Problem Set 5: Due Sunday, October 4

For this problem set, please e-mail Problem 1 and Problem 2 as separate documents.

Problem 1

Submit a 2-page outline for your Final Paper project. The syllabus, on the bottom half of page 1, describes five questions that you should answer in your project. Do your best to address these questions. Next week, for Problem Set 6, a classmate will look over your proposal and assess your answer to these questions.

Problem 2

On Monday, we'll discuss Gabaix's granular residual paper. The subsequent questions consider the draft of the paper given at

http://pages.stern.nyu.edu/~xgabaix/papers/granular.pdf.

1. What is the contribution of this paper? What gap in the literature does this paper fill?

2. In this part of the problem, we will derive the paper's Equation (19) using a specific set of assumptions on preferences/productivities. This equation follows from a more general set of assumptions, but it will be helpful to work through the calculations for a specific case.

   Consider a model with a representative consumer with preferences over labor supply L_t and consumption, C_{it}, over the goods produced by the N firms in the economy:

   $$U\left(\{C\}_{it}, L_t\right) = \log C_t - \frac{\phi}{\phi+1} L_t^{\frac{\phi+1}{\phi}}, \text{ where } \log C_t = \sum_{i=1}^{N} \xi_i \log\left(\frac{C_{it}}{\xi_i}\right) \tag{A}$$

   Each firm produces using labor and intermediate inputs

   $$Q_{it} = A_{it}\left(\frac{L_{it}}{\alpha}\right)^{\alpha}\left(\frac{M_{it}}{1-\alpha}\right)^{1-\alpha} \tag{B}$$

   Here A_{it} is the productivity of firm i at time t, L_{it} and M_{it} are labor and intermediate inputs used by firm i at time t. Feel free to assume that all firms have the same intermediate input share, $1-\alpha$. Intermediate inputs are a combination of inputs purchased from other firms

$$M_{it} = \prod_j \left(\frac{M_{j\to i,t}}{\gamma_{ji}}\right)^{\gamma_{ji}} \text{ where } \sum_j \gamma_{ji} = 1 \tag{C}$$

The market clearing conditions for labor and firm i's product are

$$L_t = \sum L_{it} \text{ and } Q_{it} = C_{it} + \sum_j M_{i\to j,t} \tag{D}$$

Write W_t and P_{it} as the Lagrange multipliers for the labor and good i market-clearing conditions, define sales as $S_i = P_i Q_i$, and set C as the numeraire so that $P_t = \prod_i P_{it}^{\xi_i} = 1$.

(a) Write out the Lagrangian of the constrained maximization problem, for a social planner trying to maximize U, subject to equations B, C, and D.

(b) Consider the problem of the representative consumer who is trying to maximize

$$\log C_t - \frac{\phi}{\phi+1} L_t^{\frac{\phi+1}{\phi}} \text{ subject to } C_t = W_t L_t$$

Solve for the utility maximizing L and C in terms of W.

(c) Take first order conditions with respect to L_{it} and $M_{j\to i,t}$. Plug these in to the production function (Equation B). From here you should be able to write out $\log P_{it}$ as a function of $\log A_{it}$, $\log W$, and the set of $\log P_{jt}$. In matrix form, you should be able to write:

$$\overrightarrow{\log P_t} = \Xi \left(\overrightarrow{\log A_t} + \alpha \log W\right)$$

for some matrix Ξ. What is the matrix Ξ?

(d) Consider the market-clearing condition for good i

$$Q_{it} = C_{it} + \sum_j M_{i\to j,t}$$

Multiply this market clearing condition by P_{it}. Then, use the first order condition for $M_{i\to j,t}$ (which should be a function of model parameters, Q_{jt}, P_{it}, and P_{jt}), and rearrange. In matrix form, you should arrive at an equation that looks like

$$\overrightarrow{S_t} = \Psi \overrightarrow{\xi} C_t$$

What is the matrix Ψ?

(e) Using your answer to parts (c) and (d), in addition to the normalization that $\xi' \overrightarrow{\log P_t} = 0$, write out C_t that takes the form given in the paper's Equation (19).