Research question and motivation

- Majority of dynamic general equilibrium models: Firm (scale) heterogeneity does not matter.
- Because some firms are so large, decisions of individual firms can have aggregate implications
  - 2004Q4: Microsoft issues $24 billion one-time dividend. Accounts for 2.1% boost in personal income growth.
  - 2000: Nokia accounts for half of Finish private R&D, 1.6 percentage points of GDP growth.
  - Are these anecdotes exceptional or common?
- Question: To what extent are firm-level shocks responsible for aggregate fluctuations?
Outline

- Some data
  - Compustat: 1960 to present.

- Theoretical results and calibration
  - The Central Limit Theorem is irrelevant when firm sizes are fat-tailed
  - The herfindahl index is a summary statistic for the importance of firm-specific shocks.

- The granular residual
The firm size distribution in 1960
The firm size distribution in 1970
The firm size distribution in 1980
The firm size distribution in 1990

![Graph showing the firm size distribution with sales in billions on the x-axis and 1-CDF on the y-axis. The graph includes data points for GM, Ford, IBM, and GE.]
The firm size distribution in 2000
The firm size distribution in 2010
Sales Herfindahl of firms in Compustat

\[ h = \left[ \sum_i \left( \frac{S_i}{S} \right)^2 \right]^{1/2} \]
Overview of the theoretical results

- If the firm size distribution is Pareto, we can show how the dispersion of GDP growth decreases in economies with more and more firms.

- Even if the firm size distribution is not Pareto, we can relate the dispersion of GDP to:
  - $\sigma$: the standard deviation of firm productivity growth rates.
  - $h$: the HHI of firm sales
  - a combination of other model parameters.
What is the relationship between micro productivity growth and aggregate output growth?

- Economy is made up of $n$ units (firms or industries)
- Utility $= C - \frac{\phi}{\phi+1} L^\frac{\phi+1}{\phi}$, where $C \equiv \prod_i \left( \frac{C_i}{\xi_i} \right)^{\xi_i}$
- Production: $Q_i = A_i \left( \frac{L_i}{\alpha} \right)^\alpha \left( \frac{M_i}{1-\alpha} \right)^{1-\alpha}$
- Intermediate input bundle: $M_i = \prod_j \left( \frac{M_{j \rightarrow i}}{\gamma_{ji}} \right)^{\gamma_{ji}}$
- Market clearing: $Q_i = C_i + \sum_j M_{i \rightarrow j}$
- Write
  - $P_i$ as the Lagrange multiplier for the good $i$ market-clearing condition, and $S_i \equiv P_i Q_i$.
  - $W$ as the Lagrange multiplier for the labor market clearing condition.
  - Set $C$ as the numeraire good: $P \equiv \prod_i (P_i)^{\xi_i} = 1$
Consider the problem of the representative consumer who is trying to maximize:

\[ C - \frac{\phi}{\phi + 1} L^\frac{\phi + 1}{\phi} \quad \text{s.t.} \quad C = WL \]

Equilibrium \( C \) and \( L \) satisfy:

\[ L = W^\phi \]
\[ C = W^{\phi + 1} = L^\frac{\phi + 1}{\phi} \quad (1) \]
Step 2: Solve for Prices

Consider the cost-minimization problem of firm/industry $i$. 

\[
\log Q_i = \log A_i + \alpha \log \frac{L_i}{\alpha} + (1 - \alpha) \sum_j \gamma_{ji} \log \left( \frac{M_{ji}}{(1 - \alpha) \gamma_{ji}} \right)
\]

\[
= \log A_i + \alpha \log \left( \frac{Q_i P_i}{W} \right) + (1 - \alpha) \sum_j \gamma_{ji} \log \left( \frac{Q_i P_i}{P_j} \right)
\]

Thus:

\[
\log P_i = - \log A_i + \alpha \log W + (1 - \alpha) \sum_j \gamma_{ji} \log P_j
\]

\[
\overrightarrow{\log P} = \left( I - ((1 - \alpha) \Gamma) \right)^{-1} \left( -\overrightarrow{\log A} + \alpha \log W \right) \quad (2)
\]
Step 3: Write out sales in each industry

Using the market clearing conditions

\[ S_i = P_i Q_i = P_i C_i + \sum_j P_i M_{i \rightarrow j} \]

Plugging in customers’ factor demand curves and re-arranging:

\[ S_i - (1 - \alpha) \sum_j \gamma_{ij} S_j = \xi_i C \]

\[ \frac{\vec{S}}{C} = (I - ((1 - \alpha) \Gamma))^{-1} \vec{\xi} \]
Step 4: Write out total consumption and labor in terms of productivity

Plug Equation (2) into Equation (1)

\[
\log \bar{P} = (I - ((1 - \alpha) \Gamma)')^{-1} \left( -\log \bar{A} + \alpha \log W \right)
\]

\[
= (I - ((1 - \alpha) \Gamma)')^{-1} \left( -\log \bar{A} + \frac{\alpha}{\phi + 1} \log C \right)
\]

Use the fact that \( \xi' \log \bar{P} = 0 \) and \( (I - ((1 - \alpha) \Gamma)')^{-1} \alpha 1 = 1 \):

\[
(\phi + 1) \xi' (I - ((1 - \alpha) \Gamma)')^{-1} \log \bar{A} = \log C
\]

Remember the equation for sales

\[
\frac{\vec{S}'}{C} = \xi' (I - ((1 - \alpha) \Gamma'))^{-1}
\]

Thus

\[
\log C = (\phi + 1) \frac{\vec{S}'}{C} \log \bar{A}
\]

and

\[
\log L = \phi \frac{\vec{S}'}{C} \log \bar{A}
\]
Hulten (1978)
The Main Results

1. Aggregate productivity is a weighted average of productivity of the individual units:

\[ A^{agg} \equiv \log \left( \frac{C}{L} \right) = \frac{\overrightarrow{S'}}{C} \log \hat{A} \]

The sum of the weights is bigger than 1.

2. Total output and labor inputs each depend on aggregate productivity and the labor supply elasticity

\[ \log C = (\phi + 1) A^{agg} \quad \text{and} \quad \log L = \phi A^{agg} \]

3. Combining (1) and (2)

\[ \sigma_{\log C} = (\phi + 1) \cdot \frac{\sum S_i}{C} \left[ \sum_i \left( \frac{S_i}{\bar{S}} \right)^2 \right]^{1/2} \quad \sigma = \mu \cdot h \cdot \sigma, \]

where \( \mu \equiv (\phi + 1) \cdot \sum \frac{S_i}{C} \)

- Calibration: \( h = 6\% \), \( \sigma = 12\% \), \( \mu = 6 \Rightarrow \sigma_{\log C} = 4.3\% \)
The Pareto Distribution

Let \( S_i \equiv P_i C_i \) be a Pareto\((\zeta, x_0)\) random variable. 
\[
P(S > x) = \left( \frac{x}{x_0} \right)^{-\zeta}.
\]

Some useful facts about the Pareto distribution:

- \( \mathbb{E}[S] = x_0 \frac{\zeta}{\zeta - 1} \) if \( \zeta > 1 \), \( \infty \) otherwise.
- \( \mathbb{E}[S^2] = (x_0)^2 \frac{\zeta}{\zeta - 2} \) if \( \zeta > 2 \), \( \infty \) otherwise.
- \( S^\alpha \) is Pareto \( \left( \frac{\zeta}{\alpha}, (x_0)^\alpha \right) \) distributed.
- \( \alpha S \) is Pareto \( (\zeta, \alpha x_0) \) distributed.
- \( r^{th} \) moment of the \( k^{th} \) largest value in a sample of \( N \) 
  \[
  \mathbb{E}[S_{k:N}^r] = (x_0)^r \frac{\Gamma\left[k - \frac{r}{\zeta}\right]}{\Gamma[k]} \frac{\Gamma[N + 1]}{\Gamma\left(N + 1 - \frac{r}{\zeta}\right)}, \text{ if } r > \zeta.
  \]
- Many other facts in Gabaix (2009, Section 2)
Classic Central Limit Theorem

Suppose $S_1, S_2, \ldots, S_N$ is a sequence of i.i.d. random variables with $\mathbb{E}[S_i] = \mu$ and $\text{Var}[S_i] = \sigma^2 < \infty$. Then, as $N$ approaches $\infty$,

$$\frac{\sqrt{N}}{\sigma} \left( \frac{\sum S_i}{N} - \mu \right) \xrightarrow{d} \mathcal{N}(0, 1)$$

"Compared to an economy with 10 firms, an economy with 10 million firms will have GDP growth with a standard deviation 0.1% as large".

What if $\text{Var}[S_i] = \infty$?
Central Limit Theorem with infinite variances

Suppose $S_1, S_2, \ldots, S_N$ is a sequence of i.i.d. nonnegative random variables with $P(S_i > x) = x^{-\zeta} L(x)$ (where $L(x)$ is a slowly-varying function, and $\zeta < 2$). Then

$$\left(\frac{\sum_i S_i - b_N}{a_N}\right) \to \mathcal{L}(\zeta),$$

where

$$a_N = \inf \left\{ x : P(S_i > x) \leq \frac{1}{N} \right\}$$

and $b_N = N \mathbb{E} \left[ S_i \cdot 1_{(X_i \leq a_N)} \right]$.

and $\mathcal{L}(\zeta)$ is a Levy distribution with exponent $\zeta$.

- A slowly-varying function, $L(x)$ is one that satisfies

  $$\lim_{x \to \infty} \frac{L(tx)}{L(x)} = 1 \quad \forall \ t > 0.$$  

- If $P(S_i > x) = \left(\frac{x}{x_0}\right)^{-\zeta}$, then

  $$a_N = \inf \left\{ x : \left(\frac{x}{x_0}\right)^{-\zeta} \leq \frac{1}{N} \right\} = x_0 N^{1/\zeta}, \quad b_N = 0$$

- Thus

  $$\frac{N^{1-1/\zeta}}{x_0} \sum_{i} S_i \to \mathcal{L}(\zeta)$$
Levy distribution

PDF of Levy distribution: \( \sqrt{\frac{\zeta}{2\pi}} \exp \left\{ -\frac{\zeta}{2x} \right\} x^{-3/2} \)
Proposition 2

"Consider a series of island economies indexed by $N$. Economy $N$ has $N$ firms whose growth rate volatility is $\sigma$ and whose sizes $S_1,..,S_N$ are independently drawn from a power law distribution."

$$P ( S > x ) = ax^{-\zeta}, \text{ with } \zeta \geq 1.$$ 

As $N \to \infty$, GDP volatility follows

$$\sigma_{GDP} \sim \frac{v_\zeta}{\log N} \sigma \text{ for } \zeta = 1$$

$$\sigma_{GDP} \sim \frac{v_\zeta}{N^{1-1/\zeta}} \sigma \text{ for } \zeta \in (1, 2)$$

$$\sigma_{GDP} \sim \frac{v_\zeta}{N^{1/2}} \sigma \text{ for } \zeta \geq 2$$

When $\zeta \geq 2$, $v_\zeta$ is a constant; when $\zeta < 2$, $v_\zeta$ is the square root of a Levy distributed (with exponent $\zeta/2$) random variable.
Intuition for Proposition 2

In our islands economy, $\sigma_{GDP} = \sigma h$. Looking across economies with different numbers of firms, how does $h$ change as $N$ changes?

Take $P(S > x) = ax^{-\zeta}$, and consider the case in which $\zeta \in (1, 2)$, and $a = 1$.

\[
\frac{\mathbb{E}[X_{k:N}]}{N \mathbb{E}[X]} = \frac{\Gamma \left[ k - \frac{1}{\zeta} \right] (\zeta - 1)}{\Gamma [k] \zeta} \frac{\Gamma [N]}{\Gamma \left( N + 1 - \frac{1}{\zeta} \right)}
\]

\[
\rightarrow_{N \to \infty} \frac{\Gamma \left[ k - \frac{1}{\zeta} \right] (\zeta - 1)}{\Gamma [k] \zeta} N^{-\left(1-1/\zeta\right)}
\]

Share of top $K$ firms is proportional to $N^{-\left(1-1/\zeta\right)} \Rightarrow h$ is proportional to $N^{-\left(1-1/\zeta\right)}$. 
Proof of Proposition 2, Part 1

If $\zeta \geq 2$, the variance of $S_i$ is finite. Can apply the formula
\[ \sigma_{GDP} = \sigma h \]

\[ h = \frac{1}{N^{1/2}} \left\{ \frac{N^{-1} \sum (S_i)^2}{N^{-1} \sum S_i} \right\}^{1/2} \]

\[ \sigma_{GDP} \rightarrow \frac{\sigma}{N^{1/2}} \cdot \left( \frac{\mathbb{E}[S^2]}{\mathbb{E}[S]} \right)^{1/2} \]
Proof of Proposition 2, Part 2

When $\zeta > 1$, $N^{-1} \sum S_i \rightarrow \mathbb{E} [S]$

$S_i^2$ has a power law exponent $\zeta/2$

$$P \left( (S_i)^2 > x \right) = ax^{-\zeta/2}$$

Use the CLT with infinite variances, if $\zeta > 1$

$$N^{-2/\zeta} \sum S_i^2 \rightarrow_d \mathcal{L} (\zeta/2)$$

$$N^{1-1/\zeta} = N^{1-1/\zeta} \frac{\left[ N^{-2/\zeta} (\sum S_i^2) \right]^{1/2}}{N^{-1} \sum S_i} \rightarrow_d \frac{(\mathcal{L} (\zeta/2))^{1/2}}{\mathbb{E} [S]}$$

Putting the pieces together

$$\sigma_{GDP} N^{1-1/\zeta} = \sigma h N^{1-1/\zeta} \rightarrow_d \sigma \frac{(\mathcal{L} (\zeta/2))^{1/2}}{\mathbb{E} [S]}$$

If $\zeta \approx 1.05 \Rightarrow N^{1-1/\zeta} \approx N^{0.05} \Rightarrow$ Compared to an economy with 10 firms, an economy with 10 million firms will have GDP growth with a standard deviation about half as large.
Digression: Is the firm size distribution Pareto?

- With moderate sample size it’s difficult to distinguish between Pareto distribution (which has infinite variance if $\zeta < 2$) and something like a lognormal distribution (for which regular CLT applies).
- Find best fit, assuming firm sizes are distributed either Pareto or lognormal.

\[ f(x) = \frac{\zeta (x_0)^\zeta}{x^{\zeta+1}} \Rightarrow \log f(x) = \log \zeta + \zeta \log x_0 - (\zeta + 1) \log x \]

\[ \frac{\partial \log L}{\partial \zeta} = \sum_{i=1}^{n} \frac{1}{\zeta} + \log \left( \frac{x_0}{x} \right) = 0 \Rightarrow \hat{\zeta} = \left[ \frac{1}{N} \sum \log \left( \frac{x}{x_0} \right) \right]^{-1} \]

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\hat{x}_0$</th>
<th>$\hat{\zeta}$</th>
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</thead>
<tbody>
<tr>
<td>80</td>
<td>2.32</td>
<td>0.87</td>
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<tr>
<td>90</td>
<td>5.62</td>
<td>1.00</td>
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<tr>
<td>95</td>
<td>11.96</td>
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<td>99</td>
<td>60.75</td>
<td>2.52</td>
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</table>
Digression: Is the firm size distribution Pareto?

![Graph showing firm size distribution](image)
Digression: Is the firm size distribution Pareto?
The dispersion of growth rates decreases with size

\[ \log(\sigma^{\text{Grow}}) = \kappa_0 - \kappa_1 \log(\text{size}); \]

\[ \kappa_1 \in [0.15, 0.25], \text{compare to benchmark of perfect correlations of shocks within firms (} \kappa_1 = 0 \text{) or no correlation (} \kappa_1 = \frac{1}{2} \text{)} \]
The dispersion of growth rates decreases with size

Lee et al. (1998)
We can extend Proposition 2 to allow for firm size and firm volatility to be related.

Consider a series of island economies indexed by \( N \). Economy \( N \) has \( N \) firms whose growth rate volatility is \( \sigma^{\text{firm}} (S) = \sigma \left( \frac{S}{x_0} \right)^{-\alpha} \) and whose sizes \( S_1, .. S_N \) are independently drawn from a power law distribution.

\[
P (S > x) = x^{-\zeta}, \text{ with } \zeta \geq 1.
\]

If \( \zeta > 1 \), the volatility of GDP, \( \sigma (Y) \), is proportional to \( N^{-\min \left\{ \frac{1}{2}, 1 - \frac{1-\alpha}{\zeta} \right\}} \).

If \( \zeta \approx 1.05 \) and \( \alpha \approx \frac{1}{6} \) \( \Rightarrow \ N^{1 - \frac{1-\alpha}{\zeta}} \approx N^{0.21} \) \( \Rightarrow \) Compared to an economy with 10 firms, an economy with 10 million firms will have GDP growth with a standard deviation about 5% as large.
Partial summary

- \( h = 6\% \) and \( \sigma = 12\% \) \( \Rightarrow \) A calibration of a simple "islands" model implies that independent firm shocks can potentially meaningfully contribute to GDP volatility

- Rest of the paper:
  - Construct a measure of productivity shocks to individual firms
  - Regress GDP growth against productivity shocks of the largest firms.
Defining the granular residual

From before

\[
\log \frac{Y_t}{Y_{t-1}} \propto \sum_i \frac{S_{i,t-1}}{Y_{it-1}} \log \left( \frac{A_{it}}{A_{i,t-1}} \right)
\]

Define

\[
\Gamma_t \equiv \sum_{i=1}^{100} \frac{S_{i,t-1}}{Y_{t-1}} \hat{\varepsilon}_{it},
\]

\[
\hat{\varepsilon}_{it} \equiv z_{it} - z_{i,t-1} - (\bar{z}_{lt} - \bar{z}_{l,t-1})
\]

where \( z_{it} = \log \left( \frac{\text{sales of } i \text{ in year } t}{\text{employees of } i \text{ in year } t} \right) \), and \( \bar{z}_{lt} \) is the corresponding average labor productivity in firm \( i \)'s industry, \( l \).
On the granular residual

\[ \log \left( \frac{A_{it}}{A_{i,t-1}} \right) \] measures changes in TFP, not in labor productivity.

- For plants in the manufacturing sector these productivity measures have pretty different patterns (Syverson 2004):
  - 90-10 (75-25) difference of log labor productivity is roughly 1.4 (0.66)
  - 90-10 (75-25) difference of log TFP is 0.7 (0.29)
GDP growth and the granular residual

<table>
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<tr>
<th>Sample</th>
<th>1952-2008</th>
<th>1952-2014</th>
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<tr>
<td>$\Gamma_t$</td>
<td>2.8</td>
<td>2.9</td>
</tr>
<tr>
<td>$\Gamma_{t-1}$</td>
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<td>3.4</td>
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<tr>
<td>$\Gamma_{t-2}$</td>
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<tr>
<td>Intercept</td>
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<td>0.02</td>
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<tr>
<td>N</td>
<td>57</td>
<td>56</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.14</td>
<td>0.32</td>
</tr>
<tr>
<td>$\tilde{R}^2$</td>
<td>0.12</td>
<td>0.29</td>
</tr>
</tbody>
</table>
GDP growth and the granular residual

Granular events

- 1970 Strike at GM (labor productivity down 18%)
- 1972 Ford and Chrsyler have a rush of subcompact sales
- 1983 Launch of IBM PC (labor productivity up 10%)
Predictive power of the granular residual

<table>
<thead>
<tr>
<th></th>
<th>(\Gamma_{t-1})</th>
<th>(\Gamma_{t-2})</th>
<th>(\hat{R}^2)</th>
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</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.015</td>
<td>0.019*</td>
<td>0.021*</td>
</tr>
<tr>
<td>(\Gamma_{t-1})</td>
<td>3.5**</td>
<td>3.3**</td>
<td></td>
</tr>
<tr>
<td>(\Gamma_{t-2})</td>
<td>1.2</td>
<td>2.3*</td>
<td></td>
</tr>
<tr>
<td>Monetary(_{t-1})</td>
<td>-0.04</td>
<td>-0.05</td>
<td></td>
</tr>
<tr>
<td>Monetary(_{t-2})</td>
<td>-0.02</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>Oil(_{t-1})</td>
<td>(-8.7 \times 10^{-5})</td>
<td>(-1.7 \times 10^{-4})</td>
<td></td>
</tr>
<tr>
<td>Oil(_{t-2})</td>
<td>(-6.9 \times 10^{-5})</td>
<td>(-1.2 \times 10^{-4})</td>
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</tr>
<tr>
<td>3-month t-bill(_{t-1})</td>
<td>-0.45</td>
<td>-0.41</td>
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<tr>
<td>3-month t-bill(_{t-2})</td>
<td>0.43</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>Term Spread(_{t-1})</td>
<td>0.38</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>Term Spread(_{t-2})</td>
<td>0.27</td>
<td>-0.38</td>
<td></td>
</tr>
</tbody>
</table>

- Oil: (Hamilton 2003): current vs. last year’s max oil price.
- Monetary policy shock: Residuals from FOMC decisions vs. FOMC forecasts (Romer and Romer 2004)
- Term spread: 5 year bond yield - 3 month bond yield.
Notes on Carvalho and Gabaix (2013): "The Great Diversification and its Undoing"
The Great Moderation
The Great Moderation and its Undoing
Motivation and question

- Why did volatility of GDP growth decrease beginning around 1980? And why did volatility increase beginning around 2005?

- Previous answers to the first question:
  - Stock and Watson (2003): It *doesn’t* seem to have to do with better inventory management or more aggressive monetary policy.
  - Arias, Hansen, and Ohanian (2007): Aggregate TFP shocks have become less volatile.
  - Jaimovich and Siu (2009): Fewer young people (those with more elastic labor supply) in the work force.

- New answer in this paper: Industry composition affects aggregate volatility.
Industries differ in their volatility
Fundamental Volatility

- Reminder from our islands economy
  - Suppose economy is made up of $n$ units (firms or industries)
  - $\log GDP_t = \sum_i \frac{S_{it}}{GDP_t} \log (A_{it})$
  - $\log \left( \frac{GDP_{t+1}}{GDP_t} \right) \approx \sum_i \frac{S_{it}}{GDP_t} \log \left( \frac{A_{i,t+1}}{A_{it}} \right)$

- Suppose $\frac{A_{i,t+1}}{A_{it}}$ are i.i.d. across time and industries, with standard deviation $\sigma_i$

$$SD \left[ \log \left( \frac{GDP_{t+1}}{GDP_t} \right) \right] \approx \left[ \sum_i \left( \frac{S_{it}}{GDP_t} \right)^{1/2} (\sigma_i)^2 \right]^{1/2}$$

- Potentially
  - productivity shocks are correlated, have volatilities that change over time.
  - things besides industries’ TFP change from one period to the next
Fundamental Volatility

\[ \sigma_{Ft} = \left[ \sum_i \left( \frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2} \]
Outline

- Definitions and data sources
  - TFP in each industry
    - $\sigma_{GDP}$
  - Fundamental volatility accounts for the break in GDP volatility.
- Sources of fundamental volatility
- Fundamental volatility and GDP volatility in other countries
Industry TFP Volatility

- KLEMS data from Dale Jorgenson (http://hdl.handle.net/1902.1/11155)

- For each industry-year define TFP growth as:

\[
\Delta TFP_{it} = \log \left( \frac{y_{it+1}}{y_{it}} \right) 
- \frac{1}{2} \left( s_{it}^k + s_{it+1}^k \right) \log \left( \frac{k_{it+1}}{k_{it}} \right) 
- \frac{1}{2} \left( s_{it}^l + s_{it+1}^l \right) \log \left( \frac{l_{it+1}}{l_{it}} \right) 
- \frac{1}{2} \left( s_{it}^m + s_{it+1}^m \right) \log \left( \frac{m_{it+1}}{m_{it}} \right)
\]

where \( s_{it}^l \) (\( s_{it}^m \), \( s_{it}^k \)) is industry \( i \)'s cost share of labor (intermediate inputs, capital) at time \( t \).

- \( \sigma_i \equiv \text{SD}(\Delta TFP_{it}) \)
GDP Volatility

Three measures:

1) Rolling standard deviation

\[ \sigma_{t}^{roll} = \text{SD} \left( y_{t-10}^{HP}, \ldots y_{t+10}^{HP} \right) \text{, where} \]
\[ y_{t}^{HP} \text{ is deviation of log GDP from trend} \]

2) Instantaneous standard deviation

\[ \Delta y_{s} = \psi + \phi \Delta y_{s-1} + \epsilon_{s} \]
\[ \sigma_{t}^{Inst} \equiv \frac{1}{2} \sqrt{\frac{\pi}{2}} \sum_{q=1}^{4} |\hat{\epsilon}_{t,q}| \]

3) \( \sigma_{t}^{HP} \) is the HP smoothed version of \( \sigma_{t}^{Inst} \)
Fundamental Volatility accounts for the break in GDP volatility.

\[
LR_T = \frac{\prod_{t=1960}^{T} f_1(\eta_t) \prod_{T+1}^{2008} f_2(\eta_t)}{\prod_{t=1960}^{2008} f_0(\eta_t)}
\]

| \( \sigma_{Y_t}^{inst} = a + \eta_t \) | \( \sigma_{Y_t}^{inst} = a + b\sigma_{Ft} + \eta_t \) |
|---|---|---|---|
| \( H_0 \) | No break in \( a \) | No break in \( b \) | No break in \( a \) or \( b \) |
| \( \max_T LR_T \) | 26.50 | 8.32 | 8.64 | 8.91 |
| Reject null? | Yes | No | No | No |
| Estimated break date | 1983 | NA | NA | NA |
Fundamental Volatility

\[
\sigma_{F_t} = \left[ \sum_i \left( \frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2}
\]
Sales weights: motor vehicles and petroleum

\[ \sigma_{Ft} = \left[ \sum_i \left( \frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2} \]
Contribution to fundamental volatility: motor vehicles and petroleum

\[ \sigma_{Ft} = \left[ \sum_i \left( \frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2} \]
Sales weights: depository and nondepository financial institutions

\[
\sigma_{Ft} = \left[ \sum_i \left( \frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2}
\]
Contribution to fundamental volatility: depository and nondepository financial institutions

\[ \sigma_{Ft} = \left[ \sum_i \left( \frac{S_{it}}{GDP_t} \right)^2 (\sigma_i)^2 \right]^{1/2} \]

![Graph showing contribution to fundamental volatility over years](image)
Fundamental volatility tracks GDP volatility in other countries

UK, Correlation=0.60

Japan, Correlation=0.60
Fundamental volatility tracks GDP volatility in other countries

Germany, Correlation=0.54

France, Correlation=0.03
Conclusion

Summary:

- GDP volatility changes over time.
- Volatility changes reflect changes in the importance of different types of firms in the economy.
  - Implies that firm/industry-level shocks are important for aggregate volatility.

Next steps:

- To what extent are the economy’s shocks independent across firms (or industries)?