## Kaldor Facts \& Kuznets Facts

- Kaldor Facts

1. $\frac{Y}{L}$ grows at a sustained rate
2. $\frac{K}{L}$ grows at a sustained rate
$(1)+(2) \Rightarrow \frac{Y}{K}$ is roughly stable.
3. $r=i-\pi$ is stable
4. The capital and labor shares of national income are stable (roughly $\frac{1}{3}$ and $\frac{2}{3}$ )
5. $Y$ per capita grows at a stable rate

- Kuznets Facts: As economies grow, the shares of income/consumption in services grow, in agriculture shrink, and in manufacturing are roughly constant (grow and then shrink).


## Labor Share of Income



## Labor Share of Income



## Labor Share of Income



## Labor Share of Income: Other Countries



## Ratio of K/Y



## Real Return of S\&P 500

| Period | Return |
| :---: | :---: |
| $1930-1950$ | $4.8 \%$ |
| $1950-1970$ | $9.2 \%$ |
| $1970-1990$ | $4.7 \%$ |
| $1990-2010$ | $6.2 \%$ |

## Kuznets Facts for the US



## Agriculture Value Added Share of GDP



## Manufacturing Value Added Share of GDP



## Services Value Added Share of GDP



## Note: We'll go over the following papers on the board

- Kongsamut, Rebelo, Xie (2001), "Beyond Balanced Growth"
- Ngai and Pissarides (2007), "Structural Change in a Multisector Model of Growth"

Notes on Herrendorf et al. (2013): "Two Perspectives on Preferences and Structural Transformation"

## Review: two views of structural transformation

- Facts:
- Agriculture shrinks, manufacturing first grows and then shrinks, services grow.
- These shifts are more pronounced in nominal rather than real terms.
- Ngai and Pissarides
- Differential growth rates in sectors' productivity.
- Nonunitary elasticity of substitution across goods.
- Low-growth sector (services) has larger relative prices; draws more resources into the economy.
- Kongsamut et al.
- Identical productivity growths.
- Nonunitary income elasticity for different goods.
- Agriculture has subunitary elasticity of substitution; services has income elasticity $>1$.
- In these papers, there was little distinction between commodities and the industries that produced them.


## Contribution of Herrendorf, Rogerson, and Valentinyi

- Construct and estimate a model that nests Ngai and Pissarides and Kongsamut et al.
- Show that the attribution of transformation to income/price effects depends on how we view what consumers value:

1. "Final Consumption Expenditures": $u\left(c_{a}, c_{m}, c_{s}\right)$

- $c_{a}$ : food and beverages purchases or off-premises consumption
- $c_{m}$ : goods, excluding food and beverages...
- $c_{s}$ : services; government consumption expenditure

2. "Consumption Value Added": $u\left(c_{a}, c_{m}, c_{s}\right)$

- $c_{a}$ : farms; forestry, fishing
- $c_{m}$ : construction; manufacturing; mining
- $c_{s}$ : all other industries
- Provide a link between the two perspectives.


## Outline

1. Model
2. Data
3. Estimation using the "Final Consumption Expenditures" perspective
4. Estimation using the "Consumption Value Added" perspective
5. Linking the two perspectives.

## Model (1)

Consider the problem of a consumer who is trying to maximize:

$$
\begin{aligned}
& \qquad u\left(c_{a t}, c_{m t}, c_{s t}\right)=\left(\sum_{i \in\{a, m, s\}} \omega_{i}^{\frac{1}{\sigma}}\left(c_{i t}+\bar{c}_{i}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\
& \text { subject to } \sum_{i \in\{a, m, s\}} p_{i t} c_{i t}
\end{aligned}=C_{t} \text {. }
$$

Note:

- If $\bar{c}_{i}=0 \Rightarrow$ Preferences as in Ngai and Pissarides.
- If $\sigma=1$ and $\bar{c}_{m}=0 \Rightarrow$ Preferences as in Kongsamut et al.
- Nothing about the technology side of the economy is explicitly specified.
- Intertemporal decisions play little/no role.


## Model (2)

- Solving the static problem from the previous slide:

$$
\begin{align*}
\frac{p_{m t} c_{m t}}{C_{t}} & =-\frac{p_{m t} \bar{c}_{m}}{C_{t}}+\frac{\omega_{m} p_{m t}^{1-\sigma}}{\sum_{i \in\{a, m, s\}} \omega_{i} p_{i t}^{1-\sigma}}\left(1+\sum_{i \in\{a, m, s\}} \frac{p_{i t} \bar{c}_{i}}{C_{t}}\right)  \tag{1}\\
\frac{p_{s t} c_{s t}}{C_{t}} & =-\frac{p_{s t} \bar{c}_{s}}{C_{t}}+\frac{\omega_{s} p_{s t}^{1-\sigma}}{\sum_{i \in\{a, m, s\}} \omega_{i} p_{i t}^{1-\sigma}}\left(1+\sum_{i \in\{a, m, s\}} \frac{p_{i t} \bar{c}_{i}}{C_{t}}\right) \tag{2}
\end{align*}
$$

- The equation for $p_{a t} c_{a t} / C_{t}$ is redundant.
- Taking the model to the data
- Parameters: $\omega_{a}, \omega_{m}, \sigma, \bar{c}_{a}, \bar{c}_{s}$
- Data: Time series on $p_{m t} c_{m t}, p_{s t} c_{s t}, p_{a t}, p_{m t}$ and $p_{s t}$,
- Fit Equations (1) and (2) as best as possible.


## Data Sources

- Consumption Final Expenditure Data $\left(p_{s t}^{f} c_{s t}^{f}\right.$ and $\left.p_{s t}^{f}\right)$
- National Income Product Accounts: Values and Quantity Indices (see http://www.econstats.com/nipa/)
- Consumption Value Added Data:
- Bureau of Economic Analysis Industry Accounts: Value Added and Quantity Indices by Industry.
- Need to subtract off investment from the production value added data. (Investment goods produced by all industries, not just manufacturing)
- In previous papers $c_{m}+\dot{k}-\delta k=m$. But, after 2002 $\dot{k}-\delta k>m$ !
- BEA: 2002 Table of service shares for different types of investment goods.
- Bureau of Economic Analayis Input-Output Tables: (Useful in Linking FE and VA perspectives.)


## Final Expenditures Data

Price Indices


Quantity Indices


- Quantity goes up most for manufacturing, least for food.
- Prices goes up most for services, least for manufacturing.


## Estimating Final Consumption Expenditure Preferences

|  | $(1)$ | $(2)$ | $(3)$ |
| :--- | :---: | :---: | :---: |
| $\sigma$ | 0.85 | 1 | 0.89 |
| $\bar{c}_{a}$ | -1350 | -1316 |  |
| $\bar{c}_{s}$ | 11237 | 19748 |  |
| $\omega_{a}$ | 0.02 | 0.02 | 0.11 |
| $\omega_{m}$ | 0.17 | 0.15 | 0.24 |
| $\omega_{s}$ | 0.81 | 0.84 | 0.65 |
| $\chi^{2}\left(\bar{c}_{a}=0, \bar{c}_{s}=0\right)$ | 3867 | 4065 |  |
| AIC | -932.55 | -931.35 | -666.03 |

Note: $\mathrm{AIC}=2 k-2 \log \mathcal{L}$

## Income effects are important in fitting expenditure share data

Prices Fixed at 1947 Values


Income Fixed at 1947 Values


Nonhomotheticity terms:

|  | 1947 | 2010 |
| :--- | :---: | :---: |
| $p_{a} \bar{c}_{a} / C$ | -0.17 | -0.04 |
| $p_{s} \bar{c}_{s} / C$ | 0.73 | 0.32 |

## Fit of estimated model, $\bar{c}_{a}=\bar{c}_{s}=0$



- $\left\{\hat{\sigma}, \omega_{a}, \omega_{m}, \omega_{s}\right\}=\{0.89,0.11,0.24,0.65\}$


## Value Added Data

Price Indices


Quantity Indices


- Correlation between prices indices and quantity indices is much stronger in the value added data (89\%) than in the final expenditure data (48\%).


## Estimating Value Added Preferences

|  | $(1)$ | $(2)$ | $(3)$ |
| :--- | :---: | :---: | :---: |
| $\sigma$ | 0.00 | 0 | 0 |
| $\bar{c}_{a}$ | -138.7 | -138.9 |  |
| $\bar{c}_{s}$ | 4261.8 | 4268.1 |  |
| $\omega_{a}$ | 0.002 | 0.002 | 0.01 |
| $\omega_{m}$ | 0.15 | 0.15 | 0.18 |
| $\omega_{s}$ | 0.85 | 0.85 | 0.81 |
| $\chi^{2}\left(\bar{c}_{a}=0, \bar{c}_{s}=0\right)$ | 1424 | 216 |  |
| AIC | -837.3 | -875.4 | -739.4 |

Income and price effects are both important in fitting the value added data

Prices Fixed at 1947 Values


Income Fixed at 1947 Values


Nonhomotheticity terms:

|  | 1947 | 2010 |
| :--- | ---: | ---: |
| $p_{a} \bar{c}_{a} / C$ | -0.08 | -0.01 |
| $p_{s} \bar{c}_{s} / C$ | 0.34 | 0.12 |

## Why are the $\bar{c}_{a}, \bar{c}_{s}$ terms less important?

- Consumption over Commodities' Final Expenditure
- Food from supermarkets is an agricultural commodity ( $\bar{c}_{a}<0$ )
- Meals from restaurants is a service ( $\bar{c}_{s}>0$ )
- Consumption over Industries' Value Added
- Both food from supermarkets and food from restaurants are produced by the agriculture industry; $\bar{c}_{a} \& \bar{c}_{s}$ balance out.


## Linking the two approaches: theory

- Assume that final added consumption is a CES aggregate of value added from the three sectors:

$$
c_{i t}^{f}=\left[\sum_{j \in\{a, m, s\}}\left(A_{i t} \phi_{j \rightarrow i}\right)^{\frac{1}{\eta_{i}}}\left(c_{j \rightarrow i, t}^{v}\right)^{\frac{\eta_{i}-1}{\eta_{i}}}\right]^{\frac{\eta_{i}}{n_{i}-1}}
$$

- Cost minimization of the "final expenditure bundler" implies that:

$$
\begin{equation*}
p_{j}^{v} c_{j \rightarrow i, t}^{v}=\frac{\phi_{j \rightarrow i}\left(p_{j}^{v}\right)^{1-\eta_{i}}}{\sum_{k \in\{a, m, s\}} \phi_{k \rightarrow i}\left(p_{k}^{v}\right)^{1-\eta_{i}}} p_{i}^{f} c_{i t}^{f} \tag{3}
\end{equation*}
$$

- Taking the model to the data
- Parameters: $\eta_{i}, \phi_{j \rightarrow i} ; i, j \in\{a, m, s\}$.
- Fit Equation (3) as best as possible, separately for each $i \in\{a, m, s\}$.


## Linking the two perspectives: data

How are the $p_{j}^{v} c_{j \rightarrow i, t}^{v}$ constructed?

- Bureau of Economic Analysis "Total Requirements" Tables
- For firms producing commodity $j$, what is the total value of purchases from industry $i$ ?
- What is, $p_{i}^{\vee} c_{i}^{V}$, the value added of firms within industry i?
- What is, $p_{j}^{f} c_{j}^{f}$, the value of final consumption of commodities $j$ ?
- Define $T_{i j}=\frac{\text { purchases of commodity } j \text { for firms producing in } i}{\text { value added in } i+\text { total purchases of firms in } i}$
- $j i$ element of $(I-T)^{-1}$ : number of dollars of value added in industry $j$ for producing a dollar of final expenditure of commodity $i$. Note $(I-T)^{-1}=I+T+T^{2}+T^{3}+\ldots$.
- Using this definition:

$$
p_{j}^{\vee} c_{j \rightarrow i}^{v}=\left((I-T)^{-1}\right)_{j i} p_{i}^{f} c_{i}^{f}
$$

## An example from the data

How are the $p_{j}^{v} c_{j \rightarrow i, t}^{v}$ constructed? BEA "Total Requirements" Tables, from 1963

|  | Agric. | 17818 | 0 | 326 | 1112 | 25641 | 259 | 3410 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Min'g | 128 | 1138 | 737 | 3686 | 10949 | 46 | 2914 |
|  | Const. | 567 | 416 | 25 | 588 | 814 | 1556 | 10906 |
| IO | Durab. | 795 | 1081 | 27329 | 97129 | 8018 | 3160 | 6299 |
| Table: | N-Dur | 6851 | 588 | 4234 | 11582 | 69029 | 6683 | 17745 |
|  | Trans. | 2795 | 876 | 9789 | 11605 | 12615 | 7278 | 11526 |
|  | Serv. | 4774 | 3529 | 5814 | 14041 | 15974 | 26717 | 60931 |
|  | VA | 22702 | 11050 | 37022 | 95905 | 75063 | 112320 | 233569 |
|  |  |  |  |  |  |  |  |  |
|  | Agric. | 1.50 | 0.02 | 0.04 | 0.04 | 0.26 | 0.02 | 0.04 |
|  | Min'g | 0.02 | 1.07 | 0.03 | 0.04 | 0.08 | 0.01 | 0.02 |
|  | Const. | 0.02 | 0.03 | 1.01 | 0.01 | 0.02 | 0.02 | 0.04 |
|  | $I-T)^{-1}$ |  |  |  |  |  |  |  |
|  | Durab. | 0.08 | 0.18 | 0.57 | 1.71 | 0.13 | 0.06 | 0.07 |
|  | N-Dur | 0.29 | 0.08 | 0.14 | 0.15 | 1.52 | 0.09 | 0.11 |
|  | Trans. | 0.11 | 0.07 | 0.17 | 0.11 | 0.12 | 1.07 | 0.06 |
|  | Serv. | 0.21 | 0.26 | 0.18 | 0.17 | 0.21 | 0.23 | 1.25 |

## An example from the data

How are the $p_{j}^{v} c_{j \rightarrow i, t}^{v}$ constructed? BEA "Total Requirements" Tables, from 1963:

$(I-T)^{-1}=$|  | Agric. | 1.50 | 0.02 | 0.04 | 0.04 | 0.26 | 0.02 | 0.04 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Min'g | 0.02 | 1.07 | 0.03 | 0.04 | 0.08 | 0.01 | 0.02 |
| Const. | 0.02 | 0.03 | 1.01 | 0.01 | 0.02 | 0.02 | 0.04 |  |
| Durab. | 0.08 | 0.18 | 0.57 | 1.71 | 0.13 | 0.06 | 0.07 |  |
|  | N-Dur | 0.29 | 0.08 | 0.14 | 0.15 | 1.52 | 0.09 | 0.11 |
| Trans. | 0.11 | 0.07 | 0.17 | 0.11 | 0.12 | 1.07 | 0.06 |  |
| Serv. | 0.21 | 0.26 | 0.18 | 0.17 | 0.21 | 0.23 | 1.25 |  |

In 1963, each dollar of final expenditures in agriculture generates 0.21 dollars of value added in services, 0.11 in transport.

Since $p_{A, 1963}^{f} c_{A, 1963}^{f}=\$ 348$ per capita, we have that $p_{S \rightarrow A}^{\vee} c_{S \rightarrow A}^{\vee}=\$ 348 \cdot(0.21+0.11)=\$ 111$

## Estimates of the production commodities

Reminder:

$$
\begin{gathered}
p_{j}^{v} c_{j \rightarrow i, t}^{v}=\frac{\phi_{j \rightarrow i}\left(p_{j}^{v}\right)^{1-\eta_{i}}}{\sum_{k \in\{a, m, s\}} \phi_{k \rightarrow i}\left(p_{k}^{v}\right)^{1-\eta_{i}}} p_{i}^{f} c_{i t}^{f} \\
\hline
\end{gathered} \begin{array}{ccc} 
& \text { Food } & \text { Goods } \\
\hline \eta_{i} & 0.19^{*} & 0.00 \\
\phi_{a \rightarrow i} & 0.05^{*} & 0.02^{*} \\
\phi_{m \rightarrow i} & 0.33^{*} & 0.01^{*} \\
\phi_{s \rightarrow i} & 0.62^{*} & 0.62^{*} \\
\hline
\end{array}
$$

- Except for agriculture, production of final expenditures is Leontief.
- Services are an important input in all commodities.
- Agriculture is relatively unimportant in the production of the three commodities.


## Linking $\eta, \sigma^{V}$, and $\sigma^{D}$

## Two alternative chains of substitution

- $\sigma^{\vee}$ : elasticity of substitution between products produced in the service vs. manufacturing sectors
- $\sigma^{f}$ : elasticity of substitution between goods and services
- $\eta^{i}$ : elasticity of substitution, across different industries' value added, when making final expenditure commodity $i$
- From Oberfield and Raval:

$$
\sigma^{\vee} \approx \chi \sigma^{f}+(1-\chi) \bar{\eta}
$$

where $\chi=$ index of cross-industry heterogeneity in producing different commodities.

- In our context, $\chi, \bar{\eta} \approx 0, \sigma^{f} \approx 0.9 \Rightarrow \sigma^{\vee} \approx 0$.


## Conclusion (1)

- Summary
- To fit the growth of service FE, and the decline of food FE $\Rightarrow$ income effects are important.
- To link FE data and VA added data $\Rightarrow$ complementarity in production of fixed expenditures.
- Next Steps
- What productivity trajectories will generate the observed relative price movements?
- Look at within-sector price \& quantity paths.
- Are they similar across industries, within sectors?
- What are the within-industry productivity paths?


## Conclusion (2)

What are the underlying productivity paths?

Herrendorf, Herrington, and Valentinyi (2014)

- Production functions of the form:

$$
\begin{aligned}
G_{i t} & =\left[F_{i t}\left(K_{i t}, L_{i t}\right)\right]^{\eta_{i}}\left[X_{i t}\left(Z_{i t}\right)\right]^{1-\eta_{i}}, \text { where } \\
F_{i t} & =\left[\alpha_{i}\left[\exp \left(\gamma_{i k} t\right) K_{i t}\right]^{\frac{\sigma_{i}-1}{\sigma_{i}}}+\left(1-\alpha_{i}\right)\left[\exp \left(\gamma_{i l} t\right) L_{i t}\right]^{\frac{\sigma_{i}-1}{\sigma_{i}}}\right]^{\frac{\sigma_{i}}{\sigma_{i}-1}}
\end{aligned}
$$

- Main result: $\gamma_{A I}>\gamma_{M I}>\gamma_{S I} ; \sigma \approx 1$ fit the price data well.


## Conclusion (3)

Quantity: \%
Ann. Growth

| GDP | $3.5 \%$ | $3.4 \%$ |
| :--- | :--- | :--- |
| Wholesale | $1.9 \%$ | $4.8 \%$ |
| Retail | $2.7 \%$ | $3.6 \%$ |
| Transportation | $2.9 \%$ | $2.9 \%$ |
| Information | $2.5 \%$ | $5.3 \%$ |
| Finance \& Insurance | $5.0 \%$ | $4.0 \%$ |
| Real Estate | $3.7 \%$ | $4.0 \%$ |
| Professional Services | $5.3 \%$ | $4.5 \%$ |
| Management | $4.2 \%$ | $3.0 \%$ |
| Administration | $4.6 \%$ | $5.3 \%$ |
| Education | $5.8 \%$ | $3.0 \%$ |
| Health | $5.3 \%$ | $4.2 \%$ |
| Arts \& Entertainment | $4.2 \%$ | $3.4 \%$ |
| Accommodation | $4.0 \%$ | $3.1 \%$ |
| Other Services | $4.9 \%$ | $1.6 \%$ |

## Review: Regional Convergence



Red: South, Blue: Midwest, Green: West

## Caselli-Coleman Hypotheses

1. South has a comparative advantage in agricultural products.
2. Declining cost of education $\Rightarrow$ decreases relative labor supply in agriculture.
3. Income elasticity of food less than 1; faster technological growth in agriculture $\Rightarrow$ lowers labor demand in agriculture.
$(2)+(3) \Rightarrow$ Possible to have decline in labor share of agriculture and increase in relative wage of agriculture. Both are important components of regional convergence.

## Census Regions



## Region $\times$ Sector



## Region $\times$ Education Status



Blue: <HS, Green: HS-Some College, Red: >=College
Solid: South, Dash: North, Dot: Midwest

## Sector $\times$ Education Status



Blue: <HS, Green: HS-Some College, Red: >=College Solid: Agriculture, Dash: Manufacturing, Dot: Services

- Last two slides:
- High school dropouts: $75 \% \rightarrow 15 \%$.
- Education status is a bit lower in the South, much lower in agriculture, much higher in services.


## Income by Sector



## Income by Education Status



Red:North, Blue:South, Green:Midwest
Solid:<HS, Dash:HS-Some College, Dot:>=College

- Last two slides:
- Income lowest in agriculture; gap decreases over time.
- Income increases with education; education premia increase beginning in the 80s.
- Within education $\times$ sector : highest in Northeast, lowest in South.


## Accounting for Regional Convergence

Wages of South approach those of the North due to 3 effects

1. Industry Shares of the South $\rightarrow$ Industry Shares of the North (Labor Reallocation)
2. Wage of Agriculture $\rightarrow$ Wage of Manufacturing/Services (Between Industry)
3. Wage of Agriculture (or Manufacturing/Services) in South $\rightarrow$ Wage of Agriculture (or Manufacturing/Services) in North (Within Industry)

$$
\begin{aligned}
& \text { Total: } \Delta \frac{w^{S}-w^{N}}{w}=\Delta \frac{w_{f t}^{S} L_{f t}^{S}+w_{m t}^{S}\left(1-L_{f t}^{S}\right)-w_{f t}^{N} L_{f t}^{N}-w_{m t}^{N}\left(1-L_{f t}^{N}\right)}{w_{f t} L_{f t}+w_{m t}\left(1-L_{f t}\right)} \\
& \text { (1): } \frac{w_{f t}^{S}-w_{m t}^{S}}{w_{t}}\left(L_{f t}^{S}-L_{f, t-1}^{S}\right)-\frac{w_{f t}^{N}-w_{m t}^{N}}{w_{t}}\left(L_{f t}^{N}-L_{f, t-1}^{N}\right) \\
& \text { (2): } \Delta \frac{w_{f}-w_{m}}{w} \cdot\left(\frac{1}{2}\left[L_{f t}^{S}+\frac{1}{2} L_{f, t-1}^{S}\right]-\frac{1}{2}\left[L_{f t}^{N}+L_{f, t-1}^{N}\right]\right)
\end{aligned}
$$

## Accounting for Regional Convergence



- All three forces (reallocation, within, and between) are important in accounting for South-North convergence.
- Between industry: Early in the sample period
- Within + Reallocation: Later in the sample period


## Model: Overview

- Two regions: North and South
- Agriculture takes place in the South. Manufacturing in either region.
- Exogeneous productivity growth $\Rightarrow$ Increased income raises relative demand for manufactured goods.
- Reallocation Effect
- Decision on whether to accumulate HC (and work in manufacturing)
- Decline in cost of going to school decreases labor supply in agriculture
- Reallocation Effect + Between Industry Effect.


## Model: Production (1)

- Two goods: food $(F)$ and manufacturing ( $M$ ).
- Capital and labor are perfectly mobile across industries and across regions.
- Labor is used in manufacturing, agriculture, or accumulating human capital.

$$
\begin{gathered}
L_{f t}^{S}+\underbrace{L_{m t}^{N}+L_{m t}^{S}}_{L_{m t}}+L_{e t}=1 \\
K_{f t}^{S}+\underbrace{K_{m t}^{N}+K_{m t}^{S}}_{K_{m t}}=1
\end{gathered}
$$

- Manufactured goods can be consumed or invested:

$$
c_{m t}+K_{t+1}=M_{t}^{N}+M_{t}^{S}+(1-\delta) K_{t}
$$

- Food can only be consumed

$$
c_{f t}=F_{t}^{S}
$$

## Model: Production (2)

- Two locations: $i \in\{$ South, North $)$.

$$
\begin{aligned}
& M_{t}^{S}=A_{m t}\left(T_{m t}^{S}\right)^{\alpha_{T}}\left(L_{f t}^{S}\right)^{\alpha_{L}}\left(K_{f t}^{S}\right)^{1-\alpha_{T}-\alpha_{L}} \\
& M_{t}^{N}=A_{m t}\left(T_{m t}^{N}\right)^{\alpha_{T}}\left(L_{f t}^{N}\right)^{\alpha_{L}}\left(K_{f t}^{N}\right)^{1-\alpha_{T}-\alpha_{L}} \\
& F_{t}^{S}=A_{f t}\left(T_{f t}^{S}\right)^{\beta_{T}}\left(L_{f t}^{S}\right)^{\beta_{L}}\left(K_{f t}^{S}\right)^{1-\beta_{T}-\beta_{L}}
\end{aligned}
$$

- Let $g_{m t} \equiv \frac{A_{m, t+1}-A_{m t}}{A_{m t}} ; g_{f t} \equiv \frac{A_{f, t+1}-A_{f t}}{A_{f t}}$
- Land is perfectly mobile across industries.
- Total supply in each region is fixed: $\omega$ in the South; $1-\omega$ in the North.
- Note: Because of decreasing returns to mobile factors (capital + labor), manufacturing will occur in both regions.


## Model: Preferences

- Each individual $i$ belongs to a household; altruistic over household successors
- Preferences over food and manufactured products:

$$
\begin{aligned}
U & =\sum_{t=0}^{\infty} \beta^{t} u\left(c_{f t}^{i}, c_{m t}^{i}\right), \text { where } \\
u\left(c_{f t}^{i}, c_{m t}^{i}\right) & =\frac{\left[\left(c_{f t}^{i}-\gamma\right)^{\tau}\left(c_{m t}^{i}\right)^{1-\tau}\right]^{1-\sigma}}{1-\sigma}
\end{aligned}
$$

- As in the Kongsamut et al. paper, $\gamma>0$ generates non-unitary income elasticities.
- Consumers intertemporal BC:

$$
\sum_{t=0}^{\infty} q_{t}\left(c_{f t}^{i}+p_{t} c_{m t}^{i}\right)=H_{0}^{i}
$$

- $H_{0}^{i}$ : lifetime income
- $q_{t}$ period-0 price of one unit of the farm good in period $t$.


## Model: Human Capital Accumulation

- Workers, $i$, are born and die stochastically. $\lambda$ be the probability of death per period.
- (Only) at birth: $i$ decides whether to go to school or not.
- Benefit of school: Can work in manufacturing.
- Cost of school: Spend $\zeta^{i} \xi_{t}(<1)$ periods of time not working.
- $\zeta^{i}$ is a random variable with density $\mu(\zeta)$
- $\xi_{t}$ is the same for all individuals in a period, potentially decreases over time.


## Model: Equilibrium Conditions

- Profit maximization by the representative firm in each industry/region.

$$
F_{1}\left(T_{f t}, L_{f t}, K_{f t}, A_{f t}\right)=a_{t} ; M_{1}\left(T_{f t}, L_{f t}, K_{f t}, A_{f t}\right)=\frac{a_{t}}{p_{t}}, \text { etc } \ldots
$$

- Utility maximization over consumption of food, manufactured goods in each period.

$$
\frac{u_{2}\left(c_{f t}, c_{m t}\right)}{u_{1}\left(c_{f t}, c_{m t}\right)}=p_{t} \quad ; \beta \frac{u_{1}\left(c_{f, t+1}, c_{m, t+1}\right)}{u_{1}\left(c_{f t}, c_{m t}\right)}=\frac{q_{t+1}}{q_{t}}
$$

- Markets for land, labor, capital clear each period.
- Utility maximization over schooling choice at age 0.


## Model: Equilibrium Conditions

Utility maximization over schooling choice.

- Let $h_{j t} \equiv$ present value of wages in sector $j$

$$
h_{j t}=\sum \frac{q_{s}}{q_{t}} \lambda^{s-t} w_{j s} \text { for } j \in\{f, m\}
$$

- It is optimal to go to school provided:

$$
h_{m t}-\underbrace{\xi_{t} \zeta^{i} w_{m t}}_{\text {Lost wages in } 1^{\text {st }} \text { period }} \geq h_{f t}
$$

- Cutoff value of $\zeta^{i}$ is $\bar{\zeta}_{t}=\frac{1}{\xi_{t}} \frac{h_{m t}-h_{f t}}{w_{m t}}$


## Model: Human Capital Distribution

- From last slide: newborns go to school if $\zeta^{i} \geq \bar{\zeta}_{t}$.
- Frac. of newborns being educated is $I_{\text {et }}^{0}=\int_{0}^{\bar{\zeta} t} \xi_{t} \zeta^{i} \mu\left(\zeta^{i}\right) d \zeta^{i}$
- Frac. of newborn graduates is $I_{m t}^{0}=\int_{0}^{\bar{\zeta}_{t}}\left(1-\xi_{t} \zeta^{i}\right) \mu\left(\zeta^{i}\right) d \zeta^{i}$
- Frac. of newborn farmers is $I_{f t}^{0}=\int_{\zeta_{t}}^{\infty} \mu\left(\zeta^{i}\right) d \zeta^{i}$
- Farmers' evolution:

$$
L_{f t}=L_{f, t-1} \lambda+I_{f t}^{0}(1-\lambda)
$$

- Manufacturers' evolution:

$$
L_{m t}=\left(L_{m, t-1}+L_{e, t-1}\right) \lambda+I_{m t}^{0}(1-\lambda)
$$

- Labor spent in education:

$$
L_{e t}=l_{e t}^{0}(1-\lambda)
$$

## Calibration

| $\beta, \delta$ | $0.60,0.36$ | Discount factor, Depreciation rate |
| :--- | :--- | :--- |
| $g_{m}, g_{f 0}$ | $0.084,0.168$ | Nonfarm, Farm TFP growth |
| $\alpha_{T}, \alpha_{L}$ | $0.19,0.60$ | Cost shares in farming |
| $\beta_{T}, \beta_{L}$ | $0.06,0.60$ | Cost shares in manufacturing |
| $\omega$ | 0.75 | Land share in South |


|  | Model with constant $\xi$ |  |
| :--- | :--- | :--- |
| $\gamma$ | 0.2205 | Non-homotheticity parameter |
| $K_{0}$ | 0.0711 | Initial capital stock |
| $\xi_{0}$ and $\xi_{\infty}$ | 2.0375 | Education cost parameter |


|  | Model with declining $\xi$ |  |
| :--- | :--- | :--- |
| $\gamma$ | 0.2201 | Non-homotheticity parameter |
| $K_{0}$ | 0.0712 | Initial capital stock |
| $\xi_{0}$ | 1.8977 | Education cost in the year 1880 |
| $\xi_{\infty}$ | 0.1239 | Education cost in the year 2190 |

## Results

| Variable | Data | Constant <br> Costs | Declining <br> Costs |
| :--- | :---: | :---: | :---: |
| $\left(c_{f} / c\right)_{1880}$ | $0.31^{*}$ | 0.31 | 0.31 |
| $\left(c_{f} / c\right)_{1980}$ | 0.014 | 0.03 | 0.08 |
| $L_{f, 1880}$ | $0.50^{*}$ | 0.50 | 0.50 |
| $L_{f, 1980}$ | 0.03 | 0.33 | 0.10 |
| $p_{1880} / p_{1980}$ | $\sim_{1.0}$ | 0.16 | 1.14 |
| $\left(w_{f} / w_{m}\right)_{1880}$ | $0.20^{*}$ | 0.20 | 0.20 |
| $\left(w_{f} / w_{m}\right)_{1880}$ | $0.69^{\dagger}$ | 0.03 | 0.69 |
| $\left(w^{S} / w^{N}\right)_{1880}$ | $0.41^{*}$ | 0.41 | 0.41 |
| $\left(w^{S} / w^{N}\right)_{1980}$ | 0.90 | 0.56 | 0.97 |

## Results

| Variable <br> Growth Rates | Constant <br> Costs | Declining <br> Costs |
| :--- | :---: | :---: |
| South/North population | $0.36 \%$ | $-0.34 \%$ |
| Farm capital/labor ratio | $-0.69 \%$ | $2.43 \%$ |
| Farm land/labor ratio | $-1.47 \%$ | $0.94 \%$ |
| Nonfarm capital/labor ratio | $1.13 \%$ | $0.99 \%$ |
| Nonfarm land/labor ratio | $0.38 \%$ | $-0.34 \%$ |

## Results

| Variable <br> Growth Rates | Constant <br> Costs | Declining <br> Costs |
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| Nonfarm land/labor ratio | $0.38 \%$ | $-0.34 \%$ |

## Summary

- Agriculture is a geographically concentrated, low education activity
- Decreasing cost of education + Decreasing relative demand for food
$\Rightarrow$ Higher relative wages in agriculture + Reallocation away from agriculture
$\Rightarrow$ Regional Convergence.


## Relative Price of Education Services Is Increasing



## Metro Status by Region



Red: S outh, Black: North, Green: Midwest, Blue: West


Red: S outh, Black: North, Green: Midwest, Blue: West


Red: S outh, Black: North, Green: Midwest, Blue: West


Red: S outh, Black: North, Green: Midwest, Blue: W est

Notes on Karabarbounis and Neiman (2014) "The Global Decline of the Labor Share"

## Review: Labor Share of Income



## Relative Price of Capital Is Falling, Especially After 1980



## A complication when computing the labor share

## How do you classify entrepreneurs" income? Taxes?

| Line |  | 2012 | 2012 | 2012 | 2012 | 2013 | 2013 | 2013 | 2013 | 2014 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | I | II | III | IV | I | II | III | IV | I |
| 1 | Gross domestic income | 16,104.6 | 16,150.3 | 16,269.6 | 16,522.0 | 16,690.9 | 16,847.8 | 17,004.6 | 17,181.4 | 17,121.3 |
| 2 | Compensation of employees, paid | 8,522.3 | 8,562.6 | $8,599.5$ | 8,795.5 | 8,756.1 | 8,844.0 | 8,896.8 | 8,973.8 | 9,049.5 |
| 3 | Wages and salaries | 6,850.3 | 6,882.3 | $6,913.2$ | 7,094.6 | 7,048.2 | 7,126.1 | 7,171.3 | 7,237.7 | 7,301.2 |
| 4 | To persons | 6,836.1 | 6,867.3 | 6,898.4 | 7,080.0 | 7,033.8 | 7,111.0 | 7,156.2 | 7,222.5 | 7,286.5 |
| 5 | To the rest of the world | 14.1 | 15.0 | 14.8 | 14.6 | 14.4 | 15.1 | 15.1 | 15.2 | 14.8 |
| 6 | Supplements to wages and salaries | 1.672 .1 | 1,680.3 | 1,686.2 | 1,700.9 | 1,707.9 | 1.717 .8 | 1,725.5 | 1,736.2 | 1.748 .3 |
| 7 | Taxes on production and imports | 1,124.4 | 1,122.2 | 1,118.8 | 1,126.3 | 1,140.7 | 1,138.8 | 1,149.0 | 1,158.3 | 1,166.7 |
| 8 | Less: Subsidies 1 | 57.8 | 57.6 | 56.0 | 57.7 | 58.0 | 58.9 | 59.1 | 58.7 | 56.8 |
| 9 | Net operating surplus | 4,008.1 | 3,989.4 | 4,052.2 | 4,083.0 | 4,248.2 | 4,292.0 | 4,358.2 | 4,416.9 | 4,240.1 |
| 10 | Private enterprises | $4,032.5$ | 4,0155 | 4,080.7 | 4,114.8 | 4.283 .7 | 4,331.0 | 4,399.6 | 4,461.3 | 4,285.5 |
| 11 | Net interest and miscellaneous payments, domestic industries | 613.6 | 580.8 | 611.7 | 583.3 | 630.3 | 591.7 | 615.5 | 638.8 | 633.4 |
| 12 | Business current transfer payments (net) | 115.7 | 110.0 | 102.6 | 99.5 | 121.9 | 125.8 | 120.1 | 129.9 | 122.5 |
| 13 | Proprietors' income with inventory valuation and capital consumption adjustments | 1,214.4 | 1,217.8 | 1,220.0 | 1,247.5 | 1,334.6 | 1,341.5 | $1,360.7$ | 1,358.5 | 1,359.5 |
| 14 | Rental income of persons with capital consumption adjustment | 524.8 | 537.8 | 546.7 | 555.4 | 574.9 | 587.7 | 596.6 | 603.2 | 611.9 |
| 15 | Corporate profits with inventory valuation and capital consumption adjustments, domestic industries | 1,564.0 | 1,569.1 | 1.599 .8 | 1,629.1 | 1,622.1 | 1,684.3 | $1,706.8$ | 1,730.9 | 1,558.4 |
| 16 | Taxes on corporate income | 437.2 | 429.7 | 439.1 | 433.2 | 408.2 | 418.2 | 417.8 | 431.1 | 458.9 |
| 17 | Profits after tax with inventory valuation and capital consumption adjustments | 1,126.8 | 1,139.4 | 1,160.7 | 1,196.0 | 1,213.8 | 1,266.1 | 1,289.0 | 1,299.8 | 1,099.5 |
| 18 | Net dividends | 569.1 | 572.5 | 577.3 | 735.3 | 616.6 | 874.7 | 769.4 | 787.8 | 674.7 |
| 19 | Undistributed corporate profits with inventory valuation and capital consumption adjustments | 557.8 | 566.9 | 583.4 | 460.7 | 597.3 | 391.4 | 519.5 | 512.0 | 424.8 |
| 20 | Current surplus of government enterprises 1 | -24.5 | -26.1 | -28.5 | -31.8 | -35.5 | -39.0 | -41.4 | -44.3 | -45.5 |
| 21 | Consumption of fixed capital | 2,507.6 | 2,533.7 | 2,555.1 | 2,575.0 | 2,603.8 | 2,631.9 | 2,659.6 | 2,691.0 | 2,721.9 |
| 22 | Private | 2,018.7 | 2,041.0 | 2.059 .8 | $2,077.6$ | 2,103.3 | 2,128.5 | 2,153.5 | 2,180.5 | 2,208.6 |
| 23 | Govermment | 488.9 | 492.7 | 495.3 | 497.4 | 500.5 | 503.4 | 506.1 | 510.5 | 513.3 |
|  | Addendum: |  |  |  |  |  |  |  |  |  |
| 24 | Statistical discrepancy | -63.0 | 10.1 | 86.4 | -101.7 | -155.6 | -186.8 | -91.7 | .91.8 | -105.3 |

## Two main contributions of Karabarbounis and Neiman

 (2014)- Measurement: Compiling data for corporate labor shares for ~ 60 countries.
- Estimation: New method (using cross-sectional data) of estimating capital-labor substitutability $\left(\sigma \equiv \frac{d \log (K / N)}{d \log (w / r)}\right)$.


## Why do we care about $\sigma$ ?

- Does an increase in $\frac{K}{L}$ increase incentive to innovate in laboror capital-intensive technologies? (Acemoglu, 2002, 2003)
- How much of the GDP per capita differences between poor and rich countries is explained by differences in $\frac{K}{L}$ ? (Caselli, 2005)
- What are the welfare effects from the observed changes to the labor share? Llater today.


## Outline

- Data sources
- Stylized facts
- Labor share
- Relative price of capital
- Theory: Linking the labor share to the relative price of capital
- Estimating $\sigma$ and sources of the decline in the labor share


## Labor Share Data

- Decomposition of GDP

$$
\begin{aligned}
Y & =\underbrace{Q_{C}}_{\text {Corporate VA }}+Q_{H}+Q_{G}+\text { Tax }_{\text {products }} \\
Q_{C} & =W_{C} N_{C}+\text { Tax }_{\text {production, } \mathrm{C}}+\text { Operating Surplus }
\end{aligned}
$$

- Total labor share $=\frac{W N}{Y}$
- Corporate labor share $=\frac{W_{C} N_{C}}{Q_{C}}$
- Major data sources
- Country-specific web pages, UN + OECD websites, books
- EUKLEMS : Includes data by industry. No seperation into corporate vs. household/government.


## Investment Price Data

1. Penn World Tables

$$
\xi_{i t}=\frac{P_{I, i, t}^{P P P} / P_{I, U S, t}^{P P P}}{P_{C, i, t}^{P P P} / P_{C, U S, t}^{P P P}} \times \frac{P_{I, U S, t}^{B E A}}{P_{C, U S, t}^{B E A}}
$$

From the second term: incorporate adjustments that the BEA makes for relative improvements the quality of investment/consumption goods.
2. World Bank: World Development Indicators (Fixed Investment Deflator, CPI)
3. EUKLEMS

## Both the overall and corporate labor share are declining



## The labor share is declining for most countries



The labor share is declining for most industries


## Changes in the labor share come from "within industry" changes

$$
\Delta s_{L i}=\underbrace{\sum_{k} \bar{\omega}_{i, k} \Delta s_{L i, k}}_{\text {Within-industry }}+\underbrace{\sum_{k} \bar{s}_{L i, k} \Delta \omega_{i, k}}_{\text {Between-industry }}
$$



## Investment Price Decline, Across Data Sources



## Model: Overview

1. Goal: Account for the decline of the labor share.
2. Two sectors: Producing consumption goods and investment goods.
2.1 Produce using capital \& labor with identical production (CES) technologies.
2.2 Relative price of the two goods dictated by technology differences $(\xi)$.
2.3 Inputs are supplied by monopolistically competitive (with markup $\mu$ ) continuum of firms.
3. Household side straightforward.
4. Key parameter of interest: $\sigma$, elasticity of substitution between capital/labor

## Model: Household Problem

- Maximize

$$
\begin{aligned}
& \max _{\left\{C_{t}, L_{t}, X_{t}, K_{t+1}, B_{t+1}\right\}} \sum \beta^{t} V\left(C_{t}, N_{t} ; \chi_{t}\right) \text { subject to } \\
& W_{t} L_{t}+R_{t} K_{t}+\Pi_{t}=C_{t}+\xi X_{t}+B_{t+1}-\left(1+r_{t}\right) B_{t} \\
& K_{t+1}=(1-\delta) K_{t}+X_{t}
\end{aligned}
$$

- FOC for capital:

$$
R_{t+1}=\xi_{t}\left(1+r_{t+1}\right)-\xi_{t+1}(1-\delta)
$$

$\xi_{t}=$ price of the investment good at time $t$ (more details on the next slide).

- Euler Equation:

$$
\beta\left(1+r_{t+1}\right)=\frac{V_{C}\left(C_{t}, N_{t} ; \chi_{t}\right)}{V_{C}\left(C_{t+1}, N_{t+1} ; \chi_{t+1}\right)}
$$

## Model: Production

- Three products: intermediate inputs $z \in\{0,1\}$, final investment good $X$, final consumption good $C$.

$$
C_{t}=\left[\int_{0}^{1} c_{t}(z)^{\frac{\varepsilon_{t}-1}{\varepsilon_{t}}} d z\right]^{\frac{\varepsilon_{t}}{\varepsilon_{t}-1}} ; X_{t}=\frac{1}{\xi_{t}}\left[\int_{0}^{1} x_{t}(z)^{\frac{\varepsilon_{t}-1}{\varepsilon_{t}}} d z\right]^{\frac{\varepsilon_{t}}{\varepsilon_{t}-1}}
$$

- Intermediate input supplier:

$$
y_{t}(z)=\left(\alpha^{\frac{1}{\sigma}}\left(A_{K, t} k_{t}(z)\right)^{\frac{\sigma-1}{\sigma}}+(1-\alpha)^{\frac{1}{\sigma}}\left(A_{L, t} n_{t}(z)\right)^{\frac{\sigma-1}{\sigma}}\right)^{\sigma /(\sigma-1)}
$$

$A_{K, t}$ and $A_{L, t}$ are capital- and labor-augmenting productivity.

- Market-clearing conditions:

$$
\begin{aligned}
y_{t}(z) & =c_{t}(z)+x_{t}(z) \\
K_{t} & =\int_{0}^{1} k_{t}(z) d z \\
L_{t} & =\int_{0}^{1} n_{t}(z) d z
\end{aligned}
$$

## Model: Input choices of each intermediate input supplier

- Problem of the intermediate input supplier:

$$
\max p_{t}(z) y_{t}(z)-k_{t}(z) R_{t}-n_{t}(z) W_{t}
$$

- First order conditions (For each $z$ ):

$$
\begin{aligned}
R_{t} & =\frac{\partial\left(p_{t} y_{t}\right)}{\partial k_{t}}=\frac{\partial\left(\left(\frac{y_{t}}{Y_{t}}\right)^{-\frac{1}{\varepsilon}} y_{t}\right)}{\partial k_{t}} \\
& =k_{t}^{-\frac{1}{\sigma}} \frac{\left(Y_{t}\right)^{\frac{1}{\varepsilon_{t}}}}{\mu_{t}}(1-\alpha)^{\frac{1}{\sigma}}\left(A_{K t}\right)^{\frac{\sigma-1}{\sigma}}\left(y_{t}\right)^{\frac{\varepsilon-1}{\varepsilon}-\frac{\sigma-1}{\sigma}} \Rightarrow \\
\mu_{t} R_{t} & =\alpha^{\frac{1}{\sigma}}\left(A_{K t}\right)^{\frac{\sigma-1}{\sigma}} p_{t}\left(\frac{k_{t}}{y_{t}}\right)^{-\frac{1}{\sigma}} \Rightarrow \mu_{t} \underbrace{k_{t} R_{t}}_{s_{K, t}(z)} \frac{y_{t} p_{t}}{}=\alpha\left(\frac{A_{K t}}{\mu_{t} R_{t}}\right)^{\sigma-1}
\end{aligned}
$$

- Similarly:

$$
\mu_{t} \frac{I_{t} W_{t}}{\underbrace{y_{t} p_{t}}_{S_{L, t}(z)}}=(1-\alpha)\left(\frac{A_{L t}}{\mu_{t} W_{t}}\right)^{\sigma-1}
$$

## Model: Input choices of each intermediate input supplier

From the last slide:

$$
\mu_{t}(z) s_{K, t}=\alpha\left(\frac{A_{K t}}{\mu_{t}(z) R_{t}}\right)^{\sigma-1}
$$

But also:

$$
s_{\Pi_{t}}(z) \equiv \frac{\Pi_{t}(z)}{p_{t}(z) \cdot y_{t}(z)}=\frac{\mu_{t}-1}{\mu_{t}}
$$

Since

$$
\begin{aligned}
s_{\Pi t}(z)+s_{L t}(z)+s_{K t}(z) & =1 \\
\mu_{t} s_{L t}(z)+\mu_{t} s_{K t}(z) & =1
\end{aligned}
$$

Thus:

$$
1-\mu_{t} s_{L t}(z)=\alpha\left(\frac{A_{K t}}{\mu_{t} R_{t}}\right)^{\sigma-1}
$$

Comparing two periods:

$$
\left(\frac{1}{1-s_{L} \mu}\right)\left(1-s_{L}\left(1+\hat{s}_{L}\right) \mu(1+\hat{\mu})\right)=\left(\frac{1+\hat{A}_{K}}{1+\hat{R}}\right)^{\sigma-1}(1+\hat{\mu})
$$

## Model: Estimating Equation

From the last slide:

$$
\left(\frac{1}{1-s_{L} \mu}\right)\left(1-s_{L}\left(1+\hat{s}_{L}\right) \mu(1+\hat{\mu})\right)=\left(\frac{1+\hat{A}_{K}}{1+\hat{R}}\right)^{\sigma-1}(1+\hat{\mu})
$$

From the FOC for capital:

$$
\begin{gathered}
1+\hat{R}=(1+\hat{\xi}) \cdot\left(1-\hat{\delta} \frac{\beta \delta}{1-\beta+\beta \delta}\right) \\
\left(\frac{1}{1-s_{L} \mu}\right)\left(1-s_{L}\left(1+\hat{s}_{L}\right) \mu(1+\hat{\mu})\right) \\
=\left(\frac{1+\hat{A}_{K}}{1+\hat{\xi}}\right)^{\sigma-1}(1+\hat{\mu})^{\sigma-1}\left(1-\hat{\delta} \frac{\beta \delta}{1-\beta+\beta \delta}\right)^{1-\sigma}
\end{gathered}
$$

So, the labor share can change if $\xi, A_{K}, \mu$ or $\delta_{\square}$ change.

## Estimation

Set $\mu-1=\hat{\mu}=\hat{\delta}=0$. Take logs:

$$
\frac{s_{L}}{1-s_{L}} \hat{s}_{L}=(\sigma-1) \hat{\xi}+\underbrace{(1-\sigma) \hat{A}_{K}}_{\gamma+u}
$$

In the benchmark regressions, assume $\hat{\xi} \perp \hat{A}_{K}$.

## Estimation

$$
\frac{s_{L}}{1-s_{L}} \hat{s}_{L}=\gamma+(\sigma-1) \hat{\xi}+u
$$



- Slope: $0.28 \Rightarrow \hat{\sigma} \approx 1.28$.


## Estimation

$\left.\begin{array}{cclc}\hline \begin{array}{c}\text { Investment } \\ \text { Price }\end{array} & \begin{array}{l}\text { Labor } \\ \text { Share }\end{array} & \hat{\sigma} & \text { Obs } \\ \hline \text { PWT } & \text { KN Merged } & \begin{array}{l}1.25 \\ (0.08)\end{array} & 58 \\ \text { WDI } & \text { KN Merged } & \begin{array}{l}1.29 \\ (0.07)\end{array} & 54 \\ & & \begin{array}{l}1.20\end{array} & 50 \\ \text { PWT } & \text { OECD \& UN } & \begin{array}{l}1.208) \\ \text { WDI }\end{array} & \text { OECD \& UN }\end{array} \begin{array}{l}1.31 \\ (0.06)\end{array}\right) 47$.

## Markup Shocks?

What if $\hat{\mu}_{j} \neq 0$ or $\mu_{j} \neq 1$ ?

$$
\left(\frac{s_{L j} \mu_{j}}{1-s_{L j} \mu_{j}}\right)\left(\hat{s}_{L j}+\hat{\mu}_{j}+\hat{s}_{L j} \hat{\mu}_{j}\right)=\gamma+(\sigma-1)\left(\hat{\xi}_{j}+\hat{\mu}_{j}\right)+u_{j}
$$

- Assuming $\beta, \delta$ are constant over time, same for all countries:

$$
\begin{aligned}
& s_{K j}=\frac{R_{j} K_{j}}{Y_{j}}=\frac{\xi_{j} X_{j}}{Y_{j}}\left(\frac{1 / \beta-1+\delta}{\delta}\right) \\
& \hat{s}_{K_{j}}=\widehat{\xi_{j} X_{j} / Y_{j}}
\end{aligned}
$$

- From before $\mu s_{L j}+\mu s_{K j}=1$. And:

$$
\hat{\mu}_{j}=\frac{1}{\mu_{j}\left(s_{L j} \hat{s}_{L j}+s_{K j} \hat{s}_{K j}\right)}
$$

## Markup Shocks?


$\Rightarrow$ Countries with declining labor shares had (on average) declines in capital shares and markups.

## Markup Shocks?

| Investment <br> Price | Investment <br> Rate | $\hat{\sigma}$ | Obs |
| :---: | :---: | :---: | :---: |
| PWT | Corporate | 1.03 <br> $(0.09)$ | 55 |
| WDI | Corporate | 1.29 <br> $(0.08)$ | 52 |
|  | Total | 1.11 <br> $(0.11)$ | 54 |
| PWT | Total | 1.35 <br> $(0.08)$ | 52 |
| WDI |  |  |  |

## Capital-Augmenting Technical Change?

Again, when $\mu=\hat{\mu}-1=\hat{\delta}=0$ :

$$
\frac{s_{L}}{1-s_{L}} \hat{s}_{L}=\gamma+(\sigma-1) \hat{\xi}+(1-\sigma) \hat{A}_{K}+u
$$

Up to know, we had assumed $\operatorname{corr}\left(\hat{A}_{k}, \hat{\xi}\right)=0$. If not:

$$
\underbrace{\tilde{\sigma}-\sigma}_{\text {Bias }}=(1-\sigma) \operatorname{corr}\left(\hat{A}_{k}, \hat{\xi}\right) \frac{\operatorname{sd}\left(\hat{A}_{k}\right)}{\operatorname{sd}(\hat{\xi})}
$$

- If $\operatorname{corr}\left(\hat{A}_{k}, \hat{\xi}\right)<0$, then
- $\tilde{\sigma}>\sigma$ iff $\sigma>1$
- $\tilde{\sigma} \rightarrow \sigma$ if $\sigma \rightarrow 1$.


## Capital-Augmenting Technical Change?

- From the last slide:

$$
\underbrace{\tilde{\sigma}-\sigma}_{\text {Bias }}=(1-\sigma) \operatorname{corr}\left(\hat{A}_{k}, \hat{\xi}\right) \frac{\operatorname{sd}\left(\hat{A}_{k}\right)}{\operatorname{sd}(\hat{\xi})}
$$

- If
$-\operatorname{corr}\left(\hat{A}_{k}, \hat{\xi}\right)=-0.28$
$-\operatorname{sd}\left(\hat{A}_{k}\right)=0.10$
- $\operatorname{sd}(\tilde{\xi})=0.11$
- then if $\sigma=1.25 \Rightarrow \tilde{\sigma}=1.20$

Effect of the markup and investment price shocks

| $\sigma$ | 1 | 1.25 | 1 | 1.25 | 1 | 1.25 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\hat{\xi}$ |  | $\hat{\mu}$ |  |  |  |
| Labor share <br> (\% points) | 0.0 | -2.6 | -3.1 | -2.6 | -3.1 | -4.9 |
| Capital share | 0.0 | 2.6 | -1.9 | -2.4 | -1.9 | -0.1 |
| (\% points) | 0.0 | 0.0 | 5.0 | 5.0 | 5.0 | 5.0 |
| Profit share <br> (\% points) | -22.1 | -22.1 | 0.0 | 0.0 | -22.1 | -22.1 |
| Rental rate |  |  |  |  |  |  |
| Capital-to-output <br> Welfare-equiv. <br> consumption <br> 28.4 | 36.6 | -5.2 | -6.4 | 21.8 | 27.9 |  |

Notes on Oberfield and
Raval (2014) "Micro Data and Macro Technology"

## Two additions

From the board:

$$
\sigma_{n}^{N}=\left(1-\chi_{n}\right) \sigma+\chi_{n} \varepsilon
$$

1. Include materials in plants' production functions:

$$
F\left(K_{n i}, L_{n i}, \quad\right)=\left[\left[\left(A_{n i} K_{n i}\right)^{\frac{\sigma_{n}-1}{\sigma_{n}}}+\left(B_{n i} L_{n i}\right)^{\frac{\sigma_{n}-1}{\sigma_{n}}}\right]^{\frac{\sigma_{n}}{\sigma_{n}-1}}\right.
$$

$$
]
$$

2. Write out the aggregate elasticity in terms of industry-level terms.

## Two additions

From the board:

$$
\sigma_{n}^{N}=\left(1-\chi_{n}\right) \sigma+\chi_{n} \varepsilon
$$

1. Include materials in plants' production functions:

$$
\begin{aligned}
F\left(K_{n i}, L_{n i}, M_{n i}\right) & =\left[\left[\left(A_{n i} K_{n i}\right)^{\frac{\sigma_{n}-1}{\sigma_{n}}}+\left(B_{n i} L_{n i}\right)^{\frac{\sigma_{n}-1}{\sigma_{n}}}\right]^{\frac{\sigma_{n}}{\sigma_{n}-1} \frac{\zeta_{n}-1}{\zeta_{n}}}\right. \\
& \left.+C_{n i} M_{n i}^{\frac{\sigma_{N}}{\sigma_{N}-1} \frac{\zeta_{N}-1}{\zeta_{N}}}\right]^{\frac{\zeta_{n}}{\zeta_{n}-1}}
\end{aligned}
$$

2. Write out the aggregate elasticity in terms of industry-level terms.

## Building up to the aggregate EoS

- The industry-level elasticity of substitution equals:

$$
\begin{aligned}
\sigma_{n}^{N} & =\left(1-\chi_{n}\right) \sigma_{n}+\chi_{n}\left[\left(1-\bar{s}_{n}^{M}\right) \varepsilon_{n}+\bar{s}_{n}^{M} \zeta_{n}\right] \\
\text { where } \chi_{n} & =\sum_{i} \frac{\left(\alpha_{n i}-\alpha_{n}\right)^{2}}{\left(1-\alpha_{n}\right) \alpha_{n}} \theta_{n i}, \text { and }
\end{aligned}
$$

$\bar{s}_{n}^{M}$ is a weighted average of plants' intermediate input shares.

- The aggregate elasticity of substitution equals:

$$
\sigma^{a g g}=\left(1-\chi^{a g g}\right) \bar{\sigma}^{N}+\chi^{\operatorname{agg}}\left[\left(1-\bar{s}^{M}\right) \eta+\bar{s}^{M} \bar{\zeta}_{n}\right]
$$

where $\chi^{\text {agg }}=\sum_{i} \frac{\left(\alpha_{n}-\alpha\right)^{2}}{(1-\alpha) \alpha} \theta_{n}$, and

- $\bar{\sigma}^{N}\left(\bar{\zeta}_{n}\right)$ is a weighted average of the industry capital-labor (materials) EoS.
- $\bar{s}^{M}$ is a weighted average of industries' intermediate input shares.


## The Census of Manufacturers \& Annual Survey of Manufacturers

- Census of Manufacturers (CM)
- All plants within the US with $\geq 5$ employees (180,000 out of 350,000)
- Every five years (1972, 1977,... 2012)
- Book value of capital is imputed for non ASM plants (except for 1987, 1997)
- Materials expenditures, labor expenditures, output.
- Annual Survey of Manufacturers
- A subset of plants $(50,000)$, oversampling of larger plants
- Materials expenditures, labor expenditures, output.


## Building blocks of $\sigma_{n}^{N}$

- $\chi$ : variation in plant-level capital shares (within value added)
- $\bar{s}_{n}^{M}$ : average materials cost share
- $\sigma_{n}$ : plant-level elasticity of substitution, between capital and labor
- $\varepsilon_{n}$ : elasticity of demand
- $\zeta_{n}$ : elasticity of substitution between materials and value added


## Building blocks of $\sigma_{n}^{N}=\left(1-\chi_{n}\right) \sigma_{n}+\chi_{n}\left[\left(1-\bar{s}_{n}^{M}\right) \epsilon_{n}+\bar{s}_{n}^{M}\right]$ :

$\chi_{N}$

$\chi_{n} \approx 0 \Rightarrow \sigma_{n}^{N} \approx \sigma_{n}$

## Building blocks of

$\sigma_{n}^{N}=\left(1-\chi_{n}\right) \sigma_{n}+\chi_{n}\left[\left(1-\bar{s}_{n}^{M}\right) \varepsilon_{n}+\bar{s}_{n}^{M}\right]: \sigma_{n}$
From the plants' cost-minimization condition:

$$
\log \left(\frac{r K}{w N}\right)_{n i}=\kappa+\left(\sigma_{n}-1\right)\left(\frac{w}{R}\right)_{n i}
$$

Specification from Raval (2014):

$$
\log \left(\frac{r K}{w N}\right)_{n i}=\kappa+\left(\sigma_{n}-1\right) \log w_{n i}^{M S A}+\text { Controls }+\epsilon_{n i}
$$

- $w_{n i}^{M S A}$ : hourly wage in the MSA of plant $i$, after controlling for worker education, experience, industry, occupation, demographics.
- Controls: age of the plant, indicator for whether it is part of a multi-unit firm.
- Key Assumptions: $R_{n i} \perp w_{n i}^{M S A}$ (or more generally, $\left.w_{n i}^{M S A} \perp \epsilon_{n i}\right)$


## Building blocks of

$\sigma_{n}^{N}=\left(1-\chi_{n}\right) \sigma_{n}+\chi_{n}\left[\left(1-\bar{s}_{n}^{M}\right) \varepsilon_{n}+\bar{s}_{n}^{M}\right]: \sigma_{n}$


Average of $\sigma_{n} \approx 0.5$.

## Building blocks of

$\sigma_{n}^{N}=\left(1-\chi_{n}\right) \sigma_{n}+\chi_{n}\left[\left(1-\bar{s}_{n}^{M}\right) \varepsilon_{n}+\bar{s}_{n}^{M}\right]: \zeta_{n}$

From the plants' cost-minimization condition:
Similar specification to identify $\zeta$ :

$$
\log \left(\frac{q M}{w N+r K}\right)_{n i}=(\zeta-1)\left(1-\alpha_{i}\right) \log w_{n i}^{M S A}+\text { Controls }+\epsilon_{n i}
$$

Results from pooled regression


## Building blocks of

$\sigma_{n}^{N}=\left(1-\chi_{n}\right) \sigma_{n}+\chi_{n}\left[\left(1-\bar{s}_{n}^{M}\right) \varepsilon_{n}+\bar{s}_{n}^{M}\right]: \bar{s}_{n}^{M}$ and $\varepsilon_{N}$

- $\bar{s}_{n}^{M}$, average materials cost share: average $=0.59$.
- $\varepsilon_{n}$ : Demand elasticity.
- According to the model, the markup equals revenues divided by total costs $\Rightarrow \frac{\varepsilon_{n}}{\varepsilon_{n}-1}=\frac{P_{n i} Y_{n i}}{w L_{n i}+K K n i+q M_{n i}}$
- $\varepsilon_{n} \in[3,5]$


## Building blocks of $\sigma_{n}^{N}=\left(1-\chi_{n}\right) \sigma_{n}+\chi_{n}\left[\left(1-\bar{s}_{n}^{M}\right) \varepsilon_{n}+\bar{s}_{n}^{M}\right]$ : $\left[\left(1-\bar{s}_{n}^{M}\right) \varepsilon_{n}+\bar{s}_{n}^{M}\right]$

Elasticity of Demand
$\triangle$ Scale Elasticity

## Building blocks of $\sigma_{n}^{N}=\left(1-\chi_{n}\right) \sigma_{n}+\chi_{n}\left[\left(1-\bar{s}_{n}^{M}\right) \varepsilon_{n}+\bar{s}_{n}^{M}\right]$



- Plant $\Delta$ Industry


## Building blocks of

$\sigma^{a g g}=\left(1-\chi^{a g g}\right) \bar{\sigma}_{n}^{N}+\chi^{a g g}\left[\left(1-\bar{s}^{M}\right) \eta+\bar{s}^{M} \bar{\zeta}_{n}\right]$

- $\eta$, elasticity of demand across industries: 1
- $\bar{\sigma}_{n}^{N}, \chi^{\text {agg }}, \bar{s}^{M}$, and $\bar{\zeta}_{n}$ all come from industry-level data.
- Estimate in 1987: 0.70
- Allowing the $\chi \mathrm{s}$, $\bar{s}$ to vary across years:



## $\sigma^{a g g}$ ranges from 0.80 to 1.15 for other countries



## Reminder: The labor share has fallen



## Why has the labor share fallen? A decomposition

$$
\begin{aligned}
d s^{v, L} & =\frac{\partial s^{v, L}}{\partial \log w / r} d \log w / r+\left[d s^{v, L}-\frac{\partial s^{v, L}}{\partial \log w / r} d \log w / r\right] \\
& =\left(1-\sigma^{a g g}\right) d \log w / r+\left[d s^{v, L}-\frac{\partial s^{v, L}}{\partial \log w / r} d \log w / r\right]
\end{aligned}
$$

Data on $w, r$ :

- For $w$ : NIPA. $w=\frac{\text { Labor compensation }}{\text { Employees }}$, adjust for changes in skills.
- For $r$ :
- Capital prices from NIPA
- Real rental rate of capital 3.5\%
- Tax rates and depreciation allowances from Jorgenson
$w / r$ has gone up \& $1-\sigma^{a g g}>0 \Rightarrow$ Contribution of factor prices is positive.


## Almost none of the change in the labor share is from $\mathrm{w} / \mathrm{r}$ increasing.



## The discrepancy between Oberfield and Raval and Karabarbounis and Neiman?

- Sample: Manufacturing (OR) vs the whole economy (KN)

Primary
Construction

Manuf. Transport

| $\theta$ | 0.03 | 0.05 | 0.20 | 0.61 |
| :--- | :--- | :--- | :--- | :--- |
| $\alpha$ | 0.55 | 0.19 | 0.35 | 0.45 |


| Electricity/ <br> Gas Serv. | Wholesale/ <br> Retail | FIRE | Other <br> Services |  |
| :--- | :---: | :---: | :---: | :---: |
| $\theta$ | 0.05 | 0.15 | 0.17 | 0.28 |
| $\alpha$ | 0.58 | 0.29 | 0.67 | 0.18 |
| $\chi^{\text {full }}=0.14$. |  |  |  |  |

- Omitted variable bias? See Loukas' discussion of OR on his webpage.

