$

Cost minimization of the "final expenditure bundler" implies that:

$$p_{j}^{v}c_{j\to i,t}^{v} = \frac{\phi_{j\to i}\left(p_{j}^{v}\right)^{1-\eta_{i}}}{\sum_{k\in\{a,m,s\}}\phi_{k\to i}\left(p_{k}^{v}\right)^{1-\eta_{i}}}p_{i}^{f}c_{it}^{f}$$
(3)

- Taking the model to the data
 - Parameters: $\eta_i, \phi_{j \to i}; i, j \in \{a, m, s\}$.
 - Fit Equation (3) as best as possible, separately for each i ∈ {a, m, s}.

Linking the two perspectives: data

How are the $p_j^v c_{j \rightarrow i,t}^v$ constructed?

- Bureau of Economic Analysis "Total Requirements" Tables
 - For firms producing commodity j, what is the total value of purchases from industry i?
 - What is, $p_i^v c_i^v$, the value added of firms within industry *i*?
 - What is, $p_j^f c_j^f$, the value of final consumption of commodities j?
- Define $T_{ij} = \frac{\text{purchases of commodity } j \text{ for firms producing in } i}{\text{value added in } i + \text{ total purchases of firms in } i}$
- *ji* element of (I − T)⁻¹: number of dollars of value added in industry *j* for producing a dollar of final expenditure of commodity *i*. Note (I − T)⁻¹ = I + T + T² + T³ +
- Using this definition:

$$p_{j}^{v}c_{j\to i}^{v} = \left((I-T)^{-1}\right)_{ji}p_{j}^{f}c_{i}^{f}$$

An example from the data

How are the $p_j^v c_{j \to i,t}^v$ constructed? BEA "Total Requirements" Tables, from 1963

	Agric.	17818	0	326	1112	25641	259	3410
	Min'g	128	1138	737	3686	10949	46	2914
	Const.	567	416	25	588	814	1556	10906
10	Durab.	795	1081	27329	97129	8018	3160	6299
IU Tables	N-Dur	6851	588	4234	11582	69029	6683	17745
Table:	Trans.	2795	876	9789	11605	12615	7278	11526
	Serv.	4774	3529	5814	14041	15974	26717	60931
	VA	22702	11050	37022	95905	75063	112320	233569
	Agric.	1.50	0.02	0.04	0.04 (0.26 (0.02 0.0	4
	Min'g	0.02	1.07	0.03	0.04 (0.08	0.01 0.0	2
	Const.	0.02	0.03	1.01	0.01 (0.02 (0.02 0.0	4
$(I - T)^{-1} =$	Durab.	80.0	0.18	0.57	1.71 (0.13 (0.06 0.0)7
	N-Dur	0.29	80.0	0.14	0.15	1.52 (0.09 0.1	.1
	Trans.	0.11	0.07	0.17	0.11 (0.12 1	L.07 0.0	6
	Serv.	0.21	0.26	0.18	0.17 (0.21 (0.23 1.2	5

An example from the data

How are the $p_i^v c_{i \to i,t}^v$ constructed? BEA "Total Requirements" Tables, from 1963:

	Agric.	1.50	0.02	0.04	0.04	0.26	0.02	0.04
	Min'g	0.02	1.07	0.03	0.04	0.08	0.01	0.02
	Const.	0.02	0.03	1.01	0.01	0.02	0.02	0.04
$(I - T)^{-1} =$	Durab.	0.08	0.18	0.57	1.71	0.13	0.06	0.07
	N-Dur	0.29	0.08	0.14	0.15	1.52	0.09	0.11
	Trans.	0.11	0.07	0.17	0.11	0.12	1.07	0.06
	Serv.	0.21	0.26	0.18	0.17	0.21	0.23	1.25

In 1963, each dollar of final expenditures in agriculture generates 0.21 dollars of value added in services, 0.11 in transport.

Since $p_{A,1963}^{f}c_{A,1963}^{f} = 348 per capita, we have that $p_{S \to A}^{v}c_{S \to A}^{v} = $348 \cdot (0.21 + 0.11) = 111

Estimates of the production commodities Reminder:

$$p_{j}^{\mathsf{v}}c_{j\to i,t}^{\mathsf{v}} = \frac{\phi_{j\to i}\left(p_{j}^{\mathsf{v}}\right)^{1-\eta_{i}}}{\sum_{k\in\{a,m,s\}}\phi_{k\to i}\left(p_{k}^{\mathsf{v}}\right)^{1-\eta_{i}}}p_{i}^{\mathsf{f}}c_{it}^{\mathsf{f}}$$

	Food	Goods	Services
η_i	0.19*	0.00	0.00
$\phi_{a \rightarrow i}$	0.05*	0.02*	0.01*
$\phi_{m \rightarrow i}$	0.33*	0.36*	0.09*
$\phi_{s \rightarrow i}$	0.62*	0.62*	0.90*

- Except for agriculture, production of final expenditures is Leontief.
- Services are an important input in all commodities.
- Agriculture is relatively unimportant in the production of the three commodities.

Linking η , σ^V , and σ^D

Two alternative chains of substitution

- σ^v: elasticity of substitution between products produced in the service vs. manufacturing sectors
- σ^{f} : elasticity of substitution between goods and services
- ηⁱ: elasticity of substitution, across different industries' value added, when making final expenditure commodity i
- From Oberfield and Raval:

$$\sigma^{\nu} \approx \chi \sigma^{f} + (1 - \chi) \,\bar{\eta},$$

where $\chi = \mathrm{index}$ of cross-industry heterogeneity in producing different commodities.

• In our context,
$$\chi$$
, $\bar{\eta} \approx 0$, $\sigma^f \approx 0.9 \Rightarrow \sigma^v \approx 0$.

Conclusion (1)

Summary

- ▶ To fit the growth of service FE, and the decline of food FE \Rightarrow income effects are important.
- ► To link FE data and VA added data ⇒ complementarity in production of fixed expenditures.
- Next Steps
 - What productivity trajectories will generate the observed relative price movements?
 - Look at within-sector price & quantity paths.
 - Are they similar across industries, within sectors?
 - What are the within-industry productivity paths?

Conclusion (2)

What are the underlying productivity paths?

Herrendorf, Herrington, and Valentinyi (2014)

Production functions of the form:

$$G_{it} = [F_{it} (K_{it}, L_{it})]^{\eta_i} [X_{it}(Z_{it})]^{1-\eta_i}, \text{ where}$$

$$F_{it} = \left[\alpha_i \left[\exp\left(\gamma_{ik}t\right) K_{it}\right]^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i) \left[\exp\left(\gamma_{il}t\right) L_{it}\right]^{\frac{\sigma_i - 1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i - 1}}$$

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• Main result: $\gamma_{AI} > \gamma_{MI} > \gamma_{SI}$; $\sigma \approx 1$ fit the price data well.

Conclusion (3)

Substantial differences within Services

	Prices: %	Quantity: %
	Ann. Growth	Ann. Growth
GDP	3.5%	3.4%
Wholesale	1.9%	4.8%
Retail	2.7%	3.6%
Transportation	2.9%	2.9%
Information	2.5%	5.3%
Finance & Insurance	5.0%	4.0%
Real Estate	3.7%	4.0%
Professional Services	5.3%	4.5%
Management	4.2%	3.0%
Administration	4.6%	5.3%
Education	5.8%	3.0%
Health	5.3%	4.2%
Arts & Entertainment	4.2%	3.4%
Accommodation	4.0%	3.1%
Other Services	4.9%	1.6%
Notes on Caselli and Coleman (2001) "The U.S. Structural Transportation and Regional Convergence: A Reinterpretation"

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Review: Regional Convergence



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Red: South, Blue: Midwest, Green: West

Caselli-Coleman Hypotheses

- 1. South has a comparative advantage in agricultural products.
- Declining cost of education ⇒ decreases relative labor supply in agriculture.
- Income elasticity of food less than 1; faster technological growth in agriculture ⇒ lowers labor demand in agriculture.

 $(2) + (3) \Rightarrow$ Possible to have decline in labor share of agriculture and increase in relative wage of agriculture. Both are important components of regional convergence.

Census Regions



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Region × Sector



Black: North, Red: South, Blue: Midwest, Green: West

Region × Education Status



Sector × Education Status



Solid: Agriculture, Dash: Manufacturing, Dot: Services

Last two slides:

- High school dropouts: $75\% \rightarrow 15\%$.
- Education status is a bit lower in the South, much lower in agriculture, much higher in services.

Income by Sector



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Income by Education Status



- Last two slides:
 - Income lowest in agriculture; gap decreases over time.
 - Income increases with education; education premia increase beginning in the 80s.
 - Within education × sector : highest in Northeast, lowest in South.

Accounting for Regional Convergence

Wages of South approach those of the North due to 3 effects

- 1. Industry Shares of the South \rightarrow Industry Shares of the North (Labor Reallocation)
- Wage of Agriculture → Wage of Manufacturing/Services (Between Industry)
- Wage of Agriculture (or Manufacturing/Services) in South → Wage of Agriculture (or Manufacturing/Services) in North (Within Industry)

$$\begin{aligned} \text{Total:} \ \Delta \frac{w^{S} - w^{N}}{w} &= \Delta \frac{w_{ft}^{S} L_{ft}^{S} + w_{mt}^{S} \left(1 - L_{ft}^{S}\right) - w_{ft}^{N} L_{ft}^{N} - w_{mt}^{N} \left(1 - L_{ft}^{N}\right)}{w_{ft} L_{ft} + w_{mt} \left(1 - L_{ft}\right)} \\ (1): \ \frac{w_{ft}^{S} - w_{mt}^{S}}{w_{t}} \left(L_{ft}^{S} - L_{f,t-1}^{S}\right) - \frac{w_{ft}^{N} - w_{mt}^{N}}{w_{t}} \left(L_{ft}^{N} - L_{f,t-1}^{N}\right) \\ (2): \ \Delta \frac{w_{f} - w_{m}}{w} \cdot \left(\frac{1}{2} \left[L_{ft}^{S} + \frac{1}{2} L_{f,t-1}^{S}\right] - \frac{1}{2} \left[L_{ft}^{N} + L_{f,t-1}^{N}\right]\right) \end{aligned}$$

Accounting for Regional Convergence



- All three forces (reallocation, within, and between) are important in accounting for South-North convergence.
 - Between industry: Early in the sample period
 - Within + Reallocation: Later in the sample period

Model: Overview

- Two regions: North and South
 - Agriculture takes place in the South. Manufacturing in either region.
- ► Exogeneous productivity growth ⇒ Increased income raises relative demand for manufactured goods.
 - Reallocation Effect
- Decision on whether to accumulate HC (and work in manufacturing)
 - Decline in cost of going to school decreases labor supply in agriculture

• Reallocation Effect + Between Industry Effect.

Model: Production (1)

- Two goods: food (F) and manufacturing (M).
- Capital and labor are perfectly mobile across industries and across regions.
 - Labor is used in manufacturing, agriculture, or accumulating human capital.

$$L_{ft}^{S} + \underbrace{L_{mt}^{N} + L_{mt}^{S}}_{L_{mt}} + L_{et} = 1$$

$$K_{ft}^{S} + \underbrace{K_{mt}^{N} + K_{mt}^{S}}_{K_{mt}} = 1$$

Manufactured goods can be consumed or invested:

$$c_{mt} + K_{t+1} = M_t^N + M_t^S + (1 - \delta) K_t$$

Food can only be consumed

$$c_{ft} = F_t^S$$

Model: Production (2)

• Two locations: $i \in \{\text{South, North}\}$.

$$M_{t}^{S} = A_{mt} \left(T_{mt}^{S}\right)^{\alpha_{T}} \left(L_{ft}^{S}\right)^{\alpha_{L}} \left(K_{ft}^{S}\right)^{1-\alpha_{T}-\alpha_{L}}$$
$$M_{t}^{N} = A_{mt} \left(T_{mt}^{N}\right)^{\alpha_{T}} \left(L_{ft}^{N}\right)^{\alpha_{L}} \left(K_{ft}^{N}\right)^{1-\alpha_{T}-\alpha_{L}}$$
$$F_{t}^{S} = A_{ft} \left(T_{ft}^{S}\right)^{\beta_{T}} \left(L_{ft}^{S}\right)^{\beta_{L}} \left(K_{ft}^{S}\right)^{1-\beta_{T}-\beta_{L}}$$

• Let
$$g_{mt} \equiv \frac{A_{m,t+1}-A_{mt}}{A_{mt}}$$
; $g_{ft} \equiv \frac{A_{f,t+1}-A_{ft}}{A_{ft}}$

- Land is perfectly mobile across industries.
 - ► Total supply in each region is fixed: ω in the South; 1 − ω in the North.
- Note: Because of decreasing returns to mobile factors (capital + labor), manufacturing will occur in both regions.

Model: Preferences

- Each individual *i* belongs to a household; altruistic over household successors
- Preferences over food and manufactured products:

$$U = \sum_{t=0}^{\infty} \beta^{t} u(c_{ft}^{i}, c_{mt}^{i}), \text{ where}$$
$$u(c_{ft}^{i}, c_{mt}^{i}) = \frac{\left[\left(c_{ft}^{i} - \gamma\right)^{\tau} \left(c_{mt}^{i}\right)^{1-\tau}\right]^{1-\sigma}}{1-\sigma}$$

- ► As in the Kongsamut et al. paper, γ > 0 generates non-unitary income elasticities.
- Consumers intertemporal BC:

$$\sum_{t=0}^{\infty} q_t \left(c_{ft}^i + p_t c_{mt}^i \right) = H_0^i$$

- H_0^i : lifetime income
- q_t period-0 price of one unit of the farm good in period t.

Model: Human Capital Accumulation

- Workers, i, are born and die stochastically. λ be the probability of death per period.
- (Only) at birth: *i* decides whether to go to school or not.
- Benefit of school: Can work in manufacturing.
- Cost of school: Spend $\zeta^i \xi_t$ (< 1) periods of time not working.
 - ζ^{i} is a random variable with density $\mu\left(\zeta\right)$
 - ξ_t is the same for all individuals in a period, potentially decreases over time.

Model: Equilibrium Conditions

 Profit maximization by the representative firm in each industry/region.

$$F_1(T_{ft}, L_{ft}, K_{ft}, A_{ft}) = a_t; M_1(T_{ft}, L_{ft}, K_{ft}, A_{ft}) = \frac{a_t}{p_t}, \text{ etc...}$$

 Utility maximization over consumption of food, manufactured goods in each period.

$$\frac{u_2(c_{ft}, c_{mt})}{u_1(c_{ft}, c_{mt})} = p_t \quad ; \beta \frac{u_1(c_{f,t+1}, c_{m,t+1})}{u_1(c_{ft}, c_{mt})} = \frac{q_{t+1}}{q_t}$$

- Markets for land, labor, capital clear each period.
- Utility maximization over schooling choice at age 0.

Model: Equilibrium Conditions

Utility maximization over schooling choice.

• Let $h_{jt} \equiv$ present value of wages in sector j

$$h_{jt} = \sum rac{q_s}{q_t} \lambda^{s-t} w_{js} \ \ ext{for} \ j \in \{f, m\}$$

It is optimal to go to school provided:

$$h_{mt} - \underbrace{\xi_t \zeta^i w_{mt}}_{\text{Lost wages in } 1^{\text{st period}}} \ge h_{ft}$$

• Cutoff value of
$$\zeta'$$
 is $\zeta_t = \frac{1}{\xi_t} \frac{n_{mt} - n_f}{w_{mt}}$

Model: Human Capital Distribution

- From last slide: newborns go to school if $\zeta^i \ge \overline{\zeta}_t$.
- Frac. of newborns being educated is $I_{et}^{0} = \int_{0}^{\bar{\zeta}_{t}} \xi_{t} \zeta^{i} \mu\left(\zeta^{i}\right) d\zeta^{i}$
- Frac. of newborn graduates is $I_{mt}^{0} = \int_{0}^{\zeta_{t}} (1 \xi_{t} \zeta^{i}) \mu(\zeta^{i}) d\zeta^{i}$
- Frac. of newborn farmers is $I_{ft}^{0} = \int_{\bar{\zeta}_{t}}^{\infty} \mu\left(\zeta^{i}\right) d\zeta^{i}$
- Farmers' evolution:

$$L_{ft} = L_{f,t-1}\lambda + I_{ft}^0 \left(1 - \lambda\right)$$

Manufacturers' evolution:

$$L_{mt} = (L_{m,t-1} + L_{e,t-1})\lambda + I_{mt}^{0}(1-\lambda)$$

Labor spent in education:

$$L_{et} = I_{et}^0 \left(1 - \lambda\right)$$

Calibration

eta , δ	0.60, 0.36	Discount factor, Depreciation rat						
g_m, g_{f0}	0.084, 0.168	Nonfarm, Farm TFP growth						
α_T , α_L	0.19, 0.60	Cost shares in farming						
β_T , β_L	0.06, 0.60	Cost shares in manufacturing						
ω	0.75	Land share in South						
	Model with co	nstant ξ						
γ	0.2205	Non-homotheticity parameter						
K_0	0.0711	Initial capital stock						
ξ_0 and ξ_∞ 2.0375		Education cost parameter						
	Wodel with de	clining ξ						
γ	0.2201	Non-homotheticity parameter						
K_0	0.0712	Initial capital stock						
ξ0	1.8977	Education cost in the year 1880						
ξ_{∞}	0.1239	Education cost in the year 2190						

Results

Variable	Data	Constant	Declining		
variable	Data	Costs	Costs		
$(c_f/c)_{1880}$	0.31*	0.31	0.31		
$(c_f/c)_{1980}$	0.014	0.03	0.08		
L _{f,1880}	0.50*	0.50	0.50		
$L_{f,1980}$	0.03	0.33	0.10		
p_{1880}/p_{1980}	~1.0	0.16	1.14		
$(w_f/w_m)_{1880}$	0.20*	0.20	0.20		
$(w_f/w_m)_{1980}$	0.69†	0.03	0.69		
$(w^{S}/w^{N})_{1880}$	0.41*	0.41	0.41		
$(w^{S}/w^{N})_{1980}$	0.90	0.56	0.97		

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Results

Variable	Constant	Declining		
Growth Rates	Costs	Costs		
South/North population	0.36%	-0.34%		
Farm capital/labor ratio	-0.69%	2.43%		
Farm land/labor ratio	-1.47%	0.94%		
Nonfarm capital/labor ratio	1.13%	0.99%		
Nonfarm land/labor ratio	0.38%	-0.34%		

Results

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Growth Rates	Costs	Costs		
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Summary

- Agriculture is a geographically concentrated, low education activity
- Decreasing cost of education + Decreasing relative demand for food

 \Rightarrow Higher relative wages in agriculture + Reallocation away from agriculture

 \Rightarrow Regional Convergence.

Relative Price of Education Services Is Increasing



Metro Status by Region



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Notes on Karabarbounis and Neiman (2014) "The Global Decline of the Labor Share"

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Review: Labor Share of Income



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Relative Price of Capital Is Falling, Especially After 1980



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A complication when computing the labor share

How do you classify entrepreneurs" income? Taxes?

Line		2012	2012	2012	2012	2013	2013	2013	2013	2014
Line		I	II	III	IV	I	II	Ш	IV	I
1	Gross domestic income	16,104.6	16,150.3	16,269.6	16,522.0	16,690.9	16,847.8	17,004.6	17,181.4	17,121.3
2	Compensation of employees, paid	8,522.3	8,562.6	8,599.5	8,795.5	8,756.1	8,844.0	8,896.8	8,973.8	9,049.5
3	Wages and salaries	6,850.3	6,882.3	6,913.2	7,094.6	7,048.2	7,126.1	7,171.3	7,237.7	7,301.2
4	To persons	6,836.1	6,867.3	6,898.4	7,080.0	7,033.8	7,111.0	7,156.2	7,222.5	7,286.5
5	To the rest of the world	14.1	15.0	14.8	14.6	14.4	15.1	15.1	15.2	14.8
6	Supplements to wages and salaries	1,672.1	1,680.3	1,686.2	1,700.9	1,707.9	1,717.8	1,725.5	1,736.2	1,748.3
7	Taxes on production and imports	1,124.4	1,122.2	1,118.8	1,126.3	1,140.7	1,138.8	1,149.0	1,158.3	1,166.7
8	Less: Subsidies 1	57.8	57.6	56.0	57.7	58.0	58.9	59.1	58.7	56.8
9	Net operating surplus	4,008.1	3,989.4	4,052.2	4,083.0	4,248.2	4,292.0	4,358.2	4,416.9	4,240.1
10	Private enterprises	4,032.5	4,015.5	4,080.7	4,114.8	4,283.7	4,331.0	4,399.6	4,461.3	4,285.5
- 11	Net interest and miscellaneous payments, domestic industries	613.6	580.8	611.7	583.3	630.3	591.7	615.5	638.8	633.4
12	Business current transfer payments (net)	115.7	110.0	102.6	99.5	121.9	125.8	120.1	129.9	122.5
13	Proprietors' income with inventory valuation and capital consumption adjustments	1,214.4	1,217.8	1,220.0	1,247.5	<mark>1,334.6</mark>	1,341.5	1,360.7	1,358.5	1,359.5
14	Rental income of persons with capital consumption adjustment	524.8	537.8	546.7	555.4	574.9	587.7	596.6	603.2	611.9
15	Corporate profits with inventory valuation and capital consumption adjustments, domestic industries	1,564.0	1,569.1	1,599.8	1,629.1	1,622.1	1,684.3	1,706.8	1,730.9	1,558.4
16	Taxes on corporate income	437.2	429.7	439.1	433.2	408.2	418.2	417.8	431.1	458.9
17	Profits after tax with inventory valuation and capital consumption adjustments	1,126.8	1,139.4	1,160.7	1,196.0	1,213.8	1,266.1	1,289.0	1,299.8	1,099.5
18	Net dividends	569.1	572.5	577.3	735.3	616.6	874.7	769.4	787.8	674.7
19	Undistributed corporate profits with inventory valuation and capital consumption adjustments	557.8	566.9	583.4	460.7	597.3	391.4	519.5	512.0	424.8
20	Current surplus of government enterprises 1	-24.5	-26.1	-28.5	-31.8	-35.5	-39.0	-41.4	-44.3	-45.5
21	Consumption of fixed capital	2,507.6	2,533.7	2,555.1	2,575.0	2,603.8	2,631.9	2,659.6	2,691.0	2,721.9
22	Private	2,018.7	2,041.0	2,059.8	2,077.6	2,103.3	2,128.5	2,153.5	2,180.5	2,208.6
23	Government	488.9	492.7	495.3	497.4	500.5	503.4	506.1	510.5	513.3
	Addendum:									
24	Statistical discrepancy	-63.0	10.1	86.4	-101.7	-155.6	-186.8	-91.7	-91.8	-105.3

Two main contributions of Karabarbounis and Neiman (2014)

 Measurement: Compiling data for corporate labor shares for ~ 60 countries.

Estimation: New method (using cross-sectional data) of estimating capital-labor substitutability (σ ≡ d log(K/N)/d log(W/r)).

Why do we care about σ ?

- Does an increase in ^K/_L increase incentive to innovate in laboror capital-intensive technologies? (Acemoglu, 2002, 2003)
- ► How much of the GDP per capita differences between poor and rich countries is explained by differences in ^K/_L? (Caselli, 2005)
- What are the welfare effects from the observed changes to the labor share? Llater today.

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Outline

- Data sources
- Stylized facts
 - Labor share
 - Relative price of capital
- > Theory: Linking the labor share to the relative price of capital

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 \blacktriangleright Estimating σ and sources of the decline in the labor share

Labor Share Data

Decomposition of GDP

$$Y = \underbrace{Q_{C}}_{\text{Corporate VA}} + Q_{H} + Q_{G} + \text{Tax}_{\text{products}}$$
$$Q_{C} = W_{C}N_{C} + \text{Tax}_{\text{production},C} + \text{ Operating Surplus}_{C}$$

- Total labor share $= \frac{WN}{Y}$
- Corporate labor share= $\frac{W_C N_C}{Q_C}$
- Major data sources
 - Country-specific web pages, UN + OECD websites, books
 - EUKLEMS : Includes data by industry. No seperation into corporate vs. household/government.

Investment Price Data

1. Penn World Tables

$$\xi_{it} = \frac{P_{I,i,t}^{PPP} / P_{I,US,t}^{PPP}}{P_{C,i,t}^{PPP} / P_{C,US,t}^{PPP}} \times \frac{P_{I,US,t}^{BEA}}{P_{C,US,t}^{BEA}}$$

From the second term: incorporate adjustments that the BEA makes for relative improvements the quality of investment/consumption goods.

2. World Bank: World Development Indicators (Fixed Investment Deflator, CPI)

3. EUKLEMS

Both the overall and corporate labor share are declining



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The labor share is declining for most countries



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The labor share is declining for most industries



Changes in the labor share come from "within industry" changes



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Investment Price Decline, Across Data Sources



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Model: Overview

- 1. Goal: Account for the decline of the labor share.
- 2. Two sectors: Producing consumption goods and investment goods.
 - 2.1 Produce using capital & labor with identical production (CES) technologies.

- 2.2 Relative price of the two goods dictated by technology differences (ξ) .
- 2.3 Inputs are supplied by monopolistically competitive (with markup μ) continuum of firms.
- 3. Household side straightforward.
- 4. Key parameter of interest : σ , elasticity of substitution between capital/labor

Model: Household Problem

Maximize

$$\max_{\{C_t, L_t, X_t, K_{t+1}, B_{t+1}\}} \sum \beta^t V(C_t, N_t; \chi_t) \text{ subject to}$$
$$W_t L_t + R_t K_t + \Pi_t = C_t + \xi X_t + B_{t+1} - (1 + r_t) B_t$$
$$K_{t+1} = (1 - \delta) K_t + X_t$$

► FOC for capital:

$$R_{t+1} = \xi_t \left(1 + r_{t+1} \right) - \xi_{t+1} \left(1 - \delta \right)$$

 ξ_t =price of the investment good at time t (more details on the next slide).

Euler Equation:

$$\beta (1 + r_{t+1}) = \frac{V_C (C_t, N_t; \chi_t)}{V_C (C_{t+1}, N_{t+1}; \chi_{t+1})}$$

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Model: Production

► Three products: intermediate inputs z ∈ {0,1}, final investment good X, final consumption good C.

$$C_{t} = \left[\int_{0}^{1} c_{t}(z)^{\frac{\varepsilon_{t}-1}{\varepsilon_{t}}} dz\right]^{\frac{\varepsilon_{t}}{\varepsilon_{t}-1}} ; X_{t} = \frac{1}{\xi_{t}} \left[\int_{0}^{1} x_{t}(z)^{\frac{\varepsilon_{t}-1}{\varepsilon_{t}}} dz\right]^{\frac{\varepsilon_{t}}{\varepsilon_{t}-1}}$$

Intermediate input supplier:

$$y_t(z) = \left(\alpha^{\frac{1}{\sigma}} \left(A_{K,t} k_t(z)\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} \left(A_{L,t} n_t(z)\right)^{\frac{\sigma-1}{\sigma}}\right)^{\sigma/(\sigma-1)}$$

A_{K,t} and A_{L,t} are capital- and labor-augmenting productivity.
▶ Market-clearing conditions:

$$y_t(z) = c_t(z) + x_t(z)$$

$$K_t = \int_0^1 k_t(z) dz$$

$$L_t = \int_0^1 n_t(z) dz$$

Model: Input choices of each intermediate input supplier

Problem of the intermediate input supplier:

$$\max p_t(z)y_t(z) - k_t(z)R_t - n_t(z)W_t$$

▶ First order conditions (For each z):

$$R_{t} = \frac{\partial \left(p_{t}y_{t}\right)}{\partial k_{t}} = \frac{\partial \left(\left(\frac{y_{t}}{Y_{t}}\right)^{-\frac{1}{\varepsilon}}y_{t}\right)}{\partial k_{t}}$$
$$= k_{t}^{-\frac{1}{\sigma}} \frac{\left(Y_{t}\right)^{\frac{1}{\varepsilon_{t}}}}{\mu_{t}} \left(1-\alpha\right)^{\frac{1}{\sigma}} \left(A_{\kappa t}\right)^{\frac{\sigma-1}{\sigma}} \left(y_{t}\right)^{\frac{\varepsilon-1}{\varepsilon}-\frac{\sigma-1}{\sigma}} \Rightarrow$$
$$\mu_{t} R_{t} = \alpha^{\frac{1}{\sigma}} \left(A_{\kappa t}\right)^{\frac{\sigma-1}{\sigma}} p_{t} \left(\frac{k_{t}}{y_{t}}\right)^{-\frac{1}{\sigma}} \Rightarrow \mu_{t} \frac{k_{t} R_{t}}{\frac{y_{t} p_{t}}{s_{\kappa,t}(z)}} = \alpha \left(\frac{A_{\kappa t}}{\mu_{t} R_{t}}\right)^{\sigma-1}$$

► Similarly:

$$\mu_{t} \underbrace{\frac{J_{t} W_{t}}{y_{t} p_{t}}}_{s_{L,t}(z)} = (1 - \alpha) \left(\frac{A_{Lt}}{\mu_{t} W_{t}} \right)^{\sigma - 1}$$

Model: Input choices of each intermediate input supplier From the last slide:

$$\mu_t(z) s_{K,t} = \alpha \left(\frac{A_{Kt}}{\mu_t(z) R_t} \right)^{\sigma-1}$$

But also:

$$s_{\Pi t}(z) \equiv \frac{\Pi_t(z)}{p_t(z) \cdot y_t(z)} = \frac{\mu_t - 1}{\mu_t}$$

Since

$$s_{\Pi t}(z) + s_{Lt}(z) + s_{Kt}(z) = 1$$

 $\mu_t s_{Lt}(z) + \mu_t s_{Kt}(z) = 1$

Thus:

$$1 - \mu_t s_{Lt}(z) = \alpha \left(\frac{A_{\kappa t}}{\mu_t R_t}\right)^{\sigma - 1}$$

Comparing two periods:

$$\left(\frac{1}{1-s_L\mu}\right)\left(1-s_L\left(1+\hat{s}_L\right)\mu\left(1+\hat{\mu}\right)\right) = \left(\frac{1+\hat{A}_K}{1+\hat{R}}\right)^{\sigma-1}\left(1+\hat{\mu}\right)$$

Model: Estimating Equation

From the last slide:

$$\left(rac{1}{1-s_L\mu}
ight)\left(1-s_L\left(1+\hat{s}_L
ight)\mu\left(1+\hat{\mu}
ight)
ight)=\left(rac{1+\hat{A}_{\mathcal{K}}}{1+\hat{R}}
ight)^{\sigma-1}\left(1+\hat{\mu}
ight)$$

From the FOC for capital:

$$1 + \hat{R} = (1 + \hat{\xi}) \cdot \left(1 - \hat{\delta} rac{eta \delta}{1 - eta + eta \delta}
ight)$$

$$\begin{pmatrix} \frac{1}{1-s_L\mu} \end{pmatrix} (1-s_L(1+\hat{s}_L)\mu(1+\hat{\mu}))$$

$$= \left(\frac{1+\hat{A}_K}{1+\hat{\xi}}\right)^{\sigma-1} (1+\hat{\mu})^{\sigma-1} \left(1-\hat{\delta}\frac{\beta\delta}{1-\beta+\beta\delta}\right)^{1-\sigma}$$

So, the labor share can change if ξ, A_K, μ or δ_{\Box} change.

Estimation

Set
$$\mu - 1 = \hat{\mu} = \hat{\delta} = 0$$
. Take logs:
$$\frac{s_L}{1 - s_L} \hat{s}_L = (\sigma - 1) \hat{\xi} + \underbrace{(1 - \sigma) \hat{A}_K}_{\gamma + u}$$

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In the benchmark regressions, assume $\hat{\xi} \perp \hat{A}_{\mathcal{K}}$.

Estimation

$$\frac{s_L}{1-s_L}\hat{s}_L = \gamma + (\sigma - 1)\hat{\xi} + u$$



• Slope: $0.28 \Rightarrow \hat{\sigma} \approx 1.28$.

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Estimation

Investment Price	Labor Share	$\hat{\sigma}$	Obs
PWT	KN Merged	1.25 (0.08)	58
WDI	KN Merged	1.29 (0.07)	54
PWT	OECD & UN	1.20 (0.08)	50
WDI	OECD & UN	1.31 (0.06)	47

Markup Shocks?

What if $\hat{\mu}_j \neq 0$ or $\mu_j \neq 1$?

$$\left(\frac{s_{Lj}\mu_j}{1-s_{Lj}\mu_j}\right)\left(\hat{s}_{Lj}+\hat{\mu}_j+\hat{s}_{Lj}\hat{\mu}_j\right)=\gamma+\left(\sigma-1\right)\left(\hat{\xi}_j+\hat{\mu}_j\right)+u_j$$

 \blacktriangleright Assuming $\beta,\,\delta$ are constant over time, same for all countries:

$$s_{\mathcal{K}j} = \frac{R_j \mathcal{K}_j}{Y_j} = \frac{\xi_j X_j}{Y_j} \left(\frac{1/\beta - 1 + \delta}{\delta}\right)$$
$$\hat{s}_{\mathcal{K}j} = \widehat{\xi_j X_j/Y_j}$$

From before $\mu s_{Lj} + \mu s_{Kj} = 1$. And:

$$\hat{\mu}_{j} = rac{1}{\mu_{j}\left(s_{Lj}\hat{s}_{Lj} + s_{\mathcal{K}j}\hat{s}_{\mathcal{K}j}
ight)}$$

Markup Shocks?



 \Rightarrow Countries with declining labor shares had (on average) declines in capital shares and markups.

Markup Shocks?

Investment Price	Investment Rate	$\hat{\sigma}$	Obs
PWT	Corporate	1.03 (0.09)	55
WDI	Corporate	1.29 (0.08)	52
PWT	Total	1.11 (0.11)	54
WDI	Total	1.35 (0.08)	52

Capital-Augmenting Technical Change?

Again, when $\mu = \hat{\mu} - 1 = \hat{\delta} = 0$:

$$\frac{s_L}{1-s_L}\hat{s}_L = \gamma + (\sigma - 1)\hat{\xi} + (1-\sigma)\hat{A}_K + u$$

Up to know, we had assumed corr $(\hat{A}_k, \hat{\xi}) = 0$. If not:

$$\underbrace{\tilde{\sigma} - \sigma}_{\text{Bias}} = (1 - \sigma) \operatorname{corr} \left(\hat{A}_k, \hat{\xi} \right) \frac{\operatorname{sd} \left(\hat{A}_k \right)}{\operatorname{sd} \left(\hat{\xi} \right)}$$

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► If corr
$$(\hat{A}_k, \hat{\xi}) < 0$$
, then
► $\tilde{\sigma} > \sigma$ iff $\sigma > 1$
► $\tilde{\sigma} \to \sigma$ iff $\sigma \to 1$.

Capital-Augmenting Technical Change?

From the last slide:

$$\underbrace{\tilde{\sigma} - \sigma}_{\text{Bias}} = (1 - \sigma) \operatorname{corr} \left(\hat{A}_k, \hat{\xi} \right) \frac{\operatorname{sd} \left(\hat{A}_k \right)}{\operatorname{sd} \left(\hat{\xi} \right)}$$

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► If

•
$$\operatorname{corr}(\hat{A}_k, \hat{\xi}) = -0.28$$

• $\operatorname{sd}(\hat{A}_k) = 0.10$
• $\operatorname{sd}(\tilde{\xi}) = 0.11$

• then if $\sigma = 1.25 \Rightarrow \tilde{\sigma} = 1.20$

Effect of the markup and investment price shocks

σ	1	1.25	1	1.25	1	1.25
	Ê		$\hat{\mu}$		$(\hat{\xi}, \hat{\mu})$	
Labor share	0.0	2.6	21	2.6	2 1	10
(% points)	0.0	-2.0	-5.1	-2.0	-3.1	-4.9
Capital share	0.0	2.6	_1 0	-24	_1 0	_0 1
(% points)	0.0	2.0	-1.9	-2.4	-1.9	-0.1
Profit share	0.0	0.0	50	50	5.0	5.0
(% points)	0.0	0.0	5.0	5.0	5.0	5.0
Rental rate	-22.1	-22.1	0.0	0.0	-22.1	-22.1
Capital-to-output	28.4	36.6	-5.2	-6.4	21.8	27.9
Welfare-equiv.	18 1	22.1	-3.0	_3.4	13.2	15.8
consumption	10.1	22.1	-5.0	-3.4	15.2	15.0

Notes on Oberfield and Raval (2014) "Micro Data and Macro Technology"

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Two additions

From the board:

$$\sigma_n^N = (1 - \chi_n) \,\sigma + \chi_n \varepsilon$$

1. Include materials in plants' production functions:

$$F(K_{ni}, L_{ni}, \dots) = \left[\left[(A_{ni}K_{ni})^{\frac{\sigma_{n-1}}{\sigma_n}} + (B_{ni}L_{ni})^{\frac{\sigma_{n-1}}{\sigma_n}} \right]^{\frac{\sigma_n}{\sigma_n-1}} \right]^{\frac{\sigma_n}{\sigma_n-1}}$$

2. Write out the aggregate elasticity in terms of industry-level terms.

Two additions

From the board:

$$\sigma_n^N = (1 - \chi_n) \,\sigma + \chi_n \varepsilon$$

1. Include materials in plants' production functions:

$$F(K_{ni}, L_{ni}, M_{ni}) = \left[\left[\left(A_{ni} K_{ni} \right)^{\frac{\sigma_n - 1}{\sigma_n}} + \left(B_{ni} L_{ni} \right)^{\frac{\sigma_n - 1}{\sigma_n}} \right]^{\frac{\zeta_n - 1}{\zeta_n}} + C_{ni} M_{ni}^{\frac{\sigma_N}{\sigma_N - 1} \frac{\zeta_N - 1}{\zeta_N}} \right]^{\frac{\zeta_n}{\zeta_n - 1}}$$

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2. Write out the aggregate elasticity in terms of industry-level terms.

Building up to the aggregate EoS

The industry-level elasticity of substitution equals:

$$\sigma_n^N = (1 - \chi_n) \, \sigma_n + \chi_n \left[\left(1 - \bar{s}_n^M \right) \varepsilon_n + \bar{s}_n^M \zeta_n \right]$$

where $\chi_n = \sum_i \frac{(\alpha_{ni} - \alpha_n)^2}{(1 - \alpha_n) \, \alpha_n} \theta_{ni}$, and

s^M_n is a weighted average of plants' intermediate input shares.
► The aggregate elasticity of substitution equals:

$$\sigma^{agg} = (1 - \chi^{agg}) \,\overline{\sigma}^{N} + \chi^{agg} \left[\left(1 - \overline{s}^{M} \right) \eta + \overline{s}^{M} \overline{\zeta}_{n} \right]$$

where $\chi^{agg} = \sum_{i} \frac{(\alpha_{n} - \alpha)^{2}}{(1 - \alpha) \,\alpha} \theta_{n}$, and

- *ō^N* (*ζ̄_n*) is a weighted average of the industry capital-labor (materials) EoS.
- ► \overline{s}^M is a weighted average of industries' intermediate input shares.

The Census of Manufacturers & Annual Survey of Manufacturers

Census of Manufacturers (CM)

- All plants within the US with \geq 5 employees (180,000 out of 350,000)
- ▶ Every five years (1972, 1977,... 2012)
- Book value of capital is imputed for non ASM plants (except for 1987, 1997)
- Materials expenditures, labor expenditures, output.
- Annual Survey of Manufacturers
 - A subset of plants (50,000), oversampling of larger plants

Materials expenditures, labor expenditures, output.

Building blocks of σ_n^N

- > χ : variation in plant-level capital shares (within value added)
- \bar{s}_n^M : average materials cost share
- σ_n : plant-level elasticity of substitution, between capital and labor
- ε_n: elasticity of demand
- ζ_n: elasticity of substitution between materials and value added

Building blocks of $\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n[(1 - \bar{s}_n^M)\epsilon_n + \bar{s}_n^M]$: χ_N



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 $\chi_n \approx \mathbf{0} \Rightarrow \sigma_n^N \approx \sigma_n$

Building blocks of

$$\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n[(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M]: \sigma_n$$

From the plants' cost-minimization condition:

$$\log\left(\frac{rK}{wN}\right)_{ni} = \kappa + (\sigma_n - 1)\left(\frac{w}{R}\right)_{ni}$$

Specification from Raval (2014):

$$\log\left(\frac{rK}{wN}\right)_{ni} = \kappa + (\sigma_n - 1)\log w_{ni}^{MSA} + \text{Controls} + \epsilon_{ni}$$

- w^{MSA}_{ni}: hourly wage in the MSA of plant *i*, after controlling for worker education, experience, industry, occupation, demographics.
- Controls: age of the plant, indicator for whether it is part of a multi-unit firm.
- Key Assumptions: $R_{ni} \perp w_{ni}^{MSA}$ (or more generally, $w_{ni}^{MSA} \perp \epsilon_{ni}$)

Building blocks of $\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n[(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M]: \sigma_n$



Average of $\sigma_n \approx 0.5$.

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Building blocks of $\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n[(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M]: \zeta_n$

From the plants' cost-minimization condition: Similar specification to identify ζ :

$$\log\left(\frac{qM}{wN+rK}\right)_{ni} = (\zeta - 1)(1 - \alpha_i)\log w_{ni}^{MSA} + \text{Controls} + \epsilon_{ni}$$

Results from pooled regression

	$\hat{\zeta}$
1987	0.90
1997	0.67
Ν	140,000

Building blocks of $\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n[(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M]$: \bar{s}_n^M and ε_N

- \bar{s}_n^M , average materials cost share: average=0.59.
- ε_n : Demand elasticity.
 - According to the model, the markup equals revenues divided by total costs $\Rightarrow \frac{\varepsilon_n}{\varepsilon_n - 1} = \frac{P_{ni}Y_{ni}}{wL_{ni} + rK_{ni} + qM_{ni}}$

ε_n ∈ [3, 5]

Building blocks of $\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n[(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M]$: [$(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M$]



Building blocks of
$$\sigma_n^N = (1 - \chi_n)\sigma_n + \chi_n[(1 - \bar{s}_n^M)\varepsilon_n + \bar{s}_n^M]$$



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Building blocks of

 $\sigma^{\text{agg}} = (1 - \chi^{\text{agg}})\bar{\sigma}_n^N + \chi^{\text{agg}}[(1 - \bar{s}^M)\eta + \bar{s}^M\bar{\zeta}_n]$

- η , elasticity of demand across industries: 1
- $\bar{\sigma}_n^N$, χ^{agg} , \bar{s}^M , and $\bar{\zeta}_n$ all come from industry-level data.
- Estimate in 1987: 0.70
- Allowing the χ s, \bar{s} s to vary across years:



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$\sigma^{\rm agg}$ ranges from 0.80 to 1.15 for other countries



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Reminder: The labor share has fallen



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Why has the labor share fallen? A decomposition

$$ds^{v,L} = \frac{\partial s^{v,L}}{\partial \log w/r} d\log w/r + \left[ds^{v,L} - \frac{\partial s^{v,L}}{\partial \log w/r} d\log w/r \right]$$
$$= (1 - \sigma^{agg}) d\log w/r + \left[ds^{v,L} - \frac{\partial s^{v,L}}{\partial \log w/r} d\log w/r \right]$$

Data on w, r:

- For w: NIPA. $w = \frac{\text{Labor compensation}}{\text{Employees}}$, adjust for changes in skills.
- ► For *r*:
 - Capital prices from NIPA
 - Real rental rate of capital 3.5%
 - Tax rates and depreciation allowances from Jorgenson

w/r has gone up & $1-\sigma^{\rm agg}>0$ \Rightarrow Contribution of factor prices is positive.
Almost none of the change in the labor share is from w/r increasing.



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The discrepancy between Oberfield and Raval and Karabarbounis and Neiman?

Sample: Manufacturing (OR) vs the whole economy (KN)				
	Primary	Construc- tion	Manuf.	Transport
θ	0.03	0.05	0.20	0.61
α	0.55	0.19	0.35	0.45
	Electricity/	Wholesale/	FIRE	Other
	Gas Serv.	Retail		Services
θ	0.05	0.15	0.17	0.28
α	0.58	0.29	0.67	0.18

Sample: Manufacturing (OP) vs the whole economy (KN)

 $\chi^{\rm full} = 0.14.$

Omitted variable bias? See Loukas' discussion of OR on his webpage.