Notes on Foerser, Sarte, Watson (2011) "Sectoral vs. Aggregate Shocks: A Structural Factor Analysis of Industrial Production"

Research questions

1. How correlated are shocks to industries' productivities?
2. What fraction of industrial production volatility is due to common shocks? Industry-specific shocks?
3. Are the answers to (1) and (2) different for different points in the sample? Were common shocks or industry-specific shocks less volatile during the Great Moderation (after 1983)?

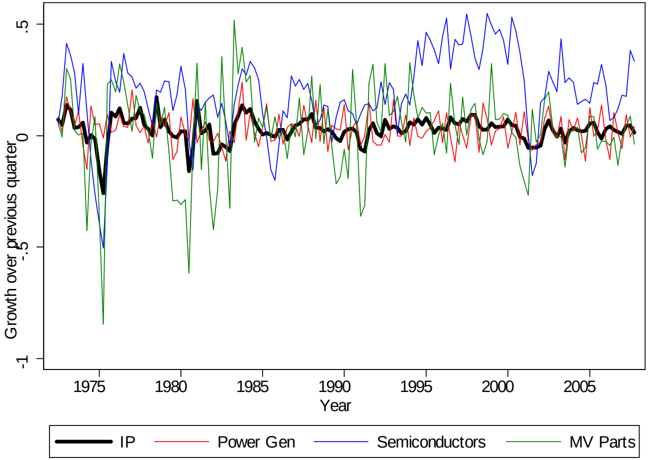
Outline

- Data
- Statistical factor analysis
- Model and structural factor analysis

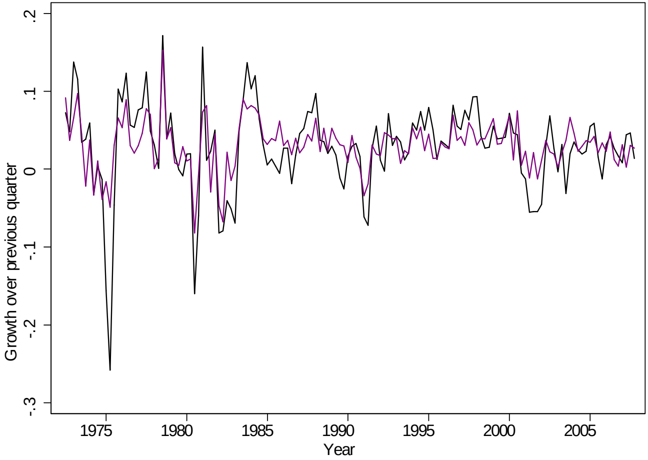
Data

- BEA: Input/Output Table & Capital Flow Table, from 1997.
- Federal Reserve Board: Quarterly data on industrial production, from 1972 to 2011.
  - Quarterly data
  - 117 industries in manufacturing, mining, energy, and publishing.

Industrial production and its components



Industrial production tracks GDP



Principal component analysis

- Define g_t as the vector of sectoral growth rates, $\log\left(\frac{Y_{t+1}}{Y_t}\right)$, and w_t as the weight of each industry within the industrial sector.
- How can we best measure the fraction of variation in $w_t' \cdot g_t$ that is due to "common shocks"?
- Suppose that

$$\underset{117\times 1}{g_t} = \underset{117\times 2}{\Lambda} \cdot \underset{2\times 1}{F_t} + \underset{117\times 1}{u_t}$$

  where F_t is some small (e.g., two) number of common factors, and u_t are idiosyncratic shocks (the covariance matrix of u has zero off-diagonal terms), and F_t and u_t are uncorrelated.
- Use principal component analysis to choose Λ, F_t so that ΛF explains the maximum possible variance of the g_t vector. These columns of F will represent the common shocks.

Principal component analysis

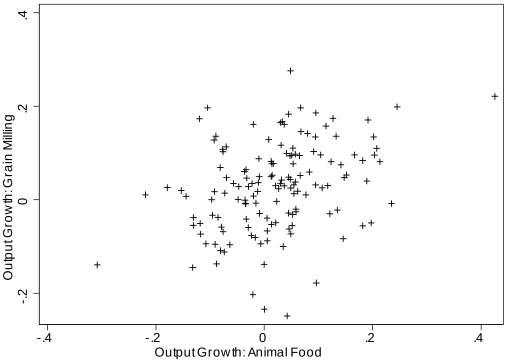
- From the last slide

$$g_t = \Lambda \cdot F_t + u_t$$

- Note that Λ and F_t are not separately identified. $\Lambda F_t = \underbrace{\Lambda \vartheta}_{\tilde{\Lambda}} \underbrace{\vartheta^{-1} F_t}_{\tilde{F}_t}$. We will normalize Λ so that the lengths of each column equal 1.
- Useful formulas:

$$\begin{aligned} \Sigma_{gg} &= \Lambda \Sigma_{FF} \Lambda' + \Sigma_{uu} \\ \sigma_g^2 &= \bar{w}' \Lambda \Sigma_{FF} \Lambda' \bar{w} + \bar{w}' \Sigma_{uu} \bar{w} \\ R^2(F) &= \frac{\bar{w}' \Lambda \Sigma_{FF} \Lambda' \bar{w}}{\sigma_g^2} \end{aligned}$$

Principal component analysis: 2-d to 1-d



Principal component analysis

The idea is to find the linear combination of the data that explains the greatest possible variance

$$
\begin{aligned}
&\max_{\|\Lambda_1\|=1} \Lambda_1' \Sigma_{\text{Animal,Grain}} \Lambda_1 \\
&= \max_{\Lambda_1} \Lambda_1' \Sigma_{\text{Animal,Grain}} \Lambda_1 + \mu_1 \left(1 - \Lambda_1' \Lambda_1\right)
\end{aligned}
$$

First order conditions:

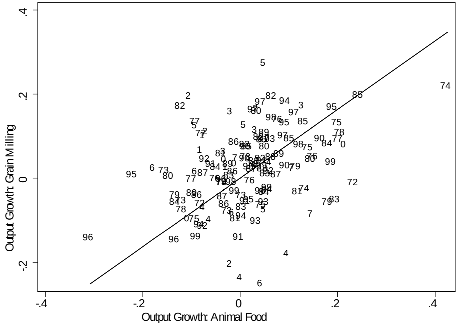
$$
\begin{aligned}
2\Sigma_{\text{Animal,Grain}} \Lambda_1 &= 2\mu_1 \Lambda_1 \\
\Sigma_{\text{Animal,Grain}} \Lambda_1 &= \mu_1 \Lambda_1
\end{aligned}
$$

Note that

$$
\Lambda_1' \Sigma_{\text{Animal,Grain}} \Lambda_1 = \Lambda_1' \mu_1 \Lambda_1 = \mu_1
$$

To maximize the left hand side, choose the unit-length eigenvector associated with the largest eigenvalue of $\Sigma_{\text{Animal,Grain}}$.

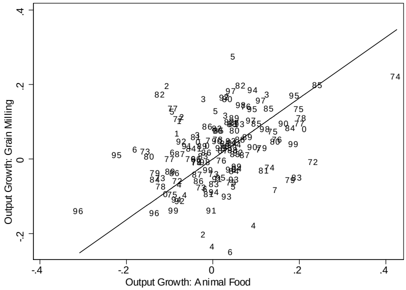
Principal component analysis: 2-d to 1-d

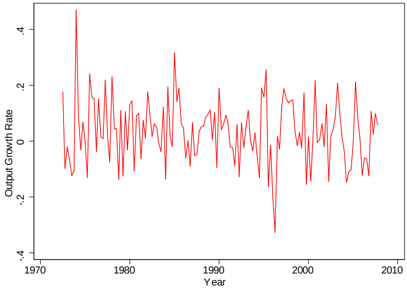


$$\Sigma_{\text{Animal,Grain}} = \begin{pmatrix} 0.010 & 0.003 \\ 0.003 & 0.009 \end{pmatrix}; \quad \Lambda_1 = \begin{pmatrix} 0.78 \\ 0.62 \end{pmatrix}$$

Principal component analysis: 2-d to 1-d

Retrieving the common factor, F





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Principal component analysis: 2-d to 1-d

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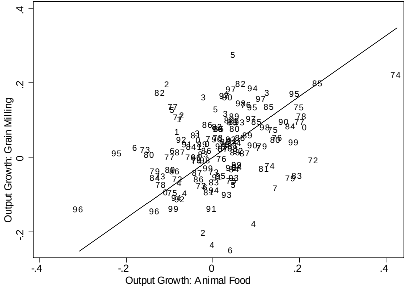

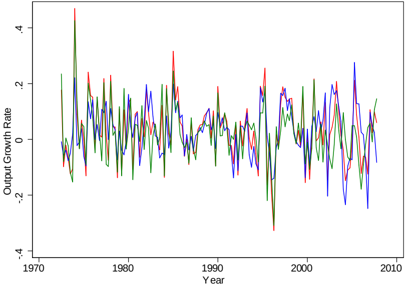

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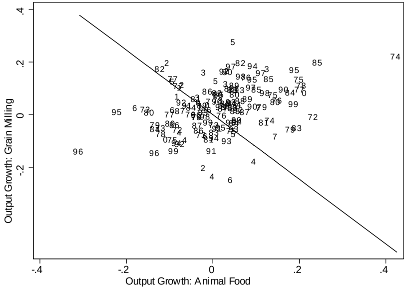
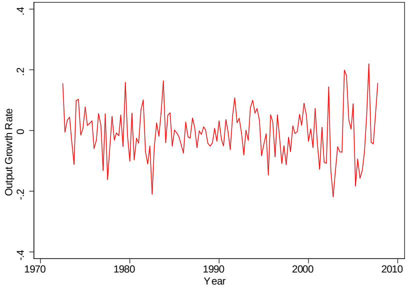
Principal component analysis: 2-d to 1-d

Retrieving the common factor, F

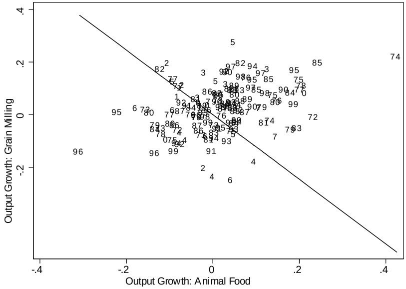


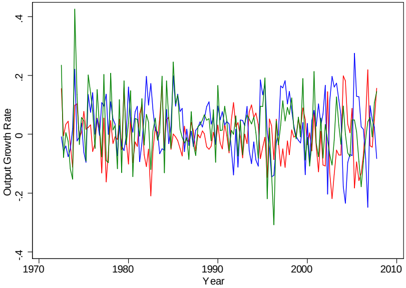


$$\Sigma_{\text{Animal,Grain}} = \begin{pmatrix} 0.010 & 0.003 \\ 0.003 & 0.009 \end{pmatrix}; \quad \Lambda_2 = -\begin{pmatrix} 0.62 \\ 0.78 \end{pmatrix}$$

Principal component analysis: 2-d to 1-d

Retrieving the common factor, F





$$\Sigma_{\text{Animal,Grain}} = \begin{pmatrix} 0.010 & 0.003 \\ 0.003 & 0.009 \end{pmatrix}; \quad \Lambda_2 = -\begin{pmatrix} 0.62 \\ 0.78 \end{pmatrix}$$

Principal component analysis

Can extend this idea to many (say 117) data series and multiple (say 2) factors.

Suppose we have 117 data series and we have computed the first factor $F_1 = g' \cdot \Lambda_1$. The problem is now to find a vector Λ_2 that is orthogonal to Λ_1 and explains the greatest possible variance:

$$\max_{\Lambda_2} \Lambda_2' \Sigma_{gg} \Lambda_2 + \mu_2 \left(1 - \Lambda_2' \Lambda_2\right) + \kappa \Lambda_2' \Lambda_1$$

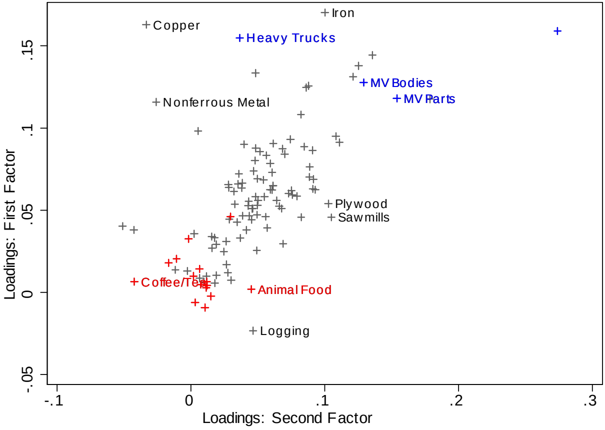
First order conditions:

$$\Sigma_{gg} \Lambda_2 = \mu_2 \Lambda_2$$

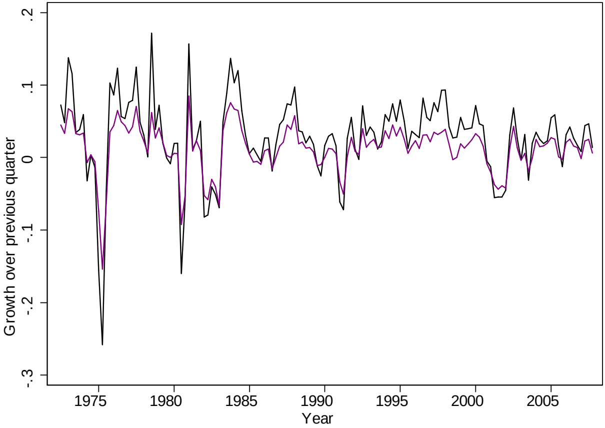
The solution to this maximization problem, Λ_2 will be the eigenvector associated with the second largest eigenvalue of Σ_{gg}

Side note: Bai and Ng (2003) look at how to choose the number of factors.

The two columns of Λ



Industrial production and its factor component



$R^2(F) \approx 0.9$

Partial summary

- The story so far: there is a strong common component to industrial production.
- Is this because there are common shocks? Or because there are independent shocks transmitted via input-output relationships?
- Rest of the paper: Use a model with input-output linkages to back out productivity shocks for each industry-quarter. Perform factor analysis on the productivity shocks.
  - N perfectly competitive sectors, which produce using capital, labor, and the output of other sectors.
  - Consumers have preferences over leisure and consumption of the goods produced by the N industries.
  - Productivity growth is distributed $\mathcal{N}(0, \Sigma_{\omega\omega})$; ω will admit an factor representation.

Model: Market Clearing

- Output can be used for consumption, as an intermediate input, or to increase one of the N capital stocks:

$$Y_{tj} = C_{tj} + \sum_{i=1}^{N} M_{t,j\to i} + \sum_{i=1}^{N} X_{t,j\to i} \quad \forall j \in \{1, ...N\}$$

Model: Preferences

- Consumers' lifetime utilities are given by:

$$U = \sum_{t=0}^{\infty} \beta^t \left[\sum_{i=1}^{N} \frac{(C_{ti})^{1-\sigma} - 1}{1-\sigma} - \psi L_{ti} \right]$$

- ψ: disutility from work
- σ: preference elasticity of substitution, intertemporal elasticity of substitution.

Model: Production

- The production technology of each sector is given by:

$$Y_{tj} = A_{tj} \left(K_{tj}\right)^{\alpha_j} M_{tj} \left(L_{tj}\right)^{1-\alpha_j-\sum_i \gamma_{ij}}$$

- The intermediate input bundle of sector j consists of the purchases from the other sectors:

$$M_{tj} = \prod_i M_{t,i\to j}^{\gamma_{ij}}$$

- γ_{ij} is the share of good i used in the production of the good-j intermediate input.

Model: Evolution of Capital, Productivity

- Industry-j-specific capital evolves according to:

$$K_{t+1,j} = (1-\delta)\, K_{tj} + Z_{tj}$$
$$\text{where } Z_{tj} = \prod_i X_{t,i\to j}^{\theta_{ij}}$$

- θ_{ij} is the share of good i used in the production of the good-j capital input.

- Productivity in each sector evolves according to a random walk:

$$\log A_{t+1,j} = \log A_{tj} + \omega_{t+1,j}, \qquad \omega_{t+1} \sim 0, \Sigma_{\omega\omega}$$

Model: Solution Outline

Goal: Recover the vector of ω shocks given data on output growth.

Solution:

- Write out Lagrangian of social planner. Take FOC
- Solve for steady state allocation
- Log linearization around the steady state
- System reduction
- Blanchard and Kahn
- Obtaining the model filter

Model: Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{j=1}^{N} \frac{C_{tj}^{1-\sigma} - 1}{1-\sigma} - \psi L_{tj} + \mu_{tj} \left[K_{t+1,j} - (1-\delta) K_{tj} + Z_{tJ} \right] \right.$$

$$\left. + P_{tJ} \left[A_{tj} (K_{tj})^{\alpha_j} M_{tj} (L_{tj})^{1-\alpha_j - \sum_i \gamma_{ij}} - C_{tj} + \sum_{i=1}^{N} M_{t,j\to i} + X_{t,j\to i} \right] \right\}$$

Write out the first order conditions for $M_{t,j\to i}$, $X_{t,j\to i}$, C_{tj}, and L_{tj}:

$$[M_{t,j\to i}] : M_{t,j\to i} = \frac{P_{ti}}{P_{tj}} \gamma_{ji} Y_{ti} \; ; \qquad [X_{t,j\to i}] : X_{t,j\to i} = \frac{\mu_{ti}}{P_{tj}} \theta_{ji} Z_{ti}$$

$$[C_{tj}] : C_{tj}^{-\sigma} = P_{tj}$$

$$[L_{tj}] : \psi = \left(1 - \alpha_j - \sum_i \gamma_{ij} \right) \cdot P_{tj} \cdot \frac{Y_{tj}}{L_{tj}}$$

$$[K_{tj}] : \mu_{tj} = \beta \mu_{t+1,j} (1-\delta) + \beta P_{t+1,j} \alpha_j \cdot \frac{Y_{tj}}{K_{tj}}$$

Model: Steady state

Plug in [K], [L], [M] FOC

$$Y_j = \underbrace{A_j}_{=1 \text{ in steady sstate}} \cdot (K_j)^{\alpha_j} \cdot M_j \cdot (L_j)^{1-\alpha_j-\sum_i \gamma_{ij}}$$

$$Y_j = \left(\frac{\alpha_j \beta Y_j P_j}{\mu_j (1-\beta(1-\delta))}\right)^{\alpha_j} M_j \left(\left(1-\alpha_j-\sum_i \gamma_{ij}\right)\frac{P_j Y_j}{\psi}\right)^{1-\alpha_j-\sum_i \gamma_{ij}}$$

$$P_j = \left(\frac{\mu_j (1-\beta(1-\delta))}{\alpha_j \beta}\right)^{\alpha_j} \prod_i \left(\frac{P_i}{\gamma_{ij}}\right)^{\gamma_{ij}} \left(\frac{\psi}{1-\alpha_j-\sum_i \gamma_{ij}}\right)^{1-\alpha_j-\sum_i \gamma_{ij}}$$

Note μ_j is also a function of the prices P_j

$$\mu_j = \prod_i \left(\frac{P_i}{\theta_{ij}}\right)^{\theta_{ij}}$$

Plug this in: we have a system of linear equations for $\log P$. $\Rightarrow$
Now use market-clearing conditions to solve for quantities.

Model: Log linearization

$$[M_{t,j\to i}] : M_{t,j\to i} = \frac{P_{ti}}{P_{tj}}\gamma_{ji} Y_{ti} \qquad \Rightarrow \quad \hat{M}_{t,j\to i} = \hat{P}_{ti} - \hat{P}_{tj} + Y_{ti}$$

$$[X_{t,j\to i}] : X_{t,j\to i} = \frac{\mu_{ti}}{P_{tj}}\theta_{ji} Z_{ti} \qquad \Rightarrow \hat{X}_{t,j\to i} = \hat{\mu}_{ti} - \hat{P}_{tj} + \hat{Z}_{ti}$$

$$[C_{tj}] : C_{tj}^{-\sigma} = P_{tj} \qquad \Rightarrow \quad -\sigma \hat{C}_{tj} = \hat{P}_{tj}$$

$$[L_{tj}] : \frac{\psi}{1 - \alpha_j - \sum_i \gamma_{ij}} = \frac{P_{tj} Y_{tj}}{L_{tj}} \qquad \Rightarrow \hat{L}_{tj} = \hat{P}_{tj} + Y_{tj}$$

$$[K_{tj}] : \mu_{tj} = \beta \mu_{t+1,j} (1 - \delta) + \beta P_{t+1,j} \alpha_j \frac{Y_{t+1,j}}{K_{t+1,j}} \Rightarrow$$

$$\hat{\mu}_{tj} = \beta \hat{\mu}_{t+1,j} (1 - \delta) + (1 - \beta (1 - \delta)) \left[\hat{P}_{t+1,j} + \hat{Y}_{t+1,j} + \hat{K}_{t+1,j} \right]$$

Also market clearing, production function, and K-evolution equatons need to be log linearized.

Then write these in vector form...

Model: System reduction

Substitute out vectors to get things down to the $2N \times 2N$ system of equations:

$$\mathbb{E}_t \begin{bmatrix} \hat{C}_{t+1} \\ \hat{K}_{t+1} \end{bmatrix} = \mathbf{M}_1 \cdot \begin{bmatrix} \hat{C}_t \\ \hat{K}_t \end{bmatrix} + \mathbf{M}_3 \cdot \mathbb{E}_t \hat{A}_{t+1} + \mathbf{M}_4 \cdot \hat{A}_t$$

- Worries:
  - More than N eigenvalues of $\mathbf{M}_1$ are greater than 1 in absolute value. $\Rightarrow$ There is no stable solution
  - Fewer than N eigenvalues of $\mathbf{M}_1$ are greater than 1 in absolute value. $\Rightarrow$ There are multiple stable solutions
- From here on, we will use our random walk assumption

$$\mathbb{E}_t \hat{A}_{t+1} = \hat{A}_t$$

and write $\mathbf{M}_2 = \mathbf{M}_3 + \mathbf{M}_4$

Model: Blanchard and Kahn

Start with

$$\mathbb{E}_t \begin{bmatrix} \hat{C}_{t+1} \\ \hat{K}_{t+1} \end{bmatrix} = \mathbf{M}_1 \cdot \begin{bmatrix} \hat{C}_t \\ \hat{K}_t \end{bmatrix} + \mathbf{M}_2 \hat{A}_t$$

Write

$$\mathbf{M}_1 = VDV^{-1}$$

- D is diagonal
- N explosive (>1) eigenvalues are ordered first

$$V^{-1}\mathbb{E}_t \begin{bmatrix} \hat{C}_{t+1} \\ \hat{K}_{t+1} \end{bmatrix} = DV^{-1} \cdot \begin{bmatrix} \hat{C}_t \\ \hat{K}_t \end{bmatrix} + \mathbf{M}_2 \hat{A}_t$$

$$\mathbb{E}_t \begin{bmatrix} \tilde{C}_{t+1} \\ \tilde{K}_{t+1} \end{bmatrix} = D \cdot \begin{bmatrix} \tilde{C}_t \\ \tilde{K}_t \end{bmatrix} + V^{-1}\mathbf{M}_2 \hat{A}_t$$

Model: Blanchard and Kahn

From last slide

$$\mathbb{E}_t \left[\begin{array}{c} \tilde{C}_{t+1} \\ \tilde{K}_{t+1} \end{array} \right] = \left(\begin{array}{cc} D_1 & 0 \\ 0 & D_2 \end{array} \right) \cdot \left[\begin{array}{c} \tilde{C}_t \\ \tilde{K}_t \end{array} \right] + V^{-1} \mathbf{M}_2 \hat{A}_t$$

Writing each of the components:

$$\begin{aligned} \tilde{C}_t &= D_1^{-1} \tilde{C}_{t+1} + D_1^{-1} V^{-1} \mathbf{M}_2 \hat{A}_t \\ &= D_1^{-2} \tilde{C}_{t+2} + D_1^{-2} V^{-1} \mathbf{M}_2 \hat{A}_t + D_1^{-1} V^{-1} \mathbf{M}_2 \hat{A}_t \\ &= \sum_{\tau=1}^{\infty} D_1^{-\tau} V^{-1} \mathbf{M}_2 \hat{A}_t = D_1^{-1} \left(I - D_1^{-1} \right) V^{-1} \mathbf{M}_2 \hat{A}_t \\ \tilde{K}_{t+1} &= D_2 \cdot \tilde{K}_t + V^{-1} \mathbf{M}_2 \hat{A}_t \end{aligned}$$

Using $\left[\begin{array}{c} \hat{C}_t \\ \hat{K}_t \end{array} \right] = V \cdot \left[\begin{array}{c} \tilde{C}_t \\ \tilde{K}_t \end{array} \right]$, we can solve for $\hat{K}_{t+1}$ (and also $\hat{C}_t$) in terms of $\hat{K}_t$ and $\hat{A}_t$.

Model: Obtaining the model filter

From before, these equations describe model dynamics

$$\hat{K}_{t+1} = \Pi_{kk}\hat{K}_t + \Pi_{ak}\hat{A}_t; \qquad \hat{C}_t = \Pi_{kc}\hat{K}_t + \Pi_{ac}\hat{A}_t$$

Within our log linearized equations we can also write out output in terms of the state/co-state

$$\begin{aligned}\hat{Y}_t &= \Pi_{ky}\hat{K}_t + \Pi_{ay}\hat{A}_t + \Pi_{cy}\hat{C}_t \\ &= \Pi_{ky}\hat{K}_t + \Pi_{ay}\hat{A}_t + \Pi_{cy}\left[\Pi_{kc}\hat{K}_t + \Pi_{ac}\hat{A}_t\right]\end{aligned}$$

Look at an adjacent period

$$\begin{aligned}\hat{Y}_{t+1} &= [\Pi_{ky} + \Pi_{cy}\Pi_{kc}]\,\hat{K}_{t+1} + [\Pi_{ay} + \Pi_{cy}\Pi_{ac}]\,\hat{A}_{t+1} \\ &= [\Pi_{ky} + \Pi_{cy}\Pi_{kc}]\left[\Pi_{kk}\hat{K}_t + \Pi_{ak}\hat{A}_t\right] + [\Pi_{ay} + \Pi_{cy}\Pi_{ac}]\,\hat{A}_{t+1} \\ &= [\Pi_{ky} + \Pi_{cy}\Pi_{kc}]\,\Pi_{kk}\hat{K}_t \\ &\quad + [\Pi_{ky} + \Pi_{cy}\Pi_{kc}]\,\Pi_{ak}\hat{A}_t + [\Pi_{ay} + \Pi_{cy}\Pi_{ac}]\,\hat{A}_{t+1}\end{aligned}$$

Now we can sub out $\hat{K}_t$

Solution: Summary

- The model yields the following (log-linear-approximate) expression for the evolution of output:

$$\begin{pmatrix} \Delta \log Y_{t+1,1} \\ \Delta \log Y_{t+1,2} \\ \ldots. \end{pmatrix} = Q \begin{pmatrix} \Delta_{t,1} \\ \Delta_{t,2} \\ \ldots. \end{pmatrix} + R \begin{pmatrix} \omega_{t1} \\ \omega_{t2} \\ \ldots \end{pmatrix} + S \begin{pmatrix} \omega_{t+1,1} \\ \omega_{t+1,2} \\ \ldots \end{pmatrix}$$

- Q, R, and S are specified, given $(\beta, \delta, \psi, \sigma, \alpha_i, \theta_{ij}, \gamma_{ij})$
- Given these parameters, one can back out innovations to productivity (setting $\omega_0 = 0$):

$$\begin{pmatrix} \omega_{t+1,1} \\ \omega_{t+1,2} \\ \ldots \end{pmatrix} = S^{-1} \left[\begin{pmatrix} \Delta \log Y_{t+1,1} \\ \Delta \log Y_{t+1,2} \\ \ldots. \end{pmatrix} - Q \begin{pmatrix} \Delta \log Y_{t1} \\ \Delta \log Y_{t2} \\ \ldots. \end{pmatrix} - R \begin{pmatrix} \omega_{t1} \\ \omega_{t2} \\ \ldots \end{pmatrix} \right]$$

Solution: Summary

From last slide

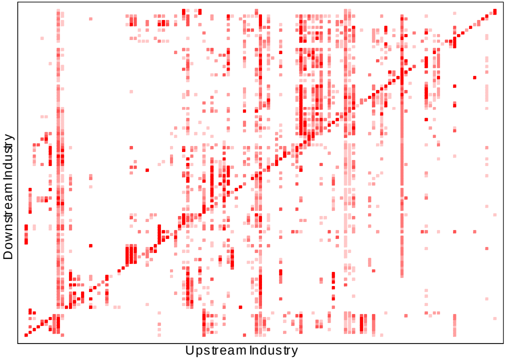
$$\begin{pmatrix} \omega_{t+1,1} \\ \omega_{t+1,2} \\ \dots \end{pmatrix} = S^{-1} \left[\begin{pmatrix} \Delta \log Y_{t+1,1} \\ \Delta \log Y_{t+1,2} \\ \dots\text{.} \end{pmatrix} - Q \begin{pmatrix} \Delta \log Y_{t1} \\ \Delta \log Y_{t2} \\ \dots\text{.} \end{pmatrix} - R \begin{pmatrix} \omega_{t1} \\ \omega_{t2} \\ \dots \end{pmatrix} \right]$$

- Worries:
  - The first few ω's will depend on our (ad-hoc) imposition that $\omega_0 = 0$.
  - Some of the eigenvalues of R may be greater than 1 in absolute value.
- Next step: Calibrate model's parameters, which will allow us to back out ω_{tj}.

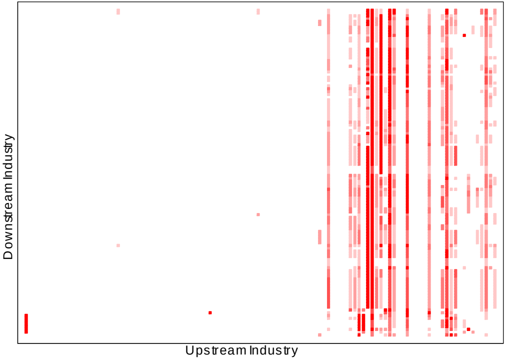
Calibration

Parameter	Value/Source
β-discount factor	0.99
δ-capital depreciation rate	0.025
ψ-disutility from work	1
σ-consumers' elasticity, across goods	1
α_i-capital share in production of i	1997 BEA I.O. Tables
$1-\alpha_i-\sum_j \gamma_{ij}$-share of capital/labor in production of i	1997 BEA I.O. Tables
γ_{ij}-share of good i in production of j's intermediate input	1997 BEA I.O. Tables
θ_{ij}-share of good i in production of j's capital input	1998 Capital Flow Tables

Calibration of Θ



Calibration of Γ



McGrattan and Schmitz: Add 0.25 to the diagonal elements of Θ.

Calibration of $\Sigma_{\omega\omega}$

- Given the other parameters, we know ω_{tj} for all time periods industries.
- Two calibrations:
  - $\Sigma_{\omega\omega}$ is diagonal, with the j, j^{th} entry equal to the sample variance of ω_{tj}
  - Perform principal component analysis on the ω_{tj}: $\Sigma_{\omega\omega} = \Lambda_S \Sigma_{SS} \Lambda_S + \Sigma_{uu}$, with a 2-dim. common factor, S_t
- With $\Sigma_{\omega\omega}$ in hand, we compute the following statistics:
  - $\bar{\rho}_{ij}$: average correlation in the growth rates for two industries
  - σ_g: standard deviation of the growth rate of industrial production
  - $R^2(S)$: fraction of the variation in industrial production growth explained by the common factors.

Results

	Period	$\bar{\rho}_{ij}$	σ_g	$R^2\ (S)$
Data	72-83	0.27	8.8	
	84-07	0.11	3.6	
Uncorrelated Shocks	72-83	0.05	5.1	
	84-07	0.04	3.1	
2 Common Factors	72-83	0.26	9.5	0.81
	84-07	0.10	4.1	0.50

Comparison to other models

- Foerster et al.:

$$g_{t+1} = Qg_t + S\omega_{t+1} - R\omega_t$$

- Long and Plosser (1983): Materials arrive with a one-period lag, no capital, log preferences for consumption and leisure.

$$g_{t+1} = \Gamma' g_t + \omega_{t+1}$$
$$\Sigma_{gg} = \sum_{i=0}^{\infty} \left(\Gamma'\right)^i \Sigma_{\omega\omega}\Gamma'$$

- Carvalho (2007), Acemoglu et al (2012): No capital, log preferences for consumption, perfectly elastic labor supply

$$g_{t+1} = \left(I - \Gamma'\right)\omega_{t+1}$$
$$\Sigma_{gg} = \left(I - \Gamma'\right)\Sigma_{\omega\omega}\left(I - \Gamma\right)$$

Model performance with independent productivity shocks

	$\bar{\rho}_{ij}$	σ_g	σ_g(diag)	σ_g(scaled)$\div$ $\sigma_{g,bench}$(scaled)
Data	0.19	5.80	1.85	
Benchmark	0.04	3.87	1.88	1.00
Long-Plosser	0.01	2.66	2.07	0.39
Carvalho	0.04	3.15	1.64	0.87
Benchmark, $\theta = I$	0.02	3.86	2.43	0.59
Benchmark, $\Sigma_{\omega\omega} = \sigma^2 I$	0.04	5.72	2.99	0.86
Benchmark, Γ, α: average	0.05	3.30	1.71	0.87

σ_g(scaled) is defined as the σ_g computed in an alternative calibration in which $\Sigma_{\omega\omega}$ is chosen so that "model-implied variance of IP growth associated with the diagonal elements of Σ_{gg} correspond to the value in the data."

Do the industry definitions matter?

	Period	2-digit 26 inds.	3-digit 88 inds.	4-digit 117 inds.
Data $\bar{\rho}_{ij}$	72-83	0.38	0.29	0.27
	84-07	0.22	0.13	0.11
Independent Error $\bar{\rho}_{ij}$	72-83	0.09	0.05	0.05
	84-07	0.07	0.05	0.04
R^2 (S)	72-83	0.76	0.85	0.81
	84-07	0.53	0.53	0.50

Conclusion

- Summary:
  - Industry-specific shocks explain about 40% of the variation in industrial production
  - Lower (20%) in the pre-Great Moderation period; higher (50%) in the Great Moderation. Common shocks became less volatile during the great moderation
- Extensions:
  - Apply this model to the whole economy, not just the goods-producing sectors (Ando 2014)
  - Decompose output variation into firm-specific, industry-specific, and common shocks.