How Important Are Sectoral Shocks?

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Abstract

I quantify the contribution of sectoral shocks to business cycle fluctuations in aggregate output. I develop and estimate a multi-industry general equilibrium model in which each industry employs the material and capital goods produced by other sectors. Using data on U.S. industries’ input prices and input choices, I find that the goods produced by different industries are complements as inputs in downstream industries’ production functions. These complementarities indicate that industry-specific shocks are substantially more important than previously thought, accounting for more than half of aggregate volatility.

1 Introduction

What are the origins of business cycle fluctuations? Do idiosyncratic micro shocks—disturbances at individual firms or industries—have an important role in explaining short-run macroeconomic fluctuations? Or are shocks that prevail on all industries the predominant source?

I address these questions by constructing and estimating a multi-industry dynamic general equilibrium model in which both common and industry-specific shocks have the potential to contribute to aggregate output volatility. I find that sectoral shocks are important, accounting for considerably more than half of the variation in aggregate output growth.

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A challenge in identifying the relative importance of industry-specific shocks is that, because of input-output linkages, both aggregate and industry-specific shocks have similar implications for data on industries’ sales. To see why, consider the following two scenarios. In the first, some underlying event (e.g., a surprise increase in the federal funds rate) reduces the demand faced by all industries, including the auto parts manufacturing, steel manufacturing (a supplier of auto parts), and auto assembly industries. In the second scenario, a strike occurs in the auto parts manufacturing industry, which temporarily reduces the demand faced by sheet-metal manufacturers, and increases the cost of establishments engaged in auto assembly. Even if industry-specific shocks are independent of one another, input-output linkages will induce comovements in these industries’ output and employment growth rates, just as in the first scenario. Intuitively, though, the more correlation across industry output growth that is observed, the more likely it is that common shocks are responsible for aggregate fluctuations.

But the extent to which industry activity co-moves depends on how easily consumers can substitute across the goods they consume and how easily the firms within an industry can substitute across different factors of production. Given a particular observed amount of comovement in output it could be for one of two reasons. Either production elasticities are low and common shocks are relatively unimportant. Or, elasticities of substitution are large, and common shocks are relatively important. The second challenge, then, emerges from the paucity of reliable, precise estimates of how easily industries can substitute across their inputs.\(^1\)

In this paper, I confront these two related challenges sequentially. First, using data from the 1997-2013 BEA (Bureau of Economic Analysis) Annual Input-Output Tables, I estimate the relevant elasticities of substitution. In the data, the expenditure share of an industry on particular intermediate inputs are both volatile and positively correlated to the input’s price. From these patterns, I estimate a relatively low value for the elasticity of substitution (which I call \(\varepsilon_M\)) among the intermediate inputs produced by different upstream industries: My benchmark estimates of \(\varepsilon_M\) range between 0 and 0.2, depending on the specification, and are always significantly less than 1. In other words, different intermediate inputs are highly complementary to one another.

Second, armed with estimates of \(\varepsilon_M\) and the model’s other salient elasticities of substitution, I construct a multi-industry dynamic general equilibrium model with which to infer industry productivity shocks. This model is an extension of that introduced in Foerster, Sarte, and Watson (2011), allowing for sectoral production functions that have non-unitary elasticities of substitution across inputs. Using the model, in conjunction with

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\(^1\)I discuss existing estimates of the relevant elasticities in Section 3.
data on industries output levels from 1960-2013, I back out the productivity that each industry experienced over this sample period. I then extract the common component of these productivity shocks.

From here, I compute the fraction of the variation in aggregate output growth that can be explained by industry-specific (versus common) shocks. I find that most of the variation in aggregate output growth is attributable to the industry-specific components: close to 80 percent in my benchmark estimates, and safely above 50 percent for reasonable assumptions on the extent to which consumers can substitute across products. These findings are in contrast to Foerster, Sarte, and Watson (2011), who conclude that common shocks are more important. When I impose unitary elasticities of substitution on my model, I estimate that slightly less than 20 percent of the variation in output volatility comes from industry-specific shocks.\(^2\) In sum, these results indicate that sectoral shocks are more important than previously thought, and that the difference is largely due to past papers’ imposition of a unitary elasticity of substitution across different inputs in sectoral production functions. These results are robust to industry classification schemes, treatment of trends, and countries.

This paper resolves the hypothesis, first advanced in Long and Plosser (1983), that independent industry-specific shocks generate patterns characteristic of modern business cycles. Models of business cycles typically portray fluctuations as the result of economy-wide, aggregate disturbances to production technologies and preferences. These disturbances, however, are difficult to justify independently, and may simply represent "a measure of our ignorance."\(^3\) Given the results of the current paper, future research on the sources of business cycle fluctuations would benefit from moving beyond the predominant one-sector framework.

**Related Literature:** The current paper falls within the literature on multi-industry real business cycle models, first introduced in Long and Plosser (1983). Long and Plosser present a model in which the economy is comprised of a collection of perfectly competitive industries. Each industry produces its output by employing a combination of capital, labor, and intermediate inputs. The capital and intermediate input bundles of each industry are, in turn, combinations of goods that are purchased from other industries. Long and Plosser (1983) and others in this literature (e.g., Horvath 1998, 2000, Dupor 1999, Acemoğlu et al. 2012, and Acemoğlu, Özdağlar, and Tahbaz-Salehi 2015) use this framework to argue that idiosyncratic shocks to industries’ productivities, by themselves, have the potential

\(^2\)See Section 4 for a discussion of why the numbers reported in Foerster, Sarte, and Watson (2011) may differ from the estimates I provide here.

\(^3\)This phrase was coined by Abramovitz (1956), when discussing the sources of long-run growth, but applies to our understanding of short-run aggregate fluctuations, as well. More recently, Summers (1986) and Cochrane (1994) have argued that it is \textit{a priori} implausible that aggregate shocks can exist at the scale needed to engender the business cycle fluctuations that we observe.
to generate substantial aggregate fluctuations. These papers, however, do not allow for aggregate shocks; they are not attempting to assess the relative importance of industry-specific and aggregate shocks.

With an appreciation of this issue, Foerster, Sarte, and Watson (2011) present a methodology that allows them to recover the underlying productivity shocks from data on industries’ output growth. The authors perform a factor analysis on the recovered productivity shocks. They find that aggregate productivity shocks—the first two common factors from the factor analysis—represent most of the variation in the first part of their sample (1972 to 1983), and decline in volatility during the Great Moderation (1984 to 2007). As a result, industry-specific shocks account for 20 percent and 50 percent of the variation in industrial production during the two parts of their sample.

Compared to the Long and Plosser literature in general—and Foerster, Sarte, and Watson (2011) in particular—the current paper makes three advances, all of which are necessary to properly gauge the contribution of sectoral shocks to aggregate volatility. First, I allow for flexible substitution patterns in industries’ production technologies. In particular, Foerster, Sarte, and Watson (2011) and earlier papers impose unitary elasticities of substitution across inputs industries’ production functions. In contrast, as a second contribution, I estimate these production elasticities. Allowing for a non-unitary elasticity of substitution among intermediate inputs turns out to be critical: With the unit elasticity assumption, the model understates the relative importance of industry-specific shocks. Third, I make a sequence of smaller advances: I allow for consumption good durability, consider a dataset that covers the entire economy, and examine data from several developed economies.

4Within this literature, Dupor (1999) is somewhat unique: Instead of arguing that industry-specific shocks have the potential to produce business cycle fluctuations, he does the converse: He provides conditions on the input-output matrix under which industry-specific shocks are irrelevant.

5Horvath (2000) accommodates non-unitary elasticities of substitution in consumers’ preferences (across goods) and in the production of the intermediate input bundle (across inputs purchased from upstream industries). A key difference between the current paper and Horvath (2000) is that the earlier paper does not attempt to estimate the values of these elasticities of substitution. There are papers in other fields which focus more closely on these elasticities: Johnson (2014) and Boehm, Flaaen, and Pandalai Nayar (2014), study the transmission of shocks across international borders as a mechanism for generating cross-country comovement and examine how the extent of model-predicted comovement varies with production and preference elasticities.

6Foerster, Sarte, and Watson (2011) is unique in its application of the Federal Reserve Board’s dataset on industrial production, a dataset that spans only the goods-producing sectors of the U.S. economy. Other papers (e.g., Long and Plosser 1983, Horvath 2000, and Ando 2014), employ datasets that cover the entire economy.

7A parallel literature attempts to gauge the relative importance of industry-specific shocks by estimating vector autoregressions (see Long and Plosser 1987, Stockman 1988, Shea 2002, or Holly and Petrella 2012). Yet another line of research constructs simple summary statistics of shocks to the largest firms or industries, relates these summary statistics to aggregate output movements, and in this way establishes the importance of micro shocks. Gabaix (2011) defines the granular residual—changes in productivity to the largest 100
Outline: In the remainder of the paper, I spell out the multi-sector real business cycle model (Section 2); estimate how easily industries can substitute across inputs (Section 3); apply these estimated elasticities to the real business cycle model to re-examine the relative importance of industry-specific shocks (Section 4); and conclude (Section 5). In Appendix A, I provide some additional details on the datasets used in the paper, and re-estimate one of the model’s elasticities of substitution using plant-level data on manufacturers’ input choices in Appendix B.

2 Model

In this section, I present a multi-industry general equilibrium model, along the lines of Foerster, Sarte, and Watson (2011). Compared to the Foerster et al. model, the only new elements are durability of certain consumption goods (which turns out not be so important for assessing the importance of sectoral shocks) and non-unitary elasticities of substitution in sectoral production functions (which do turn out to be important). This is the simplest model that can be used to compare the importance of industry-specific and aggregate disturbances and to estimate the elasticities of substitution in preferences and production. The model is populated by a representative consumer and $N$ perfectly competitive industries. I first describe the representative consumer’s preferences, then the production technology of each industry, and finally the evolution of the exogeneous variables.

2.1 Preferences

The consumer has balanced-growth-consistent preferences over leisure and the services provided by the $N$ different consumption goods.

The preferences of the consumer are given by the following utility function:

$$U = \sum_{t=0}^{\infty} \beta^t \left[ \log \left( \sum_{J=1}^{N} (\xi_J)^{\varepsilon_D^{-1}} \cdot (\delta_{C_J} \cdot C_{tJ})^{\varepsilon_D^{-1}} \right) \right] - \frac{\varepsilon LS}{\varepsilon LS + 1} \cdot \left( \sum_{J=1}^{N} L_{tJ} \right)^{\varepsilon LS+1} \right].$$ (1)

The demand parameters, $\xi_J$, reflect the time-invariant differences in the importance
of industries’ goods in the consumer’s preferences. \( C_{tJ} \) equals the stock of durable goods when \( J \) is a durable-good-producing industry and equals the expenditures on good/service \( J \) otherwise. For durable goods, \( J \), the evolution of the stock of each consumption good, \( C_{tJ} \), is given by

\[
C_{t+1,J} = C_{tJ} \cdot (1 - \delta_{C,J}) + \tilde{C}_{tJ},
\]

where \( \tilde{C}_{tJ} \) equals the consumer’s new purchases on good \( J \) at time \( t \) and \( \delta_{C,J} \) parameterizes the depreciation rate of good \( J \). The elasticities of substitution parameterize how easily the representative consumer substitutes across the different consumption goods (\( \varepsilon_D \)) and how responsive the consumer’s desired labor supply is to the prevailing wage (\( \varepsilon_{LS} \)).

### 2.2 Production and market clearing

Each industry produces a quantity \( (Q_{tJ}) \) of good \( J \) at date \( t \) using capital \( (K_{tJ}) \), labor \( (L_{tJ}) \), and intermediate inputs \( (M_{tJ}) \) according to the following constant-returns-to-scale production function:

\[
Q_{tJ} = A_{tJ} \cdot \left[ (1 - \mu_J) \frac{1}{\varepsilon_Q} \left( \left( \frac{K_{tJ}}{\alpha_J} \right)^{\alpha_J} \left( \frac{B_{tJ} \cdot L_{tJ}}{1 - \alpha_J} \right)^{1 - \alpha_J} \right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} + (\mu_J) \frac{1}{\varepsilon_Q} \left( M_{tJ} \right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} \right]^{\frac{\varepsilon_Q}{\varepsilon_Q - 1}}.
\]

The parameters \( \mu_J \) and \( \alpha_J \) reflect long-run averages in each industry’s usage of intermediate inputs, labor, and capital. These parameters will eventually be inferred from the factor cost shares of each industry. \( A_{tJ} \) and \( B_{tJ} \) are, respectively, the factor-neutral and the labor-augmenting productivity of industry \( J \) at time \( t \). For now, these productivity terms can be correlated, across industries, in any arbitrary fashion.

The parameter \( \varepsilon_Q \) dictates how easily factors of production are substituted. From the cost-minimization condition of the industry \( J \) representative firm, the relationship between

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Horvath (2000) and Kim and Kim (2006) use a more flexible specification regarding the disutility from supplying labor. In their specification, the second line of Equation 1 is replaced by

\[
-\frac{\varepsilon_{LS}}{\varepsilon_{LS} + 1} \cdot \left( \sum_{J=1}^{N} (L_{tJ}) \frac{\tau}{\varepsilon_{LS}} \right)^{\frac{\varepsilon_{LS} + 1}{\varepsilon_{LS}}}
\]

The idea behind this specification is to "capture some degree of sector specificity to labor while not deviating from the representative consumer/worker assumption." Horvath (2000, p. 76) As it turns out, neither the volatility of aggregate economic activity nor the covariances of output across industries are particularly sensitive to the value of \( \tau \) (see Table 9 of that paper). Moreover, since wages and hours are not among the observable variables that I am trying to match, the data that I employ in the following sections would have trouble identifying \( \tau \). For these reasons, I use the simpler specification of the disutility from labor supply.
the intermediate input cost share of industry $J$ and the industry $J$ specific intermediate
input price (denoted $P_{tJ}^{\text{in}}$) is log-linear, with slope $1 - \varepsilon_Q$:\footnote{The equivalence between sales and costs in the denominator of the left-hand side of Equation 4 comes from the assumption that each industry is perfectly competitive, with a constant returns-to-scale production function. To derive Equation 4, take first-order conditions of Equation 3 with respect to intermediate input purchases:

\[
\begin{align*}
P_{tJ}^{\text{in}} &= P_{tJ} \cdot \frac{\partial Q_{tJ}}{\partial M_{tJ}} \\
&= P_{tJ} \cdot (A_{tJ})^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} (M_{tJ})^{-\frac{1}{\varepsilon_Q}} (\mu_J \cdot Q_{tJ})^{\frac{1}{\varepsilon_Q}}. \\
(P_{tJ}^{\text{in}})^{\varepsilon_Q} &= (P_{tJ})^{\varepsilon_Q} (A_{tJ})^{\varepsilon_Q - 1} (M_{tJ})^{-1} \mu_J \cdot Q_{tJ}. 
\end{align*}
\]}

\[
\Delta \log \left( \frac{P_{tJ} \cdot M_{tJ}}{P_{tJ} \cdot Q_{tJ}} \right) = (\varepsilon_Q - 1) \cdot \Delta \log \left( \frac{P_{tJ}^{\text{in}}}{P_{tJ}} \right) + (\varepsilon_Q - 1) \cdot \Delta \log A_{tJ}. \tag{4}
\]

A similar set of calculations yields the following relationship describing changes in an
industry’s purchases of a specific intermediate input:

\[
\Delta \log \left( \frac{P_{tJ} M_{t,J \rightarrow J}}{P_{tJ}^{\text{in}} M_{tJ}} \right) = (1 - \varepsilon_M) \cdot \Delta \log \left( \frac{P_{tJ}}{P_{tJ}^{\text{in}}} \right). \tag{5}
\]

When $\varepsilon_Q = \varepsilon_M = 1$, as assumed in previous papers, an industry’s input cost shares are
constant, independent of the price of input prices, a prediction that I will show to be at odds
with the data.

The evolution of capital, for each industry $J$, is given in Equation 6.

\[
K_{t+1,J} = (1 - \delta_K) K_{t,J} + X_{t,J}. \tag{6}
\]

The capital stock is augmented via an industry-specific investment good, $X_{t,J}$, and
depreciates at a rate $\delta_K$ that is common across industries.

The industry-specific investment good is produced by combining the goods produced
by other industries. The $\Gamma_{tJ}^X$ indicate how important industry $I$ is in the production of the
industry $J$ specific investment good, while $\varepsilon_X$ parameterizes the substitutability of different
inputs in the production of each industry’s investment bundle:

\[
X_{t,J} = \left( \sum_{I=1}^{N} \left( \Gamma_{tJ}^X \right)^{\frac{1}{\varepsilon_X}} (X_{t,I \rightarrow J})^{\frac{\varepsilon_X - 1}{\varepsilon_X}} \right)^{\frac{\varepsilon_X}{\varepsilon_X - 1}}. \tag{7}
\]

Analogously, the intermediate input bundle of industry $J$ is produced through a
combination of the goods purchased from other industries:

\[
M_{t,J} = \left( \sum_{I=1}^{N} \left( \Gamma_{IJ}^{M} \frac{1}{\varepsilon_{M}} \left( M_{t,I,J} \right)^{\frac{\varepsilon_{M}-1}{\varepsilon_{M}}} \right) \right)^{\frac{\varepsilon_{M}}{\varepsilon_{M}-1}}.
\]  

(8)

In Equation 8, \( \varepsilon_{M} \) parameterizes the substitutability of different goods in the production of each industry’s intermediate input bundle. The \( \Gamma_{IJ}^{M} \) indicate how important industry \( I \) is in the production of the industry \( J \) specific intermediate input.

To emphasize, the parameters \( \Gamma_{IJ}^{M}, \Gamma_{IJ}^{X}, \alpha_{J}, \text{ and } \mu_{J} \) are time invariant. As such, movements in the share of \( J \)’s expenditures spent on different factors of production are due, only, to the shocks to industries’ productivity.

Finally, the market-clearing condition for each industry states that output can be used for consumption, as an intermediate input, or to increase one of the \( N \) capital stocks:

\[
Q_{tI} = C_{t+1,I} - (1 - \delta_{CI}) C_{tI} + \sum_{J=1}^{N} (M_{t,I,J} + X_{t,I,J}) .
\]  

(9)

### 2.3 Evolution of the exogeneous variables and the model filter

As in Foerster, Sarte, and Watson (2011), productivity in each industry follows a geometric random walk:

\[
\log A_{t+1,J} = \log A_{t,J} + \omega_{t+1,J}^{A} .
\]  

(10)

\[
\log B_{t+1,J} = \log B_{t,J} + \omega_{t+1,J}^{B} .
\]  

(11)

For now, the productivity shocks’ covariance matrices are left unspecified. I will add some structure to these matrices later, in Section 4.

As in Foerster, Sarte, and Watson (2011), in a competitive equilibrium, the vector of industries’ output growth rates admits a VARMA(1,1) representation. In taking the model to data on industries’ output growth rates, I will assume that productivity changes are either factor-neutral or labor augmenting.\(^{11}\) In these two cases, the evolution of output can be written as:

\[
\Delta \log Q_{t+1} = \Pi_{1} \Delta \log Q_{t} + \Pi_{2} A_{t}^{A} \omega_{t+1}^{A} + \Pi_{3} A_{t}^{A} \omega_{t+1}^{A} \text{ or}
\]  

(12)

\[
\Delta \log Q_{t+1} = \Pi_{1} \Delta \log Q_{t} + \Pi_{2} B_{t}^{B} \omega_{t+1}^{B} + \Pi_{3} B_{t}^{B} \omega_{t+1}^{B} .
\]  

(13)

\(^{11}\)With \( \varepsilon_{Q} \) equal to 1, as in Foerster, Sarte, and Watson (2011), shocks to labor-augmenting productivity and TFP cannot be separately identified. With non-unitary elasticities of substitution, the paper’s main results could a priori be sensitive to how the exogeneous productivity term affects output.
The $N \times N$ matrices $\Pi_1, \Pi_2^A, \Pi_3^A$, and $\Pi_2^B, \Pi_3^B$ are functions of the model parameters. I solve for these matrices in Appendix F.

Solving Equation 12 yields for $\omega_{t+1}^A$ the filter when productivity shocks are assumed to be factor neutral:

$$
\omega_{t+1}^A = (\Pi_3^A)^{-1} \Delta \log Q_{t+1} - (\Pi_3^A)^{-1} \Pi_1 \Delta \log Q_t + (\Pi_3^A)^{-1} \Pi_2^A \omega_t^A. \quad (14)
$$

The analogous equation with labor-productivity shocks is

$$
\omega_{t+1}^B = (\Pi_3^B)^{-1} \Delta \log Q_{t+1} - (\Pi_3^B)^{-1} \Pi_1 \Delta \log Q_t + (\Pi_3^B)^{-1} \Pi_2^B \omega_t^B. \quad (15)
$$

With some initial condition for the productivity shock (e.g., $\omega_0^A = 0$), one could iteratively use data on sectoral growth rates to infer the productivity shocks at each point in time. I will apply this procedure in Section 4. But first, I must determine values for the model’s elasticities of substitution. The example in the following subsection explains why.

### 2.4 Why do the elasticities matter?\(^{12}\)

Before turning to the empirical analysis, I work through a special case of the model. This special case yields a relatively simple set of expressions for the relationship between the model parameters, the exogeneous productivity shocks, and each industry’s output. With this relationship in hand, I then discuss the intuition behind why imposing unitary elasticities of substitution may lead one to understate the role of industry-specific shocks.

Compared to the benchmark model, I make a number of simplifying assumptions. I assume that a) all goods depreciate fully each period; b) there is no physical capital in production; c) each industry has identical production functions; d) the consumer’s preference weight is the same for each of the $N$ goods; e) the input-output matrix has $\frac{1}{N}$ in each entry; f) and productivity is factor neutral rather than labor augmenting ($A_{tJ} = 1$ for all $t$ and $J$). Relaxing these assumptions would not overturn the example’s main message, that higher elasticities of substitution generate less correlated output for a given set of correlations among the underlying productivity shocks.

In Appendix F.5, I work out the following (log-linear, around the point at which $A_I = 1$ for all $I$) approximation for each industry’s output as a function of the productivity shocks.

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\(^{12}\)This subsection is related to the technical appendix of Carvalho and Gabaix (2013). The main difference, besides assumptions (a)-(f), is that Carvalho and Gabaix impose that $\varepsilon_M = 1$ and allow for some adjustment costs to aggregate labor.
in each industry:

\[
\log Q_{it} \approx \frac{1}{1 - \mu} \log \left( \frac{1}{1 - \mu} \right) + \left( \mu \varepsilon_M + (1 - \mu) \varepsilon_D \right) \log A_{it} \\
+ \frac{1}{N} \left[ \left( \frac{1}{1 - \mu} \right)^2 - (\mu \varepsilon_M + (1 - \mu) \varepsilon_D) \right] \sum_{j=1}^{N} \log A_{tj}
\]  

(16)

The terms 1 and 2 in Equation 16 respectively describe the impact of industry-specific and common productivity changes on industry \( I \)'s output level. Term 1 is increasing in the two elasticities of substitution, \( \varepsilon_D \) and \( \varepsilon_M \), and is minimized and equal to 0 when \( \varepsilon_D = \varepsilon_M = 0 \). In words, regardless of the underlying correlation of the productivity terms, when production and preference elasticities are low, observed output will tend to strongly co-move. By contrast, in economies with larger production and preference elasticities, output will tend to co-move less for a given degree of correlation in the \( A \) terms.

In sum, the main takeaway from this simple example is that a given amount of observed output comovement could arise either from low elasticities of substitution and correlated shocks or, alternatively, high elasticities of substitution and relatively uncorrelated shocks. So, to properly assess how important common versus independent shocks are, I must have reliable estimates parameterizing consumers’ and firms’ ease of substitution. This is the task to which I turn in the following section.

Before doing so, with the aim providing the reader with some intuition, I briefly digress to address a recurring question that I received while presenting this paper: Is the propagation of industry-specific shocks—where propagation is defined, here, as the aggregate output response following a shock in an individual sector—more severe with complementarities in production? On the one hand, when inputs are more complementary a (negative) productivity shock to a supplying industry (e.g., Steel) will lead to larger decreases in output for downstream industries (Motor Vehicles, Construction, etc...). On the other hand, the output decline in the industry experiencing the productivity shock will be smaller when its output is more complementary to the output of other industries. These two countervailing effects balance each other out in this simple example. Indeed, this must be the case, as the simple example of this subsection falls within the class of models studied in Hulten (1978) and Acemoğlu et al. (2012). For this class of models, the aggregate impact of shocks to an individual sector are only a function of the sector’s gross output share; the elasticities of substitution do not matter. Instead the elasticities matter because they alter the way in which comovement in fundamental shocks map to comovements in observable data.
3 Estimates of the production elasticities

In this section, I estimate the model’s key elasticities of substitution. Towards this goal, I will apply industries’ cost-minimization conditions, as given in Equation 4 and 5, to estimate $\varepsilon_M$ and $\varepsilon_Q$. Recognizing the endogeneity of relative prices on the right-hand-sides of these equations, I follow Young (2014) and, especially, Acemoğlu et al. (2015) and use short-run industry-specific demand shifters as instruments. These shifts in demand arise from changes in military spending.

For this section, I use data from the BEA’s GDP by Industry and Input-Output Accounts data, spanning 1997 to 2013. The main variables that I construct from these tables are changes in i) industry $J$’s output price index, $\Delta \log P_t J$ and ii) its intermediate input price index, $\Delta \log P_{in}^{ij}$; iii) its intermediate input cost share, $\Delta \log \left( \frac{P_{i} M_{t,J}^{i}}{P_{ij} Q_{t,J}} \right)$; and iv) the fraction of industry $J$’s intermediate input cost shares that are due to purchases from industry $I$, $\Delta \log \left( \frac{P_{i} M_{t,I}^{i,J}}{P_{ij} M_{t,J}^{i}} \right)$. So that I may combine estimates of preference and production with Dale Jorgenson’s KLEMS data (which will be used in the following section), I collapse the 71 industries in the BEA data down to 30 industries. Appendix A, contains a detailed description of the construction of the variables used in this section.

Figure 1 depicts the smoothed relationship between $\Delta \log \left( \frac{P_{i} M_{t,I}^{i,J}}{P_{ij} M_{t,J}^{i}} \right)$ and $\Delta \log P_{in}^{ij} - \Delta \log P_{i,t,J}$, for each industry $J$ and $J$’s most-important-supplier industry . (So, to give an example, for the Electrical Machinery industry, which is depicted in the thick solid line in the bottom-left panel, I plot Electrical Machinery’s intermediate input expenditure share of Primary Metals on the y-axis, and the price of Primary Metals relative to the price of Electrical Machinery’s intermediate input bundle on the x-axis.) The main takeaway is that the share of a particular input among total intermediate input expenditures is positively correlated to the price of that input (relative to other intermediate inputs). Absent any omitted variables, Equation 5 would yield an unbiased estimate of $1 - \varepsilon_M$: The slope of $\Delta \log \left( \frac{P_{i} M_{t,I}^{i,J}}{P_{ij} M_{t,J}^{i}} \right)$ of $\Delta \log P_{in}^{ij}$, the average of which across industries is 0.85, would yield an estimate of $\varepsilon_M = 0.15$.

Similarly, $\varepsilon_Q$, the elasticity of substitution between intermediate inputs and value added, could be identified off of the slope of the relationship between changes in the intermediate input cost share, $\Delta \log \left( \frac{P_{in} M_{t,I}^{i,J}}{P_{ij} Q_{t,I}} \right)$, and the relative price of intermediate inputs, $\Delta \log P_{t,J} - \Delta \log P_{in}^{ij}$. Figure 2 plots this. All else equal, when $\varepsilon_Q$ is less than 1, higher intermediate input prices are correlated with larger fractions of expenditures spent on intermediate inputs. For most, but not all industries, this seems to be the case. The slope between $\Delta \log \left( \frac{P_{in} M_{t}}{V} \right)$ and $\Delta \log \left( \frac{P_{in} P}{T} \right)$ is statistically distinct from zero for 12 of the 30 industries: negative for three of the commodity-related industries—Petroleum/Gas Extrac-
Figure 1: Relationship between changes in intermediate input purchases and intermediate input prices.

Notes: For each downstream industry, $J$, I take the most important (highest average intermediate input expenditure share) supplier industry, $I$. The x-axis of each panel gives $\Delta \log \left( \frac{P_{it}}{P_{itJ}} \right)$. The y-axis gives, for each industry, changes in the fraction of industry $J$’s intermediate input expenditures that go to industry $I$. I compute and plot a local polynomial curve of this relationship, for each industry.
Figure 2: Relationship between changes in purchases of the intermediate input bundle and the relative price of the intermediate input bundle.

Notes: For each industry, \( J \), I plot the relationship between changes in its cost share of intermediate inputs, on the y-axis, and changes in the difference between the price of the intermediate input bundle and the marginal cost of production on the x-axis. I compute and plot a local polynomial curve of this relationship, for each industry.

...tion, Petroleum Refining, and Primary Metal Manufacturing—and positive for nine other industries. Overall, the slope of this line, for the average industry equals 0.4.

With the aim of more formally estimating \( \varepsilon_Q \) and \( \varepsilon_M \), I employ regressions of the form given by Equation 17

\[
\Delta \log \left( \frac{P_{tI} M_{tI,J} - J}{P_{tI} Q_{tI,J}} \right) = \alpha_t + \alpha_1 \left( \Delta \log P_{tJ}^{in} - \Delta \log P_{tI} \right) + \alpha_2 \left( \Delta \log P_{t,J} - \Delta \log P_{t,J}^{in} \right) + \eta_{t,I,J}
\]  

(17)

This equation emerges from the cost-minimization condition of each industry, combining Equations 4 and 5.

Shifts in relative productivity, which are correlated with changes in relative prices...
and enter the error term in Equation 17, may lead to biased estimates of the production elasticities. According to the model presented in Section 2, shocks to industries' final demand would alter industries' demand for specific factors only through their effects on relative prices. I use one of the instruments from Acemoğlu et al. (2015) to capture these shifts in final demand.\footnote{The other demand shifter used by Acemoğlu et al. (2015) focuses on changes in industry demand resulting from China’s consequential export expansion. Between 1995 and 2011, China’s gross output exports to the United States, as a share of U.S. GDP, has increased from 0.5 percent to 2.7 percent. More importantly for the purposes of the current paper, growth in China’s exports to the U.S. dramatically differ across industries. In the first-stage estimates of 4, increased exports from China are associated with an increase in prices, counter to the motivation for the instrument.}

The instrument exploits annual variation in military spending, in combination with heterogeneity in the extent to which different industries are suppliers to, either directly or indirectly, the military. It is defined as:

\[
military \text{ spending shock}_{t,J} \equiv \sum_{I} \text{Output}^{\%}_{1997,J \rightarrow I} \cdot S_{1997,J \rightarrow \text{military}} \cdot \Delta \log (\text{Military Spending}_{t}),
\]

(18)

where, \(S_{1997,J \rightarrow \text{military}}\) is the share of industry \(I\)'s output that is purchased by the "Federal National Defense" industries.\footnote{To define \(\text{Output}^{\%}_{1997,J \rightarrow I}\), write \(S_{1997,J \rightarrow I}\) as the share of industry \(J\)'s output that is purchased by industry \(I\) and store these elements in a matrix \(S\). Then, \(\text{Output}^{\%}_{1997,J \rightarrow I}\) is the \(J,I\) element of the matrix \(I + S + S^2 + S^3 + ... = (I - S)^{-1}\).} According to Equation 18, demand for an industry \(J\)'s output will vary due to fluctuations in military spending either if it is a direct supplier to the military, if its main customers are important suppliers to the military, or if its main customers are indirect suppliers to the military. Among the industries in the sample, the "Other Transportation Industry—in which ships, airplanes, and tanks are manufactured—has the highest \(S_{1997,J \rightarrow \text{military}}\). This industry has the strongest direct relationship with the military. Industries that are, on the other hand, indirectly reliant on purchases from the military include "Instruments" and "Petroleum Refining."

Table 1 presents the coefficient estimates from regressions defined by Equation 17. The first two columns present OLS estimates. In the IV specifications, given in the final two columns, the instruments have the expected relationship with the relative price variables: Increased demand from federal spending is positively related with the price of that industry’s good. In these specifications, the instruments are sufficiently powerful to yield reliable, unbiased estimates of \(\varepsilon_M\) and \(\varepsilon_Q\). For these specifications, the estimate of \(\varepsilon_M\) is essentially zero. For \(\varepsilon_Q\), the OLS estimates result in an estimate of 1.3; the IV specifications produce estimates closer to 0.5.
### Second stage regression results

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \log P_{tI}^{in} - \Delta \log P_{tI}$</th>
<th>$\Delta \log P_{tJ} - \Delta \log P_{tJ}^{in}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1.07 (0.04)</td>
<td>0.18 (0.06)</td>
</tr>
<tr>
<td></td>
<td>-1.13 (0.04)</td>
<td>0.27 (0.06)</td>
</tr>
<tr>
<td></td>
<td>-1.19 (0.26)</td>
<td>-0.39 (0.65)</td>
</tr>
<tr>
<td></td>
<td>-1.11 (0.27)</td>
<td>-0.50 (0.66)</td>
</tr>
</tbody>
</table>

### First stage: Dependent variable is $\Delta \log P_{tJ}^{in} - \Delta \log P_{tI}$

<table>
<thead>
<tr>
<th></th>
<th>$-1.81 (0.48)$</th>
<th>$-2.56 (0.53)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>military spending shock$_{tI}$</td>
<td>7.29</td>
<td>10.14</td>
</tr>
<tr>
<td>military spending</td>
<td></td>
<td></td>
</tr>
<tr>
<td>shock$_{tI}$’s suppliers</td>
<td>(0.81)</td>
<td>(1.25)</td>
</tr>
<tr>
<td>military spending shock$_{tJ}$</td>
<td>-4.53</td>
<td>-3.88</td>
</tr>
<tr>
<td>military spending</td>
<td></td>
<td></td>
</tr>
<tr>
<td>shock$_{tJ}$’s suppliers</td>
<td>(0.54)</td>
<td>(0.57)</td>
</tr>
<tr>
<td>F-statistic</td>
<td>35.05</td>
<td>12.30</td>
</tr>
</tbody>
</table>

### First stage: Dependent variable is $\Delta \log P_{tJ} - \Delta \log P_{tJ}^{in}$

|                                | 1.71 (0.32) | 1.20 (0.36) |
| military spending shock$_{tI}$ |                |                |
| military spending              | -1.47        | 0.44          |
| shock$_{tI}$’s suppliers       | (0.55)       | (0.84)       |
| military spending shock$_{tJ}$ | -0.81        | -0.38         |
| military spending              |                |                |
| shock$_{tJ}$’s suppliers       | (0.36)       | (0.39)       |
| F-statistic                    | 12.36         | 6.54          |

|                                | 12.36$^i$ | 11.96$^i$ |
| Cragg-Donald Statistic         |            |            |
| Wu-Hausman test p-value        | 0.60       | 0.48       |
| Year Fixed Effects             | No         | Yes        |
| No                            | 4800       | 4592       |
| Yes                           | 4800       | 4592       |

Table 1: Regression results related to Equation 17.

Notes: The overall sample includes pairs of industries $J$, and, for each industry $J$, the top ten supplying industries, $I$. In the third and fourth columns, the sample size is reduced because of the exclusion of the Government industry. In the row labeled "Cragg-Donald Statistic", an "$^i$" indicates that the test for a weak instrument is rejected at the 10 percent threshold.
the relationships between input expenditures and prices for different countries; using a different, more coarse, industry classification scheme; using a longer definition of a time period; and specifications for which these slopes are separately estimated for different subsamples of industries. The results in the appendix accord with those presented in Table 1. Here, I summarize the results of these exercises. First, with more coarsely defined industries, the estimated elasticity of substitution between intermediate inputs and the capital/labor aggregate is now somewhat larger, closer to 1.3 As before, $\varepsilon_M$ is small, close to 0. Second, estimates of $\varepsilon_M$ and $\varepsilon_Q$ are nearly identical with longer time periods (two years, instead of one year), and with samples that include more upstream observations per downstream observation $\times$ year. Second, using the World Input Output Tables (WIOT), I estimate the slopes of intermediate input cost share versus intermediate input price relationships for a sample of six developed countries—Denmark, France, Italy, Japan, the Netherlands, and Spain. While, for these countries, I cannot apply variation in military spending as an instrument, the OLS estimates for these six countries are similar to those in the first column of Table 1. In sum, the results from Table 1 are broadly, but not universally, robust to different samples.

While I am not aware of any previous research aimed at estimating $\varepsilon_M$, the estimates of $\varepsilon_Q$ presented in Table 1 broadly accord with the few existing estimates for these parameters. With respect to the elasticity of substitution between intermediate inputs and the capital/labor aggregate, Rotemberg and Woodford (1996) estimate $\varepsilon_Q$ by running an panel regression of manufacturing industries’ intermediate input expenditure shares against the relative price of intermediate inputs, instrumenting the relative price of intermediate inputs using the price of crude oil. For industries within the manufacturing sector, Rotemberg and Woodford estimate that $\varepsilon_Q = 0.7$. More recent papers, using variation in the unit prices that individual plants pay for different factors, yield estimates of $\varepsilon_Q$ in a similar range. Oberfield and Raval (2015) regress plants’ intermediate inputs cost shares against the wages prevailing in their local labor market, then combine this plant-level estimate with information on within-industry dispersion in plants’ intermediate inputs intensities to build an industry-level estimate of $\varepsilon_Q$. Their estimates of $\varepsilon_Q$ lie between 0.6 and 0.9. In Appendix B, I follow a similar strategy, exploiting spatial variation in materials prices instead of spatial variation in wages. I also arrive at estimates of $\varepsilon_Q$ within this same range.

The model’s other elasticities of substitution, in particular $\varepsilon_D$ and $\varepsilon_{LS}$, will turn out to play only a secondary in determining the importance of aggregate fluctuations. For these parameters, I will choose a wide range, centered around values that have been estimated in previous papers. With respect to the estimate of $\varepsilon_D$, Herrendorf, Rogerson, and Valentenyi (2013) estimate $\varepsilon_D$, considering long-run changes in broad sectors’ relative prices and final consumption expenditure shares. Their benchmark estimate of the preference elasticity
of substitution between expenditures on agricultural products, manufactured goods, and services is 0.9.\textsuperscript{15,16} Regarding $\varepsilon_{LS}$, an extensive literature has estimated the Frisch labor supply elasticity, with estimates varying between 0.5 and 3; see Prescott (2006) and Chetty et al. (2011) for two syntheses of this literature.

To summarize, Table 1 suggests that $\varepsilon_M \approx 0.0$ and $\varepsilon_Q \approx 0.5$ faithfully describe industries’ ability to substitute across inputs. Since the standard errors of $\varepsilon_Q$ are somewhat large, and since I have not even attempted to identify $\varepsilon_D$ or $\varepsilon_{LS}$, it will be necessary to apply a range of values for these parameters. In the following section, I will compute the aggregate importance of sectoral shocks applying different reasonable combinations of $\varepsilon_Q$, $\varepsilon_M$, $\varepsilon_{LS}$ and $\varepsilon_D$ to the Section 2’s model, and compare this estimated contribution of sectoral shocks to a calibration in which $\varepsilon_Q$, $\varepsilon_M$, and $\varepsilon_D$ are all set equal to 1.

4 Estimates of the importance of sectoral shocks

This section contains the main results of the paper. In this section, I describe the calibration of certain parameters and the procedure with which I estimate the importance of common productivity shocks (Section 4.1); present the estimates of the importance of sectoral shocks for different values of the preference and production elasticities (Section 4.2); and examine the sensitivity of the benchmark results to changes in sample, industry definition, country, and other details of the estimation procedure (Section 4.3). Additional robustness checks are discussed in Appendix E.

4.1 Calibration and estimation details

Besides the preference and production elasticities, the model filter requires data on industries’ output at each point in time along with information on the long-run-average relationships across sectors. I discuss these two requirements, in turn.

\textsuperscript{15}With respect to an industry classification scheme closer to the one used in the current paper, Ngai and Pissarides (2007) argue that "the observed positive correlation between employment growth and relative price inflation across two-digit sectors" (p. 430) supports an estimate of $\varepsilon_D$ that is less than 1. Also, Oberfield and Raval (2015) estimate a preference elasticity of between 0.8 and 1.1 across two-digit manufacturing industries; see Appendix D.4 of their paper.

\textsuperscript{16}To emphasize, $\varepsilon_D$ parameterizes how easily the consumer can substitute across coarsely-defined industries’ products (for example the elasticity of substitution between automobiles and furniture, or between apparel and construction). Broda and Weinstein (2006) and Foster, Haltiwanger, and Syverson (2008), among others, estimate a much larger elasticity of substitution in consumers’ preferences. These larger elasticities of substitution are estimated using \textit{within-industry} variation, and characterize how easily consumers substitute between, for example, ready-mix concrete produced by two different plants, or between different varieties of red wine.
Regarding the data on industries’ output, I combine Dale Jorgenson’s 35-Sector KLEMS dataset (which spans the 1960 to 2005 period) with the output data from the BEA Industry Accounts (spanning 1997 to 2013) that were used in the previous section.\textsuperscript{17} From these two datasets, I take information on industries’ gross output, using industry-specific price deflators.

The parameters $\xi_J$, $\mu_J$, $\alpha_J$, $\Gamma^X_{IJ}$, and $\Gamma^M_{IJ}$ are chosen to match the model-predicted cost shares to the corresponding values in the data. These parameters contain only information about the steady-state of the equilibrium allocation. The demand shares, $\xi_J$, are chosen so that the model’s steady-state consumption choices are proportional to the amount that the industry sells; the $\xi_J$ are restricted to sum to 1. The other parameters are chosen to match factor intensities, for each industry-factor pair. For instance, $\mu_J$ is the value that equates the model-predicted intermediate input cost share with the empirical counterpart.\textsuperscript{18} The empirical values that are used to calibrate the factor intensities are described in Appendix A. Online Appendix F.1 provides additional details on the calibration of the parameters relevant to the steady state.\textsuperscript{19}

I choose $\beta$ and $\delta_K$ based on the values used in past analyses. I set the discount factor, $\beta$, to 0.96 and the capital depreciation rate, $\delta_K$, to 0.125. In the benchmark analysis, $\delta_{CJ} = 1$ for all industries. That is, all industries’ goods are nondurable. I relax this assumption in Section 4.3. Finally, I set the labor supply elasticity, $\varepsilon_{LS}$, equal to 2, and explore the sensitivity of the main results to this parameter in Table 5.

These calibrated parameters define $\Pi_1$, $\Pi^A_2$, $\Pi^B_2$, $\Pi^A_3$, and $\Pi^B_3$, the matrices that appear in Equations 14 and 15. These equations—which I reproduce for the reader’s convenience below—can be used to infer each period’s productivity shocks.

\[
\omega^A_{t+1} = (\Pi^A_3)^{-1} \Delta \log Q_{t+1} - (\Pi^A_3)^{-1} \Pi_1 \Delta \log Q_t + (\Pi^A_3)^{-1} \Pi^A_2 \omega^A_t .
\]

\[
\omega^B_{t+1} = (\Pi^B_3)^{-1} \Delta \log Q_{t+1} - (\Pi^B_3)^{-1} \Pi_1 \Delta \log Q_t + (\Pi^B_3)^{-1} \Pi^B_2 \omega^B_t .
\]

I apply two procedures to recover estimates of the $\omega$s. First, following the approach of Foerster, Sarte, and Watson (2011), I initialize the first-period productivity shocks at 0, $\omega_t = 0$, and then iteratively apply Equations 14 or 15. This procedure is infeasible for...

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\textsuperscript{17}The Jorgenson data can be found on his home page; see http://scholar.harvard.edu/jorgenson/data. Jorgenson, Gollop, and Fraumeni (1987) provide an extensive description of this dataset.

\textsuperscript{18}When $\varepsilon_Q = 1$, the intermediate input cost share and $\mu_J$ are equal to one another. Alternatively, when intermediate inputs are gross complements or gross substitutes to other factors of production, the model-predicted cost share will also depend on the relative prices of the intermediate input bundle and the price of the other factors of production.

\textsuperscript{19}In Appendix E, I examine the sensitivity of Section 4.2’s results to using 1972, instead of 1997, as the year to which the steady-state allocation is calibrated.
certain sets of parameter values. For particular parameter configurations, some eigenvalues of \((\Pi_3)^{-1}\Pi_2\) are greater than 1 in absolute value. In this case, data on output changes alone cannot fully identify the productivity shocks.\(^{20}\) A second problem arises, as some of the eigenvalues of \(\Pi_3\) continuously pass from positive to negative values (or vice versa) as the chosen calibrated parameters are continuously modified.\(^{21}\) As a result, the smallest eigenvalue of \(\Pi_3\) is close to zero for certain combinations of the calibrated parameters. When either of these two issues arise, as a second approach, I treat the initial productivity shocks as an unknown state, and apply the Kalman filter, using the output data in each period to iteratively produce estimates of each date’s productivity innovation. In the parameter configurations for which the largest eigenvalue of \((\Pi_3)^{-1}\Pi_2\) is less than 1, or where the smallest eigenvalue of \(\Pi_3\) is sufficiently large, the two approaches produce the same estimates of the productivity shocks.

4.2 Results

With the estimates of \(\omega\) in hand, I present two measures of the importance of sectoral shocks in shaping aggregate volatility. To compute the first measure, I perform factor analysis to extract the (single) common component of the \(\omega\)s. Let \(\Lambda\) denote the \(N \times 1\) vector of factor loadings, \(\Sigma_{FF}\) the variance of the single factor, and \(\Sigma_{uu}\) the diagonal \(N \times N\) matrix which contains the variances of the industry-specific components of the productivity shocks. Then, with \(S_Y\) denoting the \(N\)-dimensional vector that contains each industry’s gross-output share, the fraction of GDP volatility that is explained by the independent component of industries’ productivity shocks is given by:

\[
R^2(\text{sectoral shocks}) = \frac{S_Y' \Sigma_{uu} S_Y}{S_Y' (\Lambda \Sigma_{FF} \Lambda' + \Sigma_{uu}) S_Y}. \tag{19}
\]

The second measure of the relative importance of the common shocks is the average sample correlation of the productivity shocks

\[
\bar{\rho}(\omega) = \frac{1}{N^2} \sum_{I=1}^{N} \sum_{J=1}^{N} \text{correlation}(\omega_i, \omega_j). \tag{20}
\]

\(^{20}\)For parameter combinations for which at least one eigenvalue of \((\Pi_3)^{-1}\Pi_2\) is greater than 1 in absolute value, the "poor man’s invertibility condition" in Fernández-Villaverde et al. (2007) is violated. In ongoing work, I explore the failure of this condition in more depth in Atalay and Drautzburg (2015).

\(^{21}\)To give an example, when all of the model’s elasticities are equal to 1, and applying all of the other choices described in this subsection, the smallest eigenvalue of \(\Pi_3^B\) is 0.04. Then, increasing \(\varepsilon_Q\) from 1.0 to 1.1 or 1.2 yields minimum eigenvalues of \(\Pi_3^B\) equal to 0.02, and \(-0.01\), respectively. For \(\varepsilon_Q\) near 1.15, then, the model filter given by Equation 15 will yield unreliable estimates of the \(\omega_t^B\).
These two measures were also used by Foerster, Sarte, and Watson (2011) to summarize the importance of sectoral shocks.

Figure 3 displays these two summary measures for different values of $\varepsilon_M$ and $\varepsilon_Q$. According to the left panel of this figure, when $\varepsilon_D$, $\varepsilon_M$, and $\varepsilon_Q$ are all equal to 1—as is the case in almost all previous analyses of multisector real business cycle models—sector-specific shocks account for 20 percent of aggregate volatility.\footnote{Foerster, Sarte, and Watson (2011) also perform a factor analysis on industries’ productivity shocks. They compute the fraction of industrial production growth that is due to the first two factors. The remaining variation can be considered equivalent to the industry-specific productivity shocks in the current paper. The two common factors explain 80 percent of the variation in overall industrial production growth in the first third of the sample (1972 to 1983) and 50 percent in the latter two-thirds (1984 to 2007). Averaging over these periods, sectoral shocks contribute roughly 40 percent of industrial production volatility.

There are a few potential explanations for why my figures may differ from those in Foerster, Sarte, and Watson (2011). One important difference is that the Foerster, Sarte, and Watson (2011) analysis is restricted to the goods-producing sectors of the economy, while I study the entire private economy; Ando (2014) explores the implications of this difference in coverage on the estimated contribution of industry-specific shocks. Other differences include a difference in sample period (1960 to 2011 in the current paper, compared to 1972 to 2008 in Foerster, Sarte, and Watson 2011), and period length (one quarter in Foerster, Sarte, and Watson 2011 versus one year, here). I show in the Appendix that excluding the Great Recession somewhat increases the assessed role of industry-specific shocks: When $\varepsilon_D = \varepsilon_M = \varepsilon_Q$, $R^2$ (sectoral shocks) is 26 percent without the Great Recession, instead of 19 percent when the whole sample period is included. Decreasing the period length would, on the other hand, with $\varepsilon_M = 1$ and $\varepsilon_D = 1$ would have little effect on the relative importance of sectoral shocks; see the penultimate column of Appendix Table 15.} For these same values of $\varepsilon_M$ and $\varepsilon_Q$ the average correlation of the productivity shocks is 0.19.

Lower calibrated values for the elasticity of substitution among intermediate inputs yields industries’ productivity shocks that are less correlated with one another. This relationship, which was the main takeaway of the simple example given in 2.4, is depicted in the two panels of Table 3. With $\varepsilon_M$ and $\varepsilon_D$ as 0.1 and 1, respectively, the filter results in productivity shocks that have an average correlation of 0.02. Put differently, the correlations among industries’ output growth rates could arise either through productivity shocks that are relatively correlated and relatively high levels of substitutability, or nearly independent productivity shocks and complementarity across the goods that industries produce.

With lower estimates of the correlation among productivity shocks, the common component of these shocks will account for a smaller fraction of aggregate volatility. Indeed, for $\varepsilon_D = 1$ and $\varepsilon_Q = 0.5$, more than three-fifths of aggregate volatility is due to industry-specific shocks so long as $\varepsilon_M \leq 0.2$; see the left panel of Figure 3. (With $\varepsilon_Q = 1$, the fraction of the variation due to sectoral TFP shocks is even higher.) The right panel of this figure illustrates $R^2$ (sectoral shocks) is relatively unresponsive to the chosen value of $\varepsilon_Q$. This, too, accords with Section 2.4’s example. With $\varepsilon_M = 1$ and $\varepsilon_D = 1$, the fraction of variation explained by industry-specific TFP shocks is between 12 and 24 percent for $\varepsilon_Q \in [0.15, 1.55]$. In sum, within the range of elasticities that I have estimated in Section 3, complementarities...
among intermediate inputs are important for assessing the role of aggregate fluctuations, but the elasticity of substitution between value added and intermediate inputs is not.

Table 2 expands on these results. In this table, I compute the fraction of variation explained by sectoral shocks for different $\varepsilon_M$ and $\varepsilon_D$ combinations. As in Figure 3, fixing a unit elasticity of substitution across intermediate inputs results in relatively high correlations among filtered productivity shocks, and a low estimated importance for industry-specific shocks. Even with improbably high values for $\varepsilon_D$, industry-specific shocks account for at least 40 percent of aggregate output volatility using Section 3’s estimate of $\varepsilon_M$.

Next, I examine whether the choice of elasticities has implications for individual historical episodes. Figure 4 presents historical decompositions for two choices of $\varepsilon_M$ and $\varepsilon_D$. The hollow circles denote the figures that result from the model filter, with $\omega_0$ fixed at 0, and iteratively applying Equation 14. The "+$" signs denote the figures resulting from the Kalman filter.

Table 2: Robustness checks: $R^2$ (sectoral shocks) and $\hat{\rho}(\omega)$ for different values of $\varepsilon_M$ and $\varepsilon_D$.

<table>
<thead>
<tr>
<th>$\varepsilon_M, \varepsilon_D, \varepsilon_Q$</th>
<th>$R^2$ (sectoral shocks)</th>
<th>$\hat{\rho}(\omega)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 1, 1</td>
<td>0.20$^{ kf}$</td>
<td>0.19$^{ kf}$</td>
</tr>
<tr>
<td>1, 1, $\frac{5}{2}$</td>
<td>0.15</td>
<td>0.23</td>
</tr>
<tr>
<td>$\frac{1}{10}$, $\frac{1}{2}$, 1</td>
<td>0.98$^{ kf}$</td>
<td>0.02$^{ kf}$</td>
</tr>
<tr>
<td>$\frac{1}{5}$, $\frac{1}{2}$, 1</td>
<td>0.98$^{ kf}$</td>
<td>0.04$^{ kf}$</td>
</tr>
<tr>
<td>$\frac{1}{10}$, $\frac{1}{2}$, 1</td>
<td>0.80</td>
<td>0.02</td>
</tr>
<tr>
<td>$\frac{1}{5}$, $\frac{1}{2}$, 1</td>
<td>0.55$^{ kf}$</td>
<td>0.05$^{ kf}$</td>
</tr>
<tr>
<td>$\frac{1}{10}$, $\frac{1}{2}$, 1</td>
<td>0.67</td>
<td>0.09</td>
</tr>
<tr>
<td>$\frac{1}{5}$, $\frac{1}{2}$, 1</td>
<td>0.44$^{ kf}$</td>
<td>0.15$^{ kf}$</td>
</tr>
<tr>
<td>$\frac{1}{10}$, $\frac{1}{2}$, 1</td>
<td>0.43$^{ kf}$</td>
<td>0.18$^{ kf}$</td>
</tr>
</tbody>
</table>

Table 2: Robustness checks: $R^2$ (sectoral shocks) and $\hat{\rho}(\omega)$ for different values of $\varepsilon_M$ and $\varepsilon_D$. Notes: A $^{ kf}$ indicates the usage of the Kalman filter, as opposed to direct applications of Equations 14 or 15, to infer the $\omega$ productivity shocks.
Figure 4: Historical decompositions.
Notes: The figure presents the percentage point change in each year’s aggregate output (relative to trend) due to industry-specific and common shocks. In the left panel $\varepsilon_D$, $\varepsilon_M$, and $\varepsilon_Q$ are all equal 1. In the right panel, $\varepsilon_M = 0.1$, $\varepsilon_D = 1$, and $\varepsilon_Q = 0.5$.

$\varepsilon_D = 1$, and productivity shocks are factor neutral. In the left panel, I set $\varepsilon_M = \varepsilon_Q = 1$; and, in the right, $\varepsilon_M = 0.1$ and $\varepsilon_Q = 0.5$. With relatively high elasticities of substitution across inputs, each and every recession between 1960 and the present day is explained almost exclusively by the common shocks. The sole partial exception is the relatively mild 2001 recession. In 2001 and 2002, Non-Electrical Machinery, Motor Vehicles, and Instruments, together accounting for aggregate growth rates that were 1.6 percentage point below trend, represent more than two-fifths of the below-trend GDP growth rates.

Table 3, along with the right panel of Figure 4, presents historical decompositions, now allowing for complementarities across intermediate inputs. Here, industry-specific shocks are the primary driver, accounting for a larger fraction of most, but certainly not all of, recent recessions and booms. According to the model-inferred productivity shocks, the early 80s recession was driven, to a large extent, by common shocks. At the same time, the 1974-75 recession, the late 90s expansion, and the 2009-11 recession are each more closely-linked with industry-specific events. Motor Vehicles played an important part in the early 80s, especially in the first of the two recessions. Instruments (basically Computer and Electronic Products) had an outsize role in the 1996-2000 expansion, while Wholesale/Retail, Motor Vehicles, and Finance, Insurance, and Real Estate (F.I.R.E.) appear to have had a large role in the most recent recession. (Other Services, due to their large gross output share, appear as important industries in most periods.) These model-inferred productivity shocks align with contemporaneous historical accounts.23

23 Related to the early 80s recession, Friedlaender, Winston, and Wang (1983) characterize the auto industry as "a state in flux. Not only has the Chrysler Corporation been perilously close to bankruptcy, but Ford
<table>
<thead>
<tr>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Other Services</td>
<td>-1.2%</td>
<td></td>
<td>Other Services</td>
<td>-0.7%</td>
<td>Instruments</td>
<td>0.2%</td>
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<td></td>
</tr>
<tr>
<td>Construction</td>
<td>-0.6%</td>
<td></td>
<td>Motor Vehicles</td>
<td>-0.7%</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>F.I.R.E.</td>
<td>-0.5%</td>
<td></td>
<td>Primary Metals</td>
<td>-0.2%</td>
<td>Instruments</td>
<td>0.2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motor Vehicles</td>
<td>-0.5%</td>
<td></td>
<td>Warehousing</td>
<td>-0.2%</td>
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<tr>
<td>Wholesale/Retail</td>
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<td>Common Factor</td>
<td>-0.5%</td>
<td>Common Factor</td>
<td>-5.4%</td>
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<td></td>
</tr>
<tr>
<td>Common Factor</td>
<td>1.6%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Change</td>
<td>-5.7%</td>
<td></td>
<td>Total Change</td>
<td>-7.0%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Historical decompositions, using $\varepsilon_M = \frac{1}{4}$ and $\varepsilon_D = 1$.

Notes: For four points in the sample, I report the five industries with the largest contributions to changes in aggregate output, the contribution of the common productivity shock, and the aggregate change in GDP, relative to trend.

4.3 Sensitivity analysis

In Table 4, I examine the sensitivity of the assessed role of industry-specific shocks to the industry classification scheme, the calibration of industries’ cost shares, the period length, and the durability of goods. Except for the second column of Table 4, throughout this section I assume that productivity is factor neutral rather than labor augmenting. The first column re-iterates the benchmark estimates; the second column applies labor-augmenting productivity shocks. With $\varepsilon_Q < 1$, sectoral shocks contribute a larger fraction to aggregate volatility when productivity is assumed to be factor neutral. In the third column, I establish that the results of Figure 3 are qualitatively robust to an 9-industry partition of the economy. In the fourth column, I use data from 1972 (instead of 1997, as in the benchmark calculations).

and General Motors have suffered unprecedented losses in recent years." (pp. 1-2) Regarding the 1996-2000 expansion, Jorgenson and Stiroh (2000) analyze the role information-technology-producing and consuming industries as a source productivity acceleration during this period.

24 These industries are primary inputs (industries 1 to 3, according to Table 7), construction (industry 4), non-durable goods (industries 5 to 7 and 10 to 14), durable goods (industries 8, 9, and 15 to 23), transport and communications (industries 24 to 26), wholesale and retail (industry 27), finance, insurance, and real estate (industry 28), and personal and business services (industry 29), and government (industry 30). While it would be interesting to test the sensitivity of these results to a finer industry classification scheme, the necessary data are unavailable.
between 10 and 20 percent of aggregate volatility. In contrast, so long as " and " and " and " account for at least 48 percent of aggregate volatility with fraction of aggregate volatility when " and " now constitute a substantially larger number of periods. In this column, I set " incorporates good durability, in which I allow for certain industries’ outputs to depreciate over a number of periods. In this column, I set " = 1 for all nondurable industries and " = 0.4 for durable industries. In this column, sectoral shocks now constitute a substantially larger fraction of aggregate volatility when " = 0.1. The final column incorporates good durability, in which I allow for certain industries’ outputs to depreciate over a number of periods. In this column, I set " = 1 for all nondurable industries and " = 0.4 for durable industries. In this column, sectoral shocks now constitute a substantially larger fraction of aggregate volatility when " = 0.1. Table 5 presents the relative importance of sectoral shocks for various values of " and " and " in the specifications in which " and " = 1, industry-specific shocks contribute between 10 and 20 percent of aggregate volatility. In contrast, so long as " = 0.1, industry-specific shocks account for at least 48 percent of aggregate volatility with " ∈ \{\frac{2}{3}, 1, \frac{4}{3}\}. As a third set of robustness checks, I examine the contribution of sectoral shocks to aggregate fluctuations in different countries. For this analysis, I employ data from EU-KLEMS database, which describes industries’ output growth rates for a range of developed countries between 1970 and 2007 (see Appendix C for a description of the dataset). As I estimate in Online Appendix D.2, industries’ input choices and input prices, from the World Input Output Tables, suggest that production elasticities are higher for these six countries than they in Table 1. For this reason, I choose somewhat higher values of " = 0.25 instead of

<table>
<thead>
<tr>
<th>( R^2 ) (sectoral shocks)</th>
<th>Benchmark</th>
<th>Labor-Aug. Productivity</th>
<th>9-Industry Classification</th>
<th>Use 1972 IO Table</th>
<th>Durable Goods</th>
</tr>
</thead>
<tbody>
<tr>
<td>(( \varepsilon_M, \varepsilon_D, \varepsilon_Q )) = (1, 1, 1)</td>
<td>0.19(^{\text{KF}})</td>
<td>0.19</td>
<td>0.18(^{\text{KF}})</td>
<td>0.16(^{\text{KF}})</td>
<td>0.14(^{\text{KF}})</td>
</tr>
<tr>
<td>(( \varepsilon_M, \varepsilon_D, \varepsilon_Q )) = (1, 1, \frac{1}{2})</td>
<td>0.14(^{\text{KF}})</td>
<td>0.99</td>
<td>0.15</td>
<td>0.12</td>
<td>0.13</td>
</tr>
<tr>
<td>(( \varepsilon_M, \varepsilon_D, \varepsilon_Q )) = (\frac{3}{10}, \frac{2}{3}, \frac{1}{3})</td>
<td>0.90(^{\text{KF}})</td>
<td>0.94</td>
<td>0.98</td>
<td>0.90(^{\text{KF}})</td>
<td>0.86(^{\text{KF}})</td>
</tr>
<tr>
<td>(( \varepsilon_M, \varepsilon_D, \varepsilon_Q )) = (\frac{1}{10}, 1, \frac{1}{3})</td>
<td>0.80</td>
<td>0.95</td>
<td>0.63(^{\text{KF}})</td>
<td>0.82</td>
<td>0.84(^{\text{KF}})</td>
</tr>
<tr>
<td>(( \varepsilon_M, \varepsilon_D, \varepsilon_Q )) = (\frac{4}{10}, \frac{4}{3}, \frac{1}{3})</td>
<td>0.48</td>
<td>0.93(^{\text{KF}})</td>
<td>0.76(^{\text{KF}})</td>
<td>0.56</td>
<td>0.46(^{\text{KF}})</td>
</tr>
</tbody>
</table>

\( \hat{\rho} (\omega) \)  

| \( \varepsilon_M, \varepsilon_D, \varepsilon_Q \)) = (1, 1, 1) | 0.21\(^{\text{KF}}\) | 0.21\(^{\text{KF}}\) | 0.24\(^{\text{KF}}\) | 0.20\(^{\text{KF}}\) | 0.22\(^{\text{KF}}\) |
| (\( \varepsilon_M, \varepsilon_D, \varepsilon_Q \)) = (1, 1, \frac{1}{2}) | 0.24\(^{\text{KF}}\) | 0.14\(^{\text{KF}}\) | 0.32\(^{\text{KF}}\) | 0.26 | 0.26 |
| (\( \varepsilon_M, \varepsilon_D, \varepsilon_Q \)) = (\frac{3}{10}, \frac{2}{3}, \frac{1}{3}) | 0.02\(^{\text{KF}}\) | 0.06 | 0.08 | 0.02\(^{\text{KF}}\) | 0.01\(^{\text{KF}}\) |
| (\( \varepsilon_M, \varepsilon_D, \varepsilon_Q \)) = (\frac{1}{10}, 1, \frac{1}{3}) | 0.02 | 0.01 | 0.09 | 0.02 | 0.02\(^{\text{KF}}\) |
| (\( \varepsilon_M, \varepsilon_D, \varepsilon_Q \)) = (\frac{4}{10}, \frac{4}{3}, \frac{1}{3}) | 0.08 | 0.05\(^{\text{KF}}\) | 0.15\(^{\text{KF}}\) | 0.06 | 0.08\(^{\text{KF}}\) |

Table 4: Robustness checks: \( R^2 \) (sectoral shocks) and \( \hat{\rho} (\omega) \) for different values of \( \varepsilon_M \) and \( \varepsilon_D \). Notes: A "^\text{KF}" indicates the usage of the Kalman filter, as opposed to direct applications of Equation 15, to infer the \( \omega \) productivity shocks.

25 The Capital Flows data necessary to construct \( \Gamma^X_{IJ} \) are unavailable for 1972. For this reason, I use the 1997 Capital Flows Table to infer the \( \Gamma^X_{IJ} \) for the robustness check corresponding to the final columns of Table 4.

26 These depreciation rates are considerably larger than have been estimated elsewhere by, for example, Hulten and Wykoff (1981). Unfortunately, applying lower depreciation rates would lead to exceedingly large eigenvalues of \( (\Pi^A_3)^{-1} \Pi^A_2 \) (and similarly for \( (\Pi^B_3)^{-1} \Pi^B_2 \)), meaning that the output data alone have difficulty recovering the underlying productivity shocks.
Table 5: Robustness checks: $R^2$(sectoral shocks) and $\bar{\rho}(\omega)$ for different values of $\varepsilon_M$ and $\varepsilon_D$.

Notes: A "\(\text{kf}\)" indicates the usage of the Kalman filter, as opposed to direct applications of Equation 15, to infer the $\omega$ productivity shocks. For this table $\varepsilon_Q = 0.5$.

0.10, in the second to fourth and final three rows of Table 6. As with the U.S. data, correlations among productivity shocks tend to be lower, and the assessed role of industry-specific shocks are higher, in specifications with lower values of the preference and production elasticities of substitution. For the five foreign countries for which I could extract the common factor, the industry-specific productivity shocks account for at least 64 percent of aggregate volatility with $\varepsilon_M = 0.25$.

In the Online Appendix, I demonstrate that the benchmark results are robust to i) the de-trending method, ii) censoring outlier observations, and iii) looking at different parts of the sample separately (excluding the Great Recession, or looking at the first half and second half of the sample separately).

5 Conclusion

In the short run, industries have limited ability to substitute across their inputs. This paper extends a standard multi-industry real business cycle model to explore the role of limited substitutability on the assessed role of sectoral shocks. A worked out example of this elaborate model indicates that observed relationships among industries’ output growth rates could either be rationalized with high elasticities of substitution in production (or preferences) along with correlated shocks, or low elasticities and uncorrelated shocks. Using data on industries’ input choices and their input prices, I find that production elasticities of are, on balance, small. As a result, I find that sectoral shocks are more important than previously thought. Whereas previous assessments of multisector real business cycle models,
Table 6: Robustness checks: $R^2$ (sectoral shocks) and $\hat{\rho}(\omega)$ for different values of $\varepsilon_M$ and $\varepsilon_D$.

Notes: A $^{\text{Kf}}$ indicates the usage of the Kalman filter, as opposed to direct application of Equation 15, to infer the $\omega$ productivity shocks. I could not compute $R^2$ (sectoral shocks) for Japan, as there are fewer time periods than there are industries in the sample.

based on unitary elasticities of substitution across inputs and consumption products, have concluded that industry-specific shocks account for less than half of aggregate volatility, the current paper indicates that sectoral shocks are the primary source of GDP fluctuations.

A Details of the U.S. data

This section clarifies the sample construction and defines the variables used to estimate the model's elasticities of substitution. The main data sources are the 1997-2013 "Use" tables and the 1997 Capital Flows Table, from the Bureau of Economic Analysis; and Dale Jorgenson’s KLEMS dataset.

Table 7 characterizes the way in which I classify industries. The NAICS codes refer to those in the Annual IO Tables. The third through fifth columns of Table 7 give the cost shares of capital, labor, and intermediate inputs. These are computed from the BEA GDP by Industry dataset. The intermediate input cost share is computed as the ratio of intermediate input expenditures relative to total gross output. The labor share is the ratio of labor compensation to total gross output. The residual is defined as the capital cost share. The final column of Table 7 gives the consumption expenditure share of each industry. The consumption expenditures are taken from the BEA 1997 Input-Output Table, as sales to the following industry codes: F010 (personal consumption expenditures), F02R (residential private fixed investment), and F04 (exports). To compute consumption expenditures by the government sector, I combine F06C (Federal national defense: Consumption expenditures),

<table>
<thead>
<tr>
<th>Country</th>
<th>Denmark</th>
<th>Spain</th>
<th>France</th>
<th>Italy</th>
<th>Japan</th>
<th>Netherlands</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$ (sectoral shocks)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = (1, 1, 1)$</td>
<td>0.59$^{\text{Kf}}$</td>
<td>0.79$^{\text{Kf}}$</td>
<td>0.60</td>
<td>0.29$^{\text{Kf}}$</td>
<td>0.37</td>
<td>0.34</td>
</tr>
<tr>
<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = (1, 1, \frac{1}{2})$</td>
<td>0.46$^{\text{Kf}}$</td>
<td>0.66$^{\text{Kf}}$</td>
<td>0.41$^{\text{Kf}}$</td>
<td>0.24</td>
<td>0.16</td>
<td>0.26$^{\text{Kf}}$</td>
</tr>
<tr>
<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{2}\right)$</td>
<td>0.98$^{\text{Kf}}$</td>
<td>0.99$^{\text{Kf}}$</td>
<td>1.00$^{\text{Kf}}$</td>
<td>0.98</td>
<td>0.95</td>
<td>0.99$^{\text{Kf}}$</td>
</tr>
<tr>
<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = \left(\frac{1}{2}, 1, \frac{1}{2}\right)$</td>
<td>0.91</td>
<td>0.99$^{\text{Kf}}$</td>
<td>1.00</td>
<td>0.95</td>
<td>0.82</td>
<td>0.79</td>
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<tr>
<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = \left(\frac{1}{3}, \frac{4}{3}, \frac{1}{2}\right)$</td>
<td>0.81$^{\text{Kf}}$</td>
<td>0.99$^{\text{Kf}}$</td>
<td>0.96</td>
<td>0.67$^{\text{Kf}}$</td>
<td>0.64</td>
<td>0.66$^{\text{Kf}}$</td>
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</table>

<table>
<thead>
<tr>
<th>$\hat{\rho}(\omega)$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = (1, 1, 1)$</td>
<td>0.10$^{\text{Kf}}$</td>
<td>0.10$^{\text{Kf}}$</td>
<td>0.11</td>
<td>0.15$^{\text{Kf}}$</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = (1, 1, \frac{1}{2})$</td>
<td>0.11$^{\text{Kf}}$</td>
<td>0.11$^{\text{Kf}}$</td>
<td>0.13$^{\text{Kf}}$</td>
<td>0.18</td>
<td>0.17</td>
<td>0.14$^{\text{Kf}}$</td>
</tr>
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<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{2}\right)$</td>
<td>0.02$^{\text{Kf}}$</td>
<td>0.06$^{\text{Kf}}$</td>
<td>0.02$^{\text{Kf}}$</td>
<td>0.10$^{\text{Kf}}$</td>
<td>0.06</td>
<td>0.12$^{\text{Kf}}$</td>
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<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = \left(\frac{2}{3}, 1, \frac{1}{3}\right)$</td>
<td>0.03</td>
<td>0.05$^{\text{Kf}}$</td>
<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>$(\varepsilon_M^<em>, \varepsilon_D^</em>, \varepsilon_Q^*) = \left(\frac{1}{3}, \frac{4}{3}, \frac{1}{2}\right)$</td>
<td>0.04$^{\text{Kf}}$</td>
<td>0.05$^{\text{Kf}}$</td>
<td>0.11</td>
<td>0.13$^{\text{Kf}}$</td>
<td>0.09</td>
<td>0.06$^{\text{Kf}}$</td>
</tr>
<tr>
<td>#</td>
<td>Name</td>
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<td>Capital</td>
<td>Labor</td>
<td>Intermediate Inputs</td>
<td>Consumption</td>
</tr>
<tr>
<td>----</td>
<td>-----------------------------------</td>
<td>-------</td>
<td>---------</td>
<td>-------</td>
<td>---------------------</td>
<td>-------------</td>
</tr>
<tr>
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<td>0.23</td>
<td>0.44</td>
<td>0.001</td>
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<td>4</td>
<td>Construction</td>
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<td>0.16</td>
<td>0.33</td>
<td>0.51</td>
<td>0.034</td>
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<td>5</td>
<td>Food &amp; Kindred Products</td>
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<td>0.13</td>
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<td>0.23</td>
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<td>Apparel, Leather</td>
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<td>0.24</td>
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<td>8</td>
<td>Lumber</td>
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<td>0.23</td>
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<td>0.63</td>
<td>0.009</td>
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<td>14</td>
<td>Rubber &amp; Plastics</td>
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<td>0.23</td>
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<td>Non-metallic Minerals</td>
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<td>16</td>
<td>Primary Metals</td>
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<td>Fabric. Metal Products</td>
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<td>Non-Electrical Machinery</td>
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<td>0.27</td>
<td>0.59</td>
<td>0.009</td>
</tr>
<tr>
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<td>Electrical Machinery</td>
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<td>0.25</td>
<td>0.65</td>
<td>0.005</td>
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<td>Motor Vehicles</td>
<td>3361-3363</td>
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<td>0.72</td>
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</tr>
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<td>Other Transport. Equip.</td>
<td>3364-3369</td>
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<td>0.27</td>
<td>0.64</td>
<td>0.008</td>
</tr>
<tr>
<td>22</td>
<td>Instruments</td>
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<td>0.26</td>
<td>0.57</td>
<td>0.021</td>
</tr>
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<td>Misc. Manufacturing</td>
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<td>0.20</td>
<td>0.32</td>
<td>0.48</td>
<td>0.008</td>
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<td>Warehousing</td>
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<td>Government</td>
<td>G</td>
<td>0.15</td>
<td>0.54</td>
<td>0.31</td>
<td>0.186</td>
</tr>
</tbody>
</table>

Table 7: Industry Definitions, Factor Shares, and Preference Weights
Section 3’s analysis requires information on purchases across industries, the prices of each industry’s good, and the price of each industry’s intermediate input bundle. For each year between 1997 and 2013, the Annual IO Tables contain information on the value of commodities that are used by different industries. The output price of each industry is taken from the BEA GDP by Industry dataset, using the Fisher ideal price index to aggregate up to the Table 7’s classification. To compute each industry’s intermediate input price, I follow a similar procedure: For each downstream industry, I require information on changes in its intermediate input bundle’s price, for each year (This variable appears on the right-hand side of Equation 17). I use the Fisher ideal price index to compute change in the intermediate input prices:

$$\Delta \log P_{t+1,J}^{in} = \sum_{I=1}^{30} \frac{P_{t,I}M_{t,I\rightarrow J} + P_{t+1,I}M_{t+1,I\rightarrow J}}{P_{t,I}M_{t,I\rightarrow J} + P_{t+1,I}M_{t+1,I\rightarrow J}} \cdot \Delta \log P_{t+1,I},$$

where, as in Section 2, $M_{t,I\rightarrow J}$ represents the physical units of intermediate inputs from industry $I$ to industry $J$ and $P_{t,I}$ denotes the unit price of industry $I$’s output. So, for each downstream industry, $J$, I compute the change in its intermediate input price—between years $t$ and $t + 1$—as the weighted average in the changes in the prices of the supplying industries, $I$, with the weights set at the year $t$ and $t + 1$ share of $J$’s intermediate input purchases that come from industry $I$.

To construct $\Gamma^M$ and $\Gamma^K$, I use data from the 1997 Input Output Table and Capital Flows Table. I make two adjustments to the 1997 Capital Flows Table when producing $\Gamma^K$. First, government investment is not measured in the Capital Flows Table. As a result, I need to apply information from the Input Output Table, which does contain sales to the government investment industry. These are measured as sales to the following industries: F06S, F06E, F06N (Investment in Federal Defense); F07S, F07E, F07N (Investment in Federal Nondefense); F10S, F10E, F10N (Investment in State and Local Government). Second, to account for the maintenance and repair expenditures that are not included in the Capital Flows Table. As McGrattan and Schmitz (1999) report, maintenance expenditures are sizable, potentially accounting for 50% of total physical capital investment. Foerster, Sarte, and Watson (2011) use this finding as motivation for adding to the diagonal entries of $\Gamma^K$. I add a 40% share to the diagonal entries of $\Gamma^K$ to account for these maintenance and repair expenditures. This augmentation presumes that capital-good repairs draw on within-industry resources (e.g., firms that produce a product use their own inputs to repair
their capital equipment).\textsuperscript{27}

In Table 4, the set of durable goods are those designated as such by Basu, Fernald, and Kimball (2006), plus the Construction industry: Construction, Lumber, Furniture and Fixtures, Non-metallic Minerals, Primary Metals, Fabricated Metal Products, Non-Electrical Machinery, Electrical Machinery, Motor Vehicles, Other Transportation Equipment, Instruments, and Miscellaneous Manufacturing.

\section*{B Cross-sectional estimates}

In this section, I will apply plant-level input price variation and materials usage to provide an alternate set of estimates of $\varepsilon_Q$. To do so, I pursue the following two-part strategy. For each industry, I estimate how easily individual plants substitute across their factors of production, by relating plants’ materials purchases to their materials prices. Then, I apply the methods developed in Oberfield and Raval (2015), which allow me to combine information on a) the plant-level elasticity of substitution, b) the dispersion of materials cost shares, and c) the elasticity of plant scale to marginal costs so that I can ascertain the corroborating estimates of $\varepsilon_Q$.

To preview the main results of this section, the elasticity of substitution for the plant-level production function is approximately 0.65. Because within-industry variation in materials expenditure shares is small for each of the ten industries, the industry level production function’s elasticity of substitution is only somewhat higher, 0.75. Moreover, across the industries in the sample, the industry level elasticities of substitution are similar to one another.

\subsection*{B.1 Data source and sample}

The data source, for this section, is the Census of Manufacturers. This dataset contains plant-level information for each manufacturer in the United States, and is collected once every five years, in years ending in a "2" or a "7." For certain industries, plants with greater than five employees are asked to provide information on each of the material inputs that they consume and each of the products that they produce. Critically, for the empirical analysis of this section, the Census Bureau elicits information on both the quantities and values of these inputs and outputs, allowing me to construct plant-level prices. Additionally, the Census

\textsuperscript{27}There is a more practical rationale behind this alteration of the capital flows table. When the diagonal entries of $\Gamma^K$ are sufficiently low, several of the eigenvalues of $(\Pi^K_3)\Pi^K_2$ (and, also, $(\Pi^K_3)^{-1}\Pi^K_2$) are larger than 1 in absolute value, indicating that the calibrated models are non-invertible.
Table 8: Description of the 10 industries in the sample.
Notes: This table is a duplicate of Table 1 of Atalay (2014). The percentages that appear in the Material Inputs column are the fraction of materials expenditures that go to each particular material input. The Material Inputs column shows the inputs that represent greater than 6% of the average plant’s total material purchases.

Bureau records a plant identifier, which will allow me to compare the intermediate input purchases of the same plant across different time periods.

The sample in this section is identical to that which was used in an earlier paper (see Atalay 2014). The industries are those for which outputs and inputs are relatively homogeneous. This choice reflects a desire to, as much as possible, rule out heterogeneous quality as a source of input or output price variation. The ten industries that comprise the sample are corrugated boxes (with the years 1972-1987 and 1992-1997 analyzed separately), ground coffee, ready-mix concrete, white wheat flour, gasoline, bulk milk, packaged milk, raw cane sugar, and grey cotton yarn; see Table 8. For additional details regarding the sample, see Appendix B of Atalay (2014).

### B.2 Environment and assumptions

Each industry, $I$, is comprised of a set of plants $i \in I$, who combine capital, labor, material inputs, and purchased services to produce a single product. The production function is constant-returns to scale; separable between material inputs, $N$, and other inputs, $O$; with...
constant elasticity of substitution, $\eta_P$:

$$Q_{it}(K_{it}, L_{it}, S_{it}, N_{it}) = \left((A_{it} \cdot O_{it})^{\eta_P-1} + (B_{it} \cdot N_{it})^{\eta_P-1}\right)^{\frac{\eta_P}{\eta_P-1}}, \quad (21)$$

where $O_{it} = F(K_{it}, L_{it}, S_{it})$

Also by assumption, $F$ exhibits constant returns to scale. Plants are allowed to flexibly alter their input choices, including capital, each period. Furthermore, the factor prices that each plant faces, both for the material input and for the other input aggregate, are constant in the amount purchased. These assumptions serve a dual purpose. Not only do these assumptions greatly simplify the estimation of $\eta_P$, they also allow me to apply Oberfield and Raval (2015)’s methodology to estimate $\varepsilon_Q$ from $\eta_P$.

Use $P_{it}^{oth}$ and $P_{it}^{mat}$ to denote the factor prices for a unit of the other input aggregate and the material input, respectively. Let $A_{it}$ and $B_{it}$ represent the two plant-level productivity measures (other-input-augmenting and materials augmenting).

The demand curve faced by each plant, $i$, has constant elasticity, $\varepsilon_D$:

$$Q_{it} = \exp\{\theta_{it}\} \cdot \left(P_{it}^{oth}\right)^{-\eta_D} \quad (22)$$

In equation 22, $\theta_{it}$ represents a plant-year specific demand shifter. The assumption of a constant elasticity demand curve, while probably counterfactual, is again useful for multiple reasons. The constant-demand-elasticity assumption allows me to directly apply the Foster, Haltiwanger, and Syverson (2008) methodology to estimate $\eta_D$. Moreover, the same assumption is invoked by Oberfield and Raval (2015)—whose work I apply, here—in their aggregation of plant-level to industry level production functions.

The profit-maximizing levels of $N_{it}$ and $O_{it}$ yield the following expression for the material-output ratio:

$$\log \left[ \frac{N_{it}}{Q_{it}} \right] = -\eta_P \cdot \log \left[ \frac{P_{it}^{mat}}{P_{it}} \right] + \eta_P \cdot \log \left[ \frac{\eta_D - 1}{\eta_D} \right] + (\eta_P - 1) \log B_{it} \quad (23)$$

This equation will form the basis of the estimation of $\eta_P$, a task to which I now turn.

**B.3 The micro elasticity of substitution**

In this subsection, I estimate the plant-level elasticity of substitution between purchased inputs and other inputs. The baseline regression that I run is:

$$n_{it} - q_{it} = -\eta_P \cdot \left( P_{it}^{mat} - p_{it} \right) + \epsilon_{it} \cdot \quad (24)$$
In Equation 24, and throughout the remainder of the section, I use lower-case letters to denote the logged, de-meaned values of the variable of interest. In other words, \( \eta_P \) is estimated only using within industry year variation. To emphasize, both \( n_{it} \) and \( q_{it} \) refer to the number of physical units, and not the values, of the material good that plant \( i \) purchases and the output that it produces.

Ordinary least squares results are presented in the first column of Table 9. For most industries, the estimate of \( \eta_P \) lies between 0.5 and 0.7, with concrete and flour having two of the lower estimates and bulk milk and raw cane sugar with two of the higher estimates.

There are at least two concerns regarding the interpretation of \( \eta_P \)—from an OLS estimate of Equation 24—as an estimate of the micro elasticity of substitution. First, to the extent that the constant elasticity of demand assumption—embodied in Equation 22—is violated, Equation 24 suffers from omitted variable bias. A positive correlation between \( \log \left( \frac{n_D - 1}{n_D} \right) \) and \( (p_{mat}^{it} - p_{it}) \) will engender a positive bias in \( \eta_P \). Second, I have assumed that the materials supply curve that each \( i \) faces is flat. It is likely, however, that each plant’s factor supply curve is upward sloping. This instance of simultaneity bias—whereby a high-\( B_{it} \) plant pays a high materials price—will also engender a positive bias in \( \eta_P \).

I offer two different approaches to circumvent these problems. First, I append plant-level fixed effects to Equation 24. These fixed effects aim to capture long-run cross-sectional variation in the conditions in output and factor markets. As Foster, Haltiwanger, and Syverson (2008, 2012) argue, the factor market conditions that a plant faces are substantially more persistent than its productivity.

In a second specification, I instrument plants’ output and materials prices with the prices paid and charged by competitor plants. Specifically, the two instrumental variables, for \( p_{it}^{mat} - p_{it} \), are a) the year-\( t \) average materials price for plants that are within 50 miles of plant \( i \), and b) the year-\( t \) average output price for plants that are within 50 miles of plant \( i \). The idea behind these instruments is that the price of materials in nearby markets is correlated the price that \( i \) pays for its material inputs (if, for example, there is spatial correlation in the abundance of primary inputs used in the production of \( i \)’s intermediate inputs, or if there is a very productive, low marginal-cost supplier nearby), but should not in any other way affect the propensity for \( i \) have exceptionally high or low materials expenditure shares.\(^{28}\)

\(^{28}\)Results from first-stage regressions indicate that these instruments are relevant, at least for the four largest subsamples: materials prices and output prices are each spatially correlated.
<table>
<thead>
<tr>
<th>Sample</th>
<th>$\hat{\eta}^P$(OLS)</th>
<th>$\hat{\eta}^P$(FE)</th>
<th>$\hat{\eta}^P$(IV)</th>
<th>$\hat{\chi}$</th>
<th>$\hat{\eta}^P$</th>
<th>$\hat{\varepsilon}^Q$(OLS)</th>
<th>$\hat{\varepsilon}^Q$(FE)</th>
<th>$\hat{\varepsilon}^Q$(IV)</th>
<th>N(FE)</th>
<th>N(IV)</th>
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<td>0.683</td>
<td>0.817</td>
<td>0.398</td>
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<td>(0.021)</td>
<td>(0.203)</td>
<td>(0.118)</td>
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<td>5.744</td>
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<td>0.938</td>
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<td>(0.046)</td>
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<td>(0.045,0.052)</td>
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<td>(0.038)</td>
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<td>(0.351)</td>
<td>(0.224,0.434)</td>
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<td>Gasoline</td>
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<td>0.403</td>
<td>0.496</td>
<td>0.20</td>
<td>3.466</td>
<td>0.540</td>
<td>0.464</td>
<td>0.555</td>
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<td>(0.054)</td>
<td>(0.065)</td>
<td>(0.016,0.023)</td>
<td>(1.308)</td>
<td>(0.365,0.685)</td>
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<tr>
<td>Milk, Bulk</td>
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<td>0.656</td>
<td>0.040</td>
<td>2.380</td>
<td>0.872</td>
<td>0.725</td>
<td>65</td>
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<td>(0.085)</td>
<td>(0.171)</td>
<td>(0.028,0.048)</td>
<td>(0.553)</td>
<td>(0.687,1.063)</td>
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<tr>
<td>Milk, Packaged</td>
<td>0.605</td>
<td>0.731</td>
<td>0.420</td>
<td>0.035</td>
<td>2.555</td>
<td>0.674</td>
<td>0.797</td>
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<td>(0.020)</td>
<td>(0.025)</td>
<td>(0.032,0.039)</td>
<td>(0.145)</td>
<td>(0.629,0.728)</td>
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<td>Sugar</td>
<td>1.034</td>
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<tr>
<td>Yarn</td>
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<td>0.035</td>
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<td>(0.037)</td>
<td>(0.050)</td>
<td>(0.025,0.032)</td>
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<tr>
<td>Pooled</td>
<td>0.538</td>
<td>0.680</td>
<td>0.078</td>
<td>0.035</td>
<td>2.543</td>
<td>0.608</td>
<td>0.746</td>
<td>0.164</td>
<td>7517</td>
<td>10,503</td>
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<td>(0.010)</td>
<td>(0.012)</td>
<td>(0.033,0.036)</td>
<td>(0.080)</td>
<td>(0.582,0.636)</td>
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Table 9: Components of the industry-level elasticity of substitution.
Notes: The first three columns present $\hat{\eta}^P$, as estimated using Equation 24. The values given in the fourth, sixth, seventh, and eighth, columns are computed as in Equation 25, while $\hat{\eta}^P$ is estimated using Equation 26. Robust standard errors are included in the first, second, and fourth columns, while bootstrapped confidence intervals are provided in the third, sixth, and eighth columns. "OLS," "FE," and "IV" refer, respectively, to the values corresponding to the ordinary least squares, fixed effects, and instrumental variables specifications for the estimate of $\eta^P$. 
Results from the two sets of regressions are given in the second and third columns of Table 9. In the second column, estimates of $\eta_P$ range from 0.40 to 0.92, with the two largest estimates corresponding to two of the smaller-sample industries, coffee and sugar. The pooled estimate of $\eta_P$ is 0.68.

The instrumental variables are weak for the six smallest samples. For this reason, the IV specification is performed only on the samples of plants in the corrugated boxes, ready-mix concrete, packaged milk, and petroleum industries. In the third specification, the parameter estimates are smaller and much less precisely estimated. The biggest difference is for the ready-mix concrete industry, for which the estimate of $\eta_P$ is essentially 0.

### B.4 The industry-level elasticity of substitution

The previous subsection provided an estimate for the ease with which individual plants substitute between material inputs and other inputs. This is related to, but distinct from, how easily an industry substitutes between material inputs and other inputs.

Changes in the scale, across plants, potentially makes the industry-level elasticity of substitution larger than the corresponding plant-level elasticity. The difference between the plant-level and industry-level elasticities of substitution depends on a) the heterogeneity of materials shares, within the industry, and b) how much inputs shift across plants, in response to a change in relative factor prices.

Given the assumptions, specified in Section B.2, the industry-level elasticity of substitution has a simple expression:  

$$
\varepsilon_Q = \chi_{II} \cdot \eta_P + (1 - \chi_{II}) \cdot \eta_P, \quad \text{where}
$$

$$
\chi_{II} \equiv \frac{1}{S_{II} (1 - S_{II})} \cdot \sum_{i \in I} \left( S_{II} - \frac{M_{it} P_{it}^{\text{mat}}}{M_{it} P_{it}^{\text{mat}} + O_{it} P_{it}^{\text{oth}}} \right)^2 \cdot \frac{M_{it} P_{it}^{\text{mat}}}{\sum_{j \in I} M_{jt} P_{jt}^{\text{mat}} + O_{jt} P_{jt}^{\text{oth}}},
$$

$$
S_{II} \equiv \sum_{i \in I} \frac{M_{it} P_{it}^{\text{mat}}}{M_{it} P_{it}^{\text{mat}} + O_{it} P_{it}^{\text{oth}}},
$$

In words, the industry-level elasticity of substitution is a convex combination of the plant-level elasticity of substitution and the plant-level elasticity of demand. The demand elasticity parameterizes how sensitive the scale of the plant is to changes in its marginal cost of production. Consider, for example, an increase in the price of the material input. The marginal cost of production will increase more for plants with relatively large materials cost shares. As a result, low materials share plants will produce relatively more of the

---

29 A proof is given in Oberfield and Raval (2015). See Appendix A.1 of that paper.
total industry output following the increase of the materials price. The elasticity of demand determines how much less the high-materials-share plants will produce, following the increase in the materials price.

The scope for this across-plant factor substitution depends on the dispersion of materials intensities. According to Equation 25, the appropriate measure of the dispersion of materials intensity is a weighted, normalized variance of the materials cost shares. The fraction of total industry expenditures incurred by plant $i$ (given in term 3) is the appropriate weight for summing over the within-industry deviation in materials cost shares (given in term 2). The normalization, given in term 1, ensures the $\chi_{it}$ lies within the unit interval.

What remains, then, is to provide estimates for the normalized variance of materials shares, $\chi$, and the elasticity of demand, $\eta_D$, for the ten industries in my sample.

The normalized variance of materials shares, $\chi$, ranges from 0.019 (for flour) to 0.065 (for sugar). Given these low values, the industry elasticity of substitution will closely track the micro elasticity of substitution. In other words, the estimate of $\varepsilon_Q$ will be, for the most part, insensitive to the way in which $\eta_D$ is estimated.

I estimate $\eta_D$ via the regression defined by the following equation:

$$ q_{it} = \alpha_t + \alpha_1 \cdot \log INCOME_{\Upsilon t} + \eta_D \cdot p_{it} + \theta_{it} \tag{26} $$

This specification, and the variable definitions, follow Foster, Haltiwanger, and Syverson (2008). In Equation 26, $INCOME_{\Upsilon t}$ is the aggregate income in establishment $i$’s market, $\Upsilon$, at time $t$. This variable is included to account for any differences in establishment scale that may exist between areas of high and low density of economic activity.

A positive relationship between the demand shifter ($\theta_{it}$) and output price ($p_{it}$) potentially induces a downward bias to the OLS estimates of $\eta_D$. Like Foster, Haltiwanger, and Syverson (2008), I instrument $p_{it}$ with the marginal cost of plant $i$ in year $t$. This instrumental variable is certainly relevant: plants with lower marginal costs have significantly lower output prices. Validity of the instrument rests, then, on the orthogonality of marginal costs and $\theta_{it}$. Foster, Haltiwanger, and Syverson (2008) discuss two potential threats to the validity of the instrument (measurement error in plants’ marginal costs, and a selection bias that induces a negative relationship between demand shocks and marginal costs), propose robustness checks to assess the salience of these two threats, and find that their results are similar across the different robustness checks.

The results of these regressions are presented in the fourth column of Table 9.\(^{31}\)

---

\(^{30}\)To give the reader some idea, the (unnormalized) standard deviations of materials shares range from 4.3% to 11.4% across the ten industries, again lowest for bulk milk and highest for raw cane sugar.

\(^{31}\)The results reported here are slightly different from those in Foster, Haltiwanger, and Syverson (2008):
each of the ten industries, the estimate for elasticity of demand is greater than 1, reassuringly indicating that plants are pricing on the elastic portion of their demand curve.

Combining the estimates of $\eta_P$, $\eta_D$, and $\chi$ yields the object of interest: the industry-level elasticity of substitution, $\varepsilon_Q$. Since there are three sets of estimates of $\eta_P$, there are also three sets of estimates of $\varepsilon_Q$. For the estimates corresponding to the fixed effects regression, $\varepsilon_Q$ is 0.75 for the pooled sample.\textsuperscript{32} Except for sugar and coffee (two of the smallest industries, representing only 5 percent of the sample), the industry-level elasticities of substitution range between 0.46 (for gasoline) and 0.82 (for corrugated boxes). For seven of the ten industries in the sample (with the exceptions being the smallest three subsamples), the data would reject a null hypothesis of $\varepsilon_Q = 1$.

The estimates of $\varepsilon_Q$ that correspond to the instrumental-variables-based estimate of $\eta_P$ are smaller, though again much less precisely estimated. The point estimate for $\varepsilon_Q$ is 0.10 for the ready-mix concrete subsample, and is somewhat higher (between 0.40 and 0.55) for the other three industries.

In summation, micro data on plants’ materials usage patterns indicate that material inputs are gross complements to other factors of production. For most specifications (all except for the IV specification for the ready-mix concrete subsample, or the fixed effects specification for the smaller industries), the data indicate that $\varepsilon_Q$ ranges between 0.45 and 0.80.

References


Acemoğlu, Daron, Vasco M. Carvalho, Asuman Özdağlar, and Alireza Tahbaz-Salehi. 2012.

I restrict my sample to those plants for which I can observe materials prices, while Foster, Haltiwanger, and Syverson make no such restriction. Their estimate of $\eta_D$ is lower for petroleum ($\hat{\eta}_D = 1.42$) and higher for ready-mix concrete ($\hat{\eta}_D = 5.93$). Again, because the normalized variances of materials shares are so small, these differences have will have only a moderate impact on the estimates of $\varepsilon_Q$.

\textsuperscript{32}One dissimilarity between the analysis of the current section and that of Section 2 to 4 concerns the industry definitions that I have used: to credibly compare the material purchases and material prices, I define products narrowly in this section. At the same time, limitations of the dataset necessitate a rather coarse industry definition in Sections 2 to 4. Going from a narrow to coarse industry classification should not systematically alter the estimates of $\eta_P$ or $\eta_D$, but will cause an increase in the estimate for, $\chi$, the within-industry variation in materials cost shares. For this reason, a coarser industry classification would, in turn, lead to a larger estimate of $\varepsilon_Q$. As it turns out, the overall estimate of $\varepsilon_Q$ is not particularly sensitive to the value of $\chi$: Doubling the value of $\chi$ increases the OLS-based estimate of $\varepsilon_Q$ from 0.61 to 0.67, and increases the fixed effects-based estimate from 0.75 to 0.80.


Boehm, Christoph, Aaron Flaaen, and Nitya Pandalai Nayar. 2014 "Complementarities in Multinational Production and Business Cycle Dynamics" mimeo.


Plants: Learning about Demand?" NBER Working Paper No. 17853.


C Details of the data from outside the U.S.

The data from other countries come from two sources. The flows of intermediate inputs, flows of goods output into final consumption expenditures, and industry-level prices are collected in the World Input Output Tables (WIOT). The data on industries’ output are compiled in the European Union KLEMS Growth and Productivity Accounts (EUKLEMS).\textsuperscript{33} The EUKLEMS data are reviewed, in detail, in Timmer et al. (2007) and O’Mahony and Timmer (2009). Durable good depreciation rates and flows of investment goods are not available for other countries. For these variables, I imputed values using data from the United States.

Of the thirty countries that are included in the EUKLEMS dataset, I restrict attention to six: Denmark, France, Italy, Japan, the Netherlands, and Spain. Many of the countries that I discarded are Eastern Bloc countries—such as Latvia, Lithuania, and Poland—for which pre-1990 data are unavailable. There are other countries, such as England, for which—for at least half of the sample period—intermediate input purchases and gross output are imputed from value added data. Data from all countries span 1970 to 2007, with the exception of Japan, whose sample begins in 1973.

The industry definitions in the EUKLEMS database differ from those in the U.S. dataset. Service industries are more finely defined. For example, F.I.R.E. is now broken out between finance and insurance on the one hand and real estate on the other. Mining and manufacturing industries are more coarse. Table 10 describes the EUKLEMS industry classification, in addition to the consumption shares of each of the 28 industries. The main takeaway from this table is that the six countries are broadly similar in their industry composition.

D Sensitivity Analysis Related to Section 3

D.1 Different Samples, Changing the Period Length and Industry Classification

In this section, I re-estimate Equation 17 using different samples. First, in Table 11, I examine whether the estimates of the production elasticities, $\varepsilon_Q$ and $\varepsilon_M$, differ according to the industry classification scheme or the period length. The main takeaway from this table is that the estimates of $\varepsilon_M$, as in the original specifications, are close to 0, for seven of the eight specifications.

\textsuperscript{33}The data can be downloaded at http://www.euklems.net/. I use, in this section, the ISIC Rev. 3 edition of the data.
<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Denmark</th>
<th>France</th>
<th>Italy</th>
<th>Japan</th>
<th>Netherlands</th>
<th>Spain</th>
<th>U.S. Ind.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Agriculture</td>
<td>0.027</td>
<td>0.029</td>
<td>0.022</td>
<td>0.010</td>
<td>0.036</td>
<td>0.039</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Mining</td>
<td>0.007</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>0.010</td>
<td>0.001</td>
<td>2, 3</td>
</tr>
<tr>
<td>3</td>
<td>Food and Tobacco</td>
<td>0.103</td>
<td>0.061</td>
<td>0.065</td>
<td>0.069</td>
<td>0.098</td>
<td>0.085</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>Textiles and Leather</td>
<td>0.018</td>
<td>0.025</td>
<td>0.069</td>
<td>0.019</td>
<td>0.020</td>
<td>0.039</td>
<td>6, 7</td>
</tr>
<tr>
<td>5</td>
<td>Wood Products</td>
<td>0.007</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
<td>0.004</td>
<td>0.002</td>
<td>8, 9</td>
</tr>
<tr>
<td>6</td>
<td>Paper and Publishing</td>
<td>0.017</td>
<td>0.014</td>
<td>0.015</td>
<td>0.002</td>
<td>0.022</td>
<td>0.015</td>
<td>10, 11</td>
</tr>
<tr>
<td>7</td>
<td>Petroleum Refining</td>
<td>0.012</td>
<td>0.011</td>
<td>0.013</td>
<td>0.010</td>
<td>0.024</td>
<td>0.016</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>Chemicals</td>
<td>0.040</td>
<td>0.047</td>
<td>0.037</td>
<td>0.016</td>
<td>0.073</td>
<td>0.038</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>Rubber and Plastics</td>
<td>0.013</td>
<td>0.009</td>
<td>0.012</td>
<td>0.005</td>
<td>0.012</td>
<td>0.008</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>Stone, Clay, and Glass</td>
<td>0.007</td>
<td>0.005</td>
<td>0.010</td>
<td>0.002</td>
<td>0.006</td>
<td>0.008</td>
<td>15</td>
</tr>
<tr>
<td>11</td>
<td>Metal products</td>
<td>0.021</td>
<td>0.018</td>
<td>0.024</td>
<td>0.015</td>
<td>0.029</td>
<td>0.020</td>
<td>16, 17</td>
</tr>
<tr>
<td>12</td>
<td>Non-electrical Machinery</td>
<td>0.047</td>
<td>0.024</td>
<td>0.048</td>
<td>0.028</td>
<td>0.026</td>
<td>0.018</td>
<td>18</td>
</tr>
<tr>
<td>13</td>
<td>Electrical Machinery</td>
<td>0.038</td>
<td>0.039</td>
<td>0.027</td>
<td>0.042</td>
<td>0.040</td>
<td>0.024</td>
<td>19, 22</td>
</tr>
<tr>
<td>14</td>
<td>Transportation Equipment</td>
<td>0.026</td>
<td>0.065</td>
<td>0.040</td>
<td>0.040</td>
<td>0.034</td>
<td>0.074</td>
<td>20, 21</td>
</tr>
<tr>
<td>15</td>
<td>Misc. Manufacturing</td>
<td>0.024</td>
<td>0.013</td>
<td>0.024</td>
<td>0.007</td>
<td>0.018</td>
<td>0.020</td>
<td>23</td>
</tr>
<tr>
<td>16</td>
<td>Utilities</td>
<td>0.021</td>
<td>0.037</td>
<td>0.016</td>
<td>0.017</td>
<td>0.017</td>
<td>0.015</td>
<td>26</td>
</tr>
<tr>
<td>17</td>
<td>Construction</td>
<td>0.010</td>
<td>0.006</td>
<td>0.008</td>
<td>0.000</td>
<td>0.006</td>
<td>0.006</td>
<td>4</td>
</tr>
<tr>
<td>18</td>
<td>Wholesale and Retail</td>
<td>0.073</td>
<td>0.078</td>
<td>0.109</td>
<td>0.137</td>
<td>0.070</td>
<td>0.075</td>
<td>27</td>
</tr>
<tr>
<td>19</td>
<td>Hotels and Restaurants</td>
<td>0.020</td>
<td>0.031</td>
<td>0.051</td>
<td>0.048</td>
<td>0.021</td>
<td>0.106</td>
<td>28</td>
</tr>
<tr>
<td>20</td>
<td>Transport and Warehousing</td>
<td>0.060</td>
<td>0.038</td>
<td>0.048</td>
<td>0.043</td>
<td>0.062</td>
<td>0.037</td>
<td>24</td>
</tr>
<tr>
<td>21</td>
<td>Communications</td>
<td>0.010</td>
<td>0.010</td>
<td>0.012</td>
<td>0.014</td>
<td>0.012</td>
<td>0.012</td>
<td>25</td>
</tr>
<tr>
<td>22</td>
<td>Finance and Insurance</td>
<td>0.021</td>
<td>0.032</td>
<td>0.026</td>
<td>0.032</td>
<td>0.032</td>
<td>0.015</td>
<td>28</td>
</tr>
<tr>
<td>23</td>
<td>Real Estate</td>
<td>0.086</td>
<td>0.100</td>
<td>0.072</td>
<td>0.127</td>
<td>0.060</td>
<td>0.069</td>
<td>28</td>
</tr>
<tr>
<td>24</td>
<td>Business Services</td>
<td>0.015</td>
<td>0.030</td>
<td>0.018</td>
<td>0.009</td>
<td>0.052</td>
<td>0.017</td>
<td>29</td>
</tr>
<tr>
<td>25</td>
<td>Government</td>
<td>0.071</td>
<td>0.096</td>
<td>0.079</td>
<td>0.105</td>
<td>0.079</td>
<td>0.076</td>
<td>30</td>
</tr>
<tr>
<td>26</td>
<td>Education</td>
<td>0.058</td>
<td>0.056</td>
<td>0.053</td>
<td>0.060</td>
<td>0.035</td>
<td>0.052</td>
<td>29</td>
</tr>
<tr>
<td>27</td>
<td>Health and Social Work</td>
<td>0.111</td>
<td>0.087</td>
<td>0.065</td>
<td>0.082</td>
<td>0.070</td>
<td>0.066</td>
<td>29</td>
</tr>
<tr>
<td>28</td>
<td>Other Personal Services</td>
<td>0.038</td>
<td>0.035</td>
<td>0.032</td>
<td>0.058</td>
<td>0.030</td>
<td>0.048</td>
<td>29</td>
</tr>
</tbody>
</table>

Table 10: Industry definitions and consumption shares in the EUKLEMS dataset.
Notes: The final column shows the correspondence between the EUKLEMS industry definitions and the industry definitions for the United States data.
Second stage regression results

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \log P_{tJ}^{in} - \Delta \log P_{tI}$</th>
<th>$\Delta \log P_{tJ} - \Delta \log P_{tJ}^{in}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.87$ $-0.83$ $-0.97$ $-0.49$ $-0.71$ $-1.25$ $-0.77$</td>
<td>$0.13$ $0.08$ $-0.62$ $-0.29$ $-0.05$ $-0.05$ $1.33$ $-0.30$</td>
</tr>
<tr>
<td></td>
<td>$(0.05)$ $(0.05)$ $(0.46)$ $(0.46)$ $(0.07)$ $(0.07)$ $(0.39)$ $(0.28)$</td>
<td>$(0.08)$ $(0.08)$ $(0.94)$ $(0.80)$ $(0.12)$ $(0.12)$ $(1.12)$ $(0.73)$</td>
</tr>
</tbody>
</table>

First stage: Dependent variable is $\Delta \log P_{tJ}^{in} - \Delta \log P_{tI}$

<table>
<thead>
<tr>
<th></th>
<th>military spending shock$_{tI}$</th>
<th>military spending shock$_{tJ}$'s suppliers</th>
<th>military spending shock$_{tJ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-1.03$ $-1.53$</td>
<td>$4.26$ $6.10$</td>
<td>$-2.87$ $-2.51$</td>
</tr>
<tr>
<td></td>
<td>$(0.48)$ $(0.54)$</td>
<td>$(1.27)$</td>
<td>$(0.55)$ $(0.58)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>military spending shock$_{tI}$</th>
<th>military spending shock$_{tJ}$'s suppliers</th>
<th>military spending shock$_{tJ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1.07$ $0.51$</td>
<td>$-0.47$ $1.70$</td>
<td>$-0.79$ $-0.27$</td>
</tr>
<tr>
<td></td>
<td>$(0.33)$ $(0.37)$</td>
<td>$(0.87)$</td>
<td>$(0.37)$ $(0.40)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>F-statistic</th>
<th>Cragg-Donald Statistic</th>
<th>Wu-Hausman test p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$11.96$ $8.80$</td>
<td>$5.35$ $6.45$</td>
<td>$0.62$ $0.69$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample</th>
<th>Coarse Industries</th>
<th>Period Length =2 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year Fixed Effects</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>$N$</td>
<td>2400</td>
<td>2400</td>
</tr>
</tbody>
</table>

Table 11: Regression results related to Equation 17.

Notes: The overall sample includes pairs of industries $J$, and, for each industry $J$, the top ten supplying industries, $I$. In the row labeled "Cragg-Donald Statistic", an "i" indicates that the test for a weak instrument is rejected at the 10 percent threshold.
Next, in Table 12, I estimate the production elasticities separately for different broad sectors. The Primary sector consists of the first three industries in Table 7. The Manufacturing sector consists of the Construction and all manufacturing industries, the fourth through twenty-third industries according to Table 7. The remaining industries are in the Services sector. Estimates of $\varepsilon_M$ are similar across sectors. Estimates of $\varepsilon_Q$, though less precisely estimated, are somewhat larger for the Primary Sector and lower for the Services sector.

As a third set of robustness checks, in Table 13, I assess whether the number of upstream industries used in the sample alters our estimates of $\varepsilon_M$ and $\varepsilon_Q$. In the benchmark regressions, in Table 1, the sample included the top ten upstream industries for each downstream industry $J$. There are no clear patterns, regarding the relationship between estimates of $\varepsilon_Q$ and $\varepsilon_M$ and the broadness of the sample.

D.2 Production Elasticities of Substitution in Other Countries

In this subsection, I report on results from other countries. I apply data from the World Input Output Tables, taking data from 1997 to 2011. The industry definitions, similar those used for the U.S. data, are given in Table 10. In Table 14, I report on regressions that relate changes in the inputs’ cost shares with changes in the prices of individual inputs and prices of the intermediate input bundles. Unfortunately, for these countries, changes in military expenditures are not a sufficiently powerful source of variation to permit an IV regression. In these tables the slope of the relationship of changes in the intermediate input cost share on $\Delta \log P_{it}^{\text{inh}} - \Delta \log P_{it}$ approximately 0.3 for France and between 0.6 and 0.8 for all other countries. In addition, the slope of the relationship of intermediate input purchases on the price of $\Delta \log P_{ij} - \Delta \log P_{tij}^{\text{inh}}$ are 0.10 for Denmark and between 0.4 and 0.8 for all other countries. While the coefficient estimates reported in 14 cannot identify $\varepsilon_Q$ or $\varepsilon_M$ on their own, they are in accordance with the OLS estimates for the United States.

E Sensitivity Analysis Related to Section 4

In Table 15, I re-estimate the correlations among shocks for different parts of the sample period. For the most part, the correlations among the $\omega$ productivity shocks are similar in the first half and the second half of the sample. (Unfortunately, since there are fewer time periods in either of the two halves of the sample than there are industries, I cannot compute the first factor of the industries’ productivity shocks to assess the contribution of common productivity shocks to aggregate volatility.) In the third column, I exclude the Great-Recession period from the sample. Here, the assessed role of industry-specific shocks
## Second stage regression results

<table>
<thead>
<tr>
<th>Stages</th>
<th>Coefficients</th>
<th>Standard Errors</th>
<th>t-values</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\Delta \log P_{tJ} - \Delta \log P_{tI}$</td>
<td>$-0.98$</td>
<td>(0.11)</td>
<td>0.378</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1.25$</td>
<td>(0.33)</td>
<td>0.129</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1.21$</td>
<td>(0.32)</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1.27$</td>
<td>(0.05)</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1.27$</td>
<td>(1.05)</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-0.72$</td>
<td>(0.08)</td>
<td>0.930</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-0.74$</td>
<td>(0.60)</td>
<td>0.550</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-0.83$</td>
<td>(0.55)</td>
<td>0.930</td>
</tr>
<tr>
<td>2.</td>
<td>$\Delta \log P_{tJ} - \Delta \log P_{tJ}^m$</td>
<td>$0.41$</td>
<td>(0.10)</td>
<td>0.364</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.42$</td>
<td>(0.36)</td>
<td>0.364</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.65$</td>
<td>(0.44)</td>
<td>0.260</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.35$</td>
<td>(0.09)</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.45$</td>
<td>(1.05)</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-1.08$</td>
<td>(0.15)</td>
<td>0.950</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-0.98$</td>
<td>(0.90)</td>
<td>0.580</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$-0.31$</td>
<td>(0.93)</td>
<td>0.930</td>
</tr>
</tbody>
</table>

### First stage: Dependent variable is $\Delta \log P_{tJ} - \Delta \log P_{tJ}^m$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>Standard Errors</th>
<th>t-values</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>military spending</td>
<td>$-2.85$</td>
<td>(2.70)</td>
<td>0.285</td>
<td></td>
</tr>
<tr>
<td>shock$_{tJ}$</td>
<td>$4.34$</td>
<td>(5.65)</td>
<td>0.759</td>
<td></td>
</tr>
<tr>
<td>military spending</td>
<td>$20.19$</td>
<td>(15.43)</td>
<td>1.313</td>
<td></td>
</tr>
<tr>
<td>shock$_{tJ}$’s suppliers</td>
<td>$-1.22$</td>
<td>(0.97)</td>
<td>1.243</td>
<td></td>
</tr>
<tr>
<td>military spending</td>
<td>$-16.87$</td>
<td>(15.43)</td>
<td>1.100</td>
<td></td>
</tr>
<tr>
<td>shock$_{tJ}$</td>
<td>$-2.69$</td>
<td>(2.43)</td>
<td>1.111</td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>$19.80$</td>
<td>0.000</td>
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</tbody>
</table>

### First stage: Dependent variable is $\Delta \log P_{tJ} - \Delta \log P_{tJ}^m$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>Standard Errors</th>
<th>t-values</th>
<th>p-values</th>
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</thead>
<tbody>
<tr>
<td>military spending</td>
<td>$8.69$</td>
<td>(2.89)</td>
<td>3.000</td>
<td></td>
</tr>
<tr>
<td>shock$_{tJ}$</td>
<td>$16.74$</td>
<td>(5.32)</td>
<td>3.100</td>
<td></td>
</tr>
<tr>
<td>military spending</td>
<td>$-4.35$</td>
<td>(14.52)</td>
<td>0.297</td>
<td></td>
</tr>
<tr>
<td>shock$_{tJ}$’s suppliers</td>
<td>$-0.32$</td>
<td>(1.29)</td>
<td>0.243</td>
<td></td>
</tr>
<tr>
<td>military spending</td>
<td>$-1.91$</td>
<td>(1.15)</td>
<td>1.674</td>
<td></td>
</tr>
<tr>
<td>shock$_{tJ}$</td>
<td>$-5.00$</td>
<td>(2.27)</td>
<td>2.222</td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>$15.04$</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 12:** Regression results related to Equation 17.

Notes: The overall sample includes pairs of industries $I-J$ that, for each industry $J$, I include $J$’s top ten supplying industries, $I$. In the row labeled "Cragg-Donald Statistic", an "$i$" indicates that the test for a weak instrument is rejected at the 10 percent threshold.
## Second stage regression results

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \log P_{tI} - \Delta \log P_{tJ}$</td>
<td>-1.20</td>
<td>(0.30)</td>
</tr>
<tr>
<td></td>
<td>-1.22</td>
<td>(0.34)</td>
</tr>
<tr>
<td></td>
<td>-0.87</td>
<td>(0.23)</td>
</tr>
<tr>
<td></td>
<td>-0.84</td>
<td>(0.24)</td>
</tr>
<tr>
<td></td>
<td>-1.50</td>
<td>(0.27)</td>
</tr>
<tr>
<td></td>
<td>-1.35</td>
<td>(0.24)</td>
</tr>
</tbody>
</table>

## First stage: Dependent variable is $\Delta \log P_{tJ}^i - \Delta \log P_{tI}$

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>military spending shock$_{tI}$</td>
<td>0.44</td>
<td>(0.66)</td>
</tr>
<tr>
<td></td>
<td>-0.16</td>
<td>(0.75)</td>
</tr>
<tr>
<td></td>
<td>-1.52</td>
<td>(0.52)</td>
</tr>
<tr>
<td></td>
<td>-2.23</td>
<td>(0.59)</td>
</tr>
<tr>
<td></td>
<td>-2.28</td>
<td>(0.46)</td>
</tr>
<tr>
<td></td>
<td>-3.51</td>
<td>(0.51)</td>
</tr>
<tr>
<td>military spending</td>
<td>6.25</td>
<td>(1.09)</td>
</tr>
<tr>
<td>shock$_{tJ}$'s suppliers</td>
<td>-5.79</td>
<td>(1.76)</td>
</tr>
<tr>
<td></td>
<td>-5.57</td>
<td>(0.89)</td>
</tr>
<tr>
<td></td>
<td>-5.90</td>
<td>(1.38)</td>
</tr>
<tr>
<td></td>
<td>-5.37</td>
<td>(0.79)</td>
</tr>
<tr>
<td></td>
<td>-3.91</td>
<td>(1.21)</td>
</tr>
<tr>
<td></td>
<td>-2.84</td>
<td></td>
</tr>
<tr>
<td>F-statistic</td>
<td>23.88</td>
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</tr>
<tr>
<td></td>
<td>7.87</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>36.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11.09</td>
<td></td>
</tr>
</tbody>
</table>

## Table 13: Regression results related to Equation 17.

Notes: The overall sample includes pairs of industries $J$, and, for each industry $J$, the top two supplying industries, $I$ in the first four columns, and the top four supplying industries in the final four columns. In the row labeled "Cragg-Donald Statistic", an $i$ indicates that the test for a weak instrument is rejected at the 10 percent threshold.

## First stage: Dependent variable is $\Delta \log P_{tJ}$

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>military spending shock$_{tI}$</td>
<td>1.90</td>
<td>(0.50)</td>
</tr>
<tr>
<td></td>
<td>1.33</td>
<td>(0.58)</td>
</tr>
<tr>
<td></td>
<td>1.72</td>
<td>(0.36)</td>
</tr>
<tr>
<td></td>
<td>1.21</td>
<td>(0.40)</td>
</tr>
<tr>
<td></td>
<td>1.71</td>
<td>(0.27)</td>
</tr>
<tr>
<td></td>
<td>1.22</td>
<td>(0.30)</td>
</tr>
<tr>
<td>military spending</td>
<td>-1.42</td>
<td>(0.83)</td>
</tr>
<tr>
<td>shock$_{tJ}$'s suppliers</td>
<td>-1.02</td>
<td>(0.58)</td>
</tr>
<tr>
<td></td>
<td>-0.82</td>
<td>(0.59)</td>
</tr>
<tr>
<td></td>
<td>-0.91</td>
<td>(0.41)</td>
</tr>
<tr>
<td></td>
<td>-0.53</td>
<td>(0.43)</td>
</tr>
<tr>
<td></td>
<td>-0.55</td>
<td>(0.27)</td>
</tr>
<tr>
<td></td>
<td>-0.12</td>
<td>(0.29)</td>
</tr>
<tr>
<td>F-statistic</td>
<td>5.45</td>
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</tr>
<tr>
<td></td>
<td>2.77</td>
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<tr>
<td></td>
<td>10.16</td>
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</tr>
<tr>
<td></td>
<td>5.24</td>
<td></td>
</tr>
<tr>
<td></td>
<td>17.98</td>
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</tr>
<tr>
<td></td>
<td>9.52</td>
<td></td>
</tr>
<tr>
<td>Cragg-Donald Statistic</td>
<td>4.58</td>
<td></td>
</tr>
<tr>
<td>Wu-Hausman test p-value</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Upstream Industries per downstream industry×year</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
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<tr>
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<td>15</td>
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<tr>
<td></td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>1856</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1856</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3680</td>
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<tr>
<td></td>
<td>6832</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6832</td>
<td></td>
</tr>
</tbody>
</table>

## Table 14: Regression results related to Equation 17.

Notes: This table contains OLS specifications, using three input-supplying industries per downstream industry. This regression does not contain fixed effects per upstream industry, downstream industry pair, corresponding to the first column of Table 1.
Table 15: Robustness checks: $R^2$ (sectoral shocks) and $\tilde{\rho}(\omega)$ for different values of $\varepsilon_M$ and $\varepsilon_D$. I could not compute $R^2$ (sectoral shocks) in the second, third, and final columns, as there are fewer time periods than there are industries in these samples. A "kf" indicates the usage of the Kalman filter, as opposed to direct applications of Equation 15, to infer the $\omega$ productivity shocks.

is somewhat larger. The final column applies biennial data.

A final set of robustness check considers the sensitivity of the main results to the de-trending procedure. In the benchmark calculations, I had linearly de-trended each industry-level observable before performing the maximum likelihood estimation procedure. In Table 16, I consider three alternate de-trending procedures: not detrending the data, a Hodrick-Prescott filter, and a linear trend with a break in the trend at 1983. These detrending procedures have almost no quantitative impact on the relative contribution of sectoral vs. common shocks to aggregate volatility. Censoring outlier observations, or not detrending at all, somewhat increases the role of sectoral shocks, particularly when $\varepsilon_D$ is large.

---

34In estimations of dynamic general equilibrium models, the choice of the de-trending procedure is potentially important; see Canova (2014).

An alternative—intuitively appealing but unfortunately infeasible—way to deal with trends would be to include both transitory and permanent shocks in the model. This would obviate the need to de-trend the data before estimation; the parameters governing the permanent and transitory shock processes would be jointly estimated in a single stage. I do not pursue this approach, mainly because of the difficulty of scaling the model by the permanent shocks. Doing so requires a clean characterization of the changes in the industry-level observable variables as functions of the permanent shocks, something that exists only for a few special cases of the model (such special cases can be found in, for example, Ngai and Pissarides 2007 and Acemoglu and Guerrerri 2008).
### Detrending Method

<table>
<thead>
<tr>
<th>Detrending Method</th>
<th>Benchmark</th>
<th>None</th>
<th>Hodrick-Prescott</th>
<th>Linear, Break in 1983</th>
<th>Linear, Censor Outliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1))</td>
<td>0.19(^{kf})</td>
<td>0.19(^{kf})</td>
<td>0.18(^{kf})</td>
<td>0.18(^{kf})</td>
<td>0.19(^{kf})</td>
</tr>
<tr>
<td>((\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1))</td>
<td>0.14(^{kf})</td>
<td>0.14(^{kf})</td>
<td>0.14(^{kf})</td>
<td>0.14(^{kf})</td>
<td>0.14(^{kf})</td>
</tr>
<tr>
<td>((\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1))</td>
<td>0.90(^{kf})</td>
<td>0.92(^{kf})</td>
<td>0.87(^{kf})</td>
<td>0.89(^{kf})</td>
<td>0.91(^{kf})</td>
</tr>
<tr>
<td>((\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1))</td>
<td>0.80</td>
<td>0.80</td>
<td>0.79</td>
<td>0.79</td>
<td>0.82</td>
</tr>
<tr>
<td>((\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1))</td>
<td>0.48</td>
<td>0.54</td>
<td>0.51</td>
<td>0.48</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Table 16: Robustness checks: \(R^2\) (sectoral shocks) and \(\hat{\rho}(\omega)\) for different values of \(\varepsilon_M\) and \(\varepsilon_D\). A \(^{nfk}\) indicates the usage of the Kalman filter, as opposed to direct applications of Equation 15, to infer the \(\omega\) productivity shocks.

### F Solution of the model filter

This section spells out the solution of the model. First, I write out the constrained maximization problem of a social planner. I take first-conditions, write out the conditions that characterize the steady state, log-linearize around the steady state, solve for the policy functions, and then the model filter.

#### F.1 First order conditions and steady-state shares

Since this economy satisfies the welfare theorems, it will suffice to solve the social planner’s problem. Begin with the Lagrangian:

\[
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log \left[ \sum_{J=1}^{N} \left( \xi_{J} \cdot \varepsilon_{C,J} \cdot C_{t,J} \right) \right] e^{\varepsilon_{D} - T} \right\} 
- \frac{\varepsilon_{LS}}{\varepsilon_{LS} + 1} \left( \sum_{J=1}^{N} L_{t,J} \right) e^{\varepsilon_{LS} + 1}
+ \sum_{J=1}^{N} P_{t,J}^{inv} X_{t,J} + (1 - \delta_K) K_{t,J} - K_{t+1,J}
+ \sum_{J=1}^{N} P_{t,J} \left[ Q_{t,J} + (1 - \delta_C) C_{t,J} - C_{t+1,J} - \sum_{I=1}^{N} \left[ M_{t,J-I} + X_{t,J-I} \right] \right].
\]

Here, \(P_{t,J}^{inv}\) is the Lagrange multiplier on a unit of capital, and \(P_{t,J}\) is the Lagrange multiplier on the good-J market-clearing condition.

I re-state the expression for \(Q_{t,J}\):

---

47
\[ Q_{t, J} = A_{t, J} \cdot \left[ (1 - \mu_{J})^{\frac{1}{Q}} \left( \frac{K_{t, J}}{\alpha_{J}} \right)^{\alpha_{J}} \left( \frac{L_{t, J} \cdot B_{t, J}}{1 - \alpha_{J}} \right)^{1-\alpha_{J}} \right]^{\frac{\varepsilon_{Q}-1}{\varepsilon_{Q}}} + (\mu_{J})^{\frac{1}{Q}} (M_{t, J})^{\frac{\varepsilon_{Q}-1}{\varepsilon_{Q}}} \].

The first-order conditions for the planner are:

\[ K_{t+1, J} = X_{t, J} + (1 - \delta_{K}) \cdot K_{t, J} \]

\[ [C_{t, J}] : \frac{P_{t-1, J}}{P_{t, J}} - \beta P_{t, J} (1 - \delta_{C_{J}}) = \beta (\xi_{J})^{\frac{1}{D}} (\delta_{C_{J}})^{\frac{\varepsilon_{D}-1}{\varepsilon_{D}}} \times \]

\[ (C_{t, J})^{-\frac{1}{D}} \left( \sum_{I=1}^{N} (\xi_{I})^{\frac{1}{D}} (\delta_{C_{I}} \cdot C_{t, I})^{\frac{\varepsilon_{D}-1}{\varepsilon_{D}}} \right)^{-1}. \] (29)

\[ [M_{t, J-1, J}] : \frac{P_{t, J}}{P_{t, J}} = (A_{t, J})^{\frac{\varepsilon_{Q}-1}{\varepsilon_{Q}}} \left( \frac{Q_{t, J} \cdot \mu_{J}}{M_{t, J}} \right)^{\frac{1}{Q}} \left( \frac{M_{t, J} \cdot \Gamma_{t, J}^{M}}{M_{t, J-1, J}} \right)^{\frac{1}{M}}. \] (30)

\[ [X_{t, J-1, J}] : P_{t, J} = P_{t, J}^{inv} \left( \frac{X_{t, J} \cdot \Gamma_{t, J}^{X}}{X_{t, J-1, J}} \right)^{\frac{1}{X}}. \] (31)

\[ [L_{t, J}] : \left( \sum_{J'=1}^{N} L_{t, J'} \right)^{\frac{1}{L_{S}}} = P_{t, J} \cdot (A_{t, J})^{\frac{\varepsilon_{Q}-1}{\varepsilon_{Q}}} B_{t, J} (Q_{t, J} (1 - \mu_{J}))^{\frac{1}{Q}} \times \]

\[ \left( \frac{K_{t, J}}{\alpha_{J}} \right)^{\alpha_{J}-1} \frac{L_{t, J} \cdot B_{t, J}}{1 - \alpha_{J}}^{\frac{\varepsilon_{Q}-1}{\varepsilon_{Q}}}. \] (32)

\[ [K_{t+1, J}] : P_{t, J}^{inv} = \beta \cdot \mathbb{E}_{t} \left[ P_{t+1, J} (Q_{t+1, J} (1 - \mu_{J}))^{\frac{1}{Q}} (A_{t+1, J})^{\frac{\varepsilon_{Q}-1}{\varepsilon_{Q}}} \right] \times \]

\[ \left( \frac{K_{t+1, J}}{\alpha_{J}} \right)^{-1} \frac{L_{t+1, J} \cdot B_{t+1, J}}{1 - \alpha_{J}}^{(1-\alpha_{J}) \frac{\varepsilon_{Q}-1}{\varepsilon_{Q}}} \]

\[ + \beta (1 - \delta_{K}) \mathbb{E}_{t} \left[ P_{t+1, J}^{inv} \right]. \]

Towards the goal of solving for the steady-state, drop. time subscripts and re-arrange. Also, employ the normalization that steady-state labor is the numeraire good (so that
\[
\left( \sum_{j'=1}^{N} L_{t,j'} \right)^{1/\varepsilon_X} = 1 :
\]

\[
\delta K K_J = X_J
\]

\[
\frac{1 - \beta (1 - \delta_{CJ})}{\beta} P_J = (\xi_J)^{\frac{1}{\varepsilon_D}} \cdot (\delta_{CJ})^{\frac{\varepsilon_{D-1}}{\varepsilon_D}} (C_J)^{-\frac{1}{\varepsilon_D}} \left( \sum_{l=1}^{N} (\xi_I)^{\frac{1}{\varepsilon_D}} (\delta_{C_I} \cdot C_I)^{\frac{\varepsilon_{D-1}}{\varepsilon_D}} \right)^{-1}
\]

\[
P_I = P_J \left( \frac{Q_J \cdot \mu_J}{M_J} \right)^{\frac{1}{\varepsilon_Q}} \left( \frac{M_J \cdot \Gamma_{II}^M}{M_{I-J}} \right)^{\frac{1}{\varepsilon_M}}
\]

\[
P_{J}^{inv} = P_I \left( \frac{X_J \cdot \Gamma_{II}^X}{X_{I-J}} \right)^{-\frac{1}{\varepsilon_X}}
\]

\[
1 = P_J (Q_J (1 - \mu_J))^\frac{1}{\varepsilon_Q} \left( \frac{K_J}{\alpha_J} \right)^{\alpha_J \frac{\varepsilon_{Q-1}}{\varepsilon_Q}} \left( \frac{L_J}{1 - \alpha_J} \right)^{(1 - \alpha_J) \frac{\varepsilon_{Q-1}}{\varepsilon_Q}}
\]

\[
Q_J = \left[ (1 - \mu_J)^\frac{1}{\varepsilon_Q} \left( \frac{K_J}{\alpha_J} \right)^{\alpha_J \frac{\varepsilon_{Q-1}}{\varepsilon_Q}} \left( \frac{L_J}{1 - \alpha_J} \right)^{(1 - \alpha_J) \frac{\varepsilon_{Q-1}}{\varepsilon_Q}} + (\mu_J)^\frac{1}{\varepsilon_Q} (M_J)^{\frac{\varepsilon_{Q-1}}{\varepsilon_Q}} \right]^{\frac{1}{\varepsilon_{Q-1}}}
\]

(35)

First, I will solve for the prices of each industry’s good, in the steady state, \( P_J \). This will follow from each industry’s cost-minimization conditions.

Take the cost-minimization condition for capital, which equates the rental price of a unit of capital to the marginal revenue product of capital:

\[
\frac{1 - \beta (1 - \delta_{K})}{\beta} \left[ \sum \Gamma_{I,J}^X (P_I)^{1-\varepsilon_X} \right]^{1/(1-\varepsilon_X)} = P_J (Q_J (1 - \mu_J))^\frac{1}{\varepsilon_Q} \left( \frac{K_J}{\alpha_J} \right)^{\alpha_J \frac{\varepsilon_{Q-1}}{\varepsilon_Q}} \left( \frac{L_J}{1 - \alpha_J} \right)^{(1 - \alpha_J) \frac{\varepsilon_{Q-1}}{\varepsilon_Q}}
\]

(36)

Second, take cost-minimizing condition for industry \( J \)’s intermediate input purchases:

\[
(\mu_J)^{\frac{1}{\varepsilon_Q}} (M_J)^{\frac{\varepsilon_{Q-1}}{\varepsilon_Q}} = \mu_J (Q_J)^{\frac{\varepsilon_{Q-1}}{\varepsilon_Q}} \left( \frac{P_{J}^{inv}}{P_J} \right)^{1-\varepsilon_Q}
\]

(37)

And, third, the following equation takes the cost-minimizing choice of the capital-labor aggregate.

\[
(1 - \mu_J)^\frac{1}{\varepsilon_Q} \left( \frac{K_J}{\alpha_J} \right)^{\alpha_J \frac{\varepsilon_{Q-1}}{\varepsilon_Q}} \left( \frac{L_J}{1 - \alpha_J} \right)^{(1 - \alpha_J) \frac{\varepsilon_{Q-1}}{\varepsilon_Q}} = (1 - \mu_J) (Q_J)^{\frac{\varepsilon_{Q-1}}{\varepsilon_Q}} \times \left( \frac{1 - \beta (1 - \delta_{K})}{\beta} \right)^{\alpha_J \frac{\varepsilon_{Q-1}}{\varepsilon_Q}} \left[ \sum \Gamma_{I,J}^X (P_I)^{1-\varepsilon_X} \right]^{\alpha_J \frac{1}{1-\varepsilon_X}} \left( \frac{P_{J}^{inv}}{P_J} \right)^{1-\varepsilon_Q}
\]

(38)
Plug 36-38 into Equation 35.

\[(P_J)^{1-\varepsilon_Q} = (1 - \mu_J) \left( \beta^{-1} - (1 - \delta_K) \right)^{\alpha_J(1-\varepsilon_Q)} \left[ \sum_I \Gamma_{IJ}^X (P_I)^{1-\varepsilon_X} \right]^{\alpha_J(1-\varepsilon_Q)} (1-\varepsilon_X)^{(1-\varepsilon_Q)} \] (39)

\[+ \mu_J \left[ \sum_I \Gamma_{IJ}^M (P_I)^{1-\varepsilon_M} \right]^{1-\varepsilon_Q} \]

Equation 39 describes and \(N \times N\) system of equations for the \(N\) steady-state price levels. This completes the first part of the characterization of the steady state.

For the second part, consider the market clearing condition for good \(J\):

\[Q_J = \delta_{CJ} C_J + \sum_{I=1}^{N} M_{J-I} + X_{J-I} \] (40)

Below, I will write out the terms on the right-hand-side of Equation 40 in terms of the steady state prices (which have just been solved for):

First, write out the consumption of good \(J\).

\[\left(1 - \frac{(1 - \delta_{CJ})}{\beta} \right) P_J = (\xi_J)^{\frac{\varepsilon_D}{\varepsilon_D}} \cdot (\delta_{CJ})^{\frac{\varepsilon_D-1}{\varepsilon_D}} \cdot (C_J)^{-\frac{1}{\varepsilon_D}} \left( \sum_{I=1}^{N} (\xi_I)^{\frac{1}{\varepsilon_D}} (\delta_{CI} \cdot C_I)^{\frac{\varepsilon_D-1}{\varepsilon_D}} \right)^{-1} \]

\[\delta_{CJ} C_J = \xi_J (\delta_{CJ})^{\varepsilon_D} \left( \frac{1 - \beta (1 - \delta_{CJ})}{\beta} \right)^{-\varepsilon_D} \cdot (P_J)^{-\varepsilon_D} \cdot \bar{C}^{1-\varepsilon_D} \] (41)

Then write out the intermediate input purchases from industry \(J\) to industry \(I\)

\[M_{J-I} = (Q_I \mu_I)^{\varepsilon_M} \cdot (M_I)^{\varepsilon_Q-\varepsilon_M} \cdot \Gamma_{JI}^M \cdot \left( \frac{P_I}{P_J} \right)^{\varepsilon_M} \]

\[= Q_I \mu_I \Gamma_{JI}^M (P_J)^{-\varepsilon_M} \left( P_I^{\varepsilon_M-\varepsilon_Q} \cdot (P_I)^{\varepsilon_Q} \right) \]

\[= Q_I \mu_I \Gamma_{JI}^M (P_J)^{-\varepsilon_M} \left( \sum_{J'} \Gamma_{J'JI}^M (P_{J'})^{1-\varepsilon_M} \right)^{\varepsilon_M-\varepsilon_Q} \cdot (P_J)^{\varepsilon_Q} \] (42)

And, finally, write out the investment input purchases from industry \(J\) sold to industry \(I\). Begin by writing out the total investment purchases of industry \(J\).

\[\left( \frac{K_J}{\alpha_J} \right) = \left( \frac{1 - \beta (1 - \delta_K)}{\beta} \right) \left[ \sum_{I} \Gamma_{IJ}^X (P_I)^{1-\varepsilon_X} \right]^{\frac{1}{1-\varepsilon_X}} \cdot (1 - \mu_J) Q_J (P_J)^{\varepsilon_Q} \]

\[X_J = (1 - \mu_J) Q_J \alpha_J \delta_K \left( \frac{1 - \beta (1 - \delta_K)}{\beta} \right) \left[ \sum_{I} \Gamma_{IJ}^X (P_I)^{1-\varepsilon_X} \right]^{\frac{1}{1-\varepsilon_X}} \cdot (1 - \mu_J) Q_J (P_J)^{\varepsilon_Q} \]
So:

\[ X_{J\to I} = X_I \cdot \Gamma_{JI}^X \cdot \left( \frac{P_J}{P_{J_{inv}}} \right)^{-\varepsilon_X} \]

\[ = X_I \cdot \Gamma_{JI}^X \cdot (P_J)^{-\varepsilon_X} (P_I^{inv})^{\varepsilon_X} \]

\[ = Q_I (1 - \mu_I) \alpha_I \delta_K \left( \frac{1 - \beta (1 - \delta_K)}{\beta} \right)^{-1+\alpha_I (1-\varepsilon_Q)} \Gamma_{JI}^X \times \]

\[ \left[ \sum_{J'} \Gamma_{J'JI} (P_{J'})^{1-\varepsilon_X} \right]^{\varepsilon_X-1+\alpha_J (1-\varepsilon_Q)} (P_J)^{-\varepsilon_X} (P_I)^{\varepsilon_Q} \]  \hspace{1cm} (43)

Plug in the expressions (Equations 41-43) into the market clearing condition (Equation 40):

\[ Q_J - \sum_{I=1}^{N} \tilde{\Gamma}_{JI} Q_I = \xi_J (\delta_{C_J})^{\varepsilon_D} \left( \frac{1 - \beta (1 - \delta_{C_J})}{\beta} \right)^{-\varepsilon_D} (P_J)^{-\varepsilon_D} \tilde{C}^{1-\varepsilon_D} \]

where

\[ \tilde{\Gamma}_{JI} = (P_I)^{\varepsilon_Q} \times \left\{ \mu_I \Gamma_{JI}^M \left[ \sum_{J'} \Gamma_{J'JI}^M (P_{J'})^{1-\varepsilon_M} \right]^{\varepsilon_M-\varepsilon_Q} \right\}^{\varepsilon_M} (P_J)^{-\varepsilon_M} \]

\[ + (1 - \mu_I) \alpha_I \delta_K \left( \frac{1 - \beta (1 - \delta_K)}{\beta} \right)^{-1+\alpha_I (1-\varepsilon_Q)} \Gamma_{JI}^X \left[ \sum_{J'} \Gamma_{J'JI}^X (P_{J'})^{1-\varepsilon_X} \right]^{-1+\alpha_J (1-\varepsilon_X)} (P_J)^{-\varepsilon_X} \left\} \]

We can solve for the \( Q \) vector using linear algebra. From here, we can solve for the
steady state shares:

\[
L_J = Q_J (1 - \alpha_J) (1 - \mu_J) (P_J)^{\varepsilon_Q} \left( \frac{1 - \beta (1 - \delta_K) P_{inv}^J}{\beta} \right)^{\alpha_J (1 - \varepsilon_Q)} 
\]

\[
C_J = \xi_J \delta_{C_J}^\varepsilon \bar{C}^{1-\varepsilon} \left( \frac{1 - \beta (1 - \delta_{C_J})}{\beta} \right)^{-\varepsilon_{C_J}} (P_J)^{-\varepsilon_{C_J}}
\]

\[
S^C_I = \frac{\left( \xi_I \right)^{\varepsilon_{D_I}} \left( \delta_{C_I} \cdot C_I \right)^{\varepsilon_{D-I}}}{\sum \left( \xi_{I'} \right)^{\varepsilon_{D-I'}} \left( \delta_{C_{I'}} \cdot C_{I'} \right)^{\varepsilon_{D-I'}}} 
\]

\[
M_{J-I} = (Q_J)^{-1} Q_I \mu_I \Gamma_{J-I}^M \left( P_{inv}^{J-I} \right)^{\varepsilon_{M-I}} \left( P_J \right)^{-\varepsilon_{M-J}} \left( P_I \right)^{\varepsilon_{Q-I}}
\]

\[
X_{J-I} = (Q_J)^{-1} Q_I \left( 1 - \mu_I \right) \alpha_I \delta_K \left( \frac{1 - \beta (1 - \delta_K)}{\beta} \right)^{-1+\alpha_I (1 - \varepsilon_Q)} \Gamma_{J-I}^X 
\]

\[
\left[ S^1_X \right]_{IJ} = \Gamma_{IJ}^X \left( \frac{P_{inv}^J}{P_I} \right)^{1-\varepsilon_X} 
\]

\[
\left[ S^1_M \right]_{IJ} = \Gamma_{IJ}^M \left( \frac{P_{inv}^J}{P_I} \right)^{1-\varepsilon_M} \]

Clearly, these equations depend on \( Q_J \) and the steady-state prices. Note that, however, these figures have already been solved for. For future reference, define \( \bar{S}_M^Q \) as the matrix that has, in its \( J \), \( I \) entry, the fraction of good \( J \) that is sold to industry \( I \) as an intermediate input: \( \bar{S}_M^Q \equiv \frac{M_{I-J}}{Q_J} \). Similarly, define \( \bar{S}_X^Q \equiv \frac{X_{J-I}}{Q_J} \). Equation 44 characterizes the share of labor that is employed in industry \( J \), in the steady state. Use \( \bar{S}^L \) as the \( N \times N \) matrix that has, in its \( J \)th column, this steady-state share. Also for future reference, define \( \bar{S}_I^C \) as the matrix that has \( S_J^C \) (as given in Equation 45) in its \( I \)th column. And, finally, use \( \bar{S}_C^Q \) to denote the share of good \( J \) that is consumed (which can be computed by subtracting \( \bar{S}_M^Q \) and \( \bar{S}_M^Q \) from 1.)

Log linearization

The log linearization of the first order conditions are rather straightforward to derive. Below, I will derive Equations 48 and 49. In all of these equations a lower-case letter with
the circumflex (^) denotes log-deviation from the steady state.

\[ \hat{x}_{t+1,J} = \delta_K^{-1} \hat{k}_{t+1,J} + (1 - \delta_K^{-1}) \hat{k}_{t,J} \]

\[ \hat{q}_{t,J} = \delta_{C,J}^{-1} S_{C,J}^{Q} \hat{c}_{t+1,J} + (1 - \delta_{C,J}^{-1}) S_{C,J}^{Q} \hat{c}_{t,J} \]

\[ \frac{1}{1 - \beta (1 - \delta_{C,J})} \hat{p}_{t,J} - \beta \frac{(1 - \delta_{C,J})}{1 - \beta (1 - \delta_{C,J})} \hat{p}_{t+1,J} \approx \frac{1}{\varepsilon_D} \frac{1}{\varepsilon_D} \left( \frac{\xi_I}{\xi_D} \right) \left( \frac{\delta_{C,J}}{\varepsilon_D} \right) \left( \frac{\varepsilon_D - 1}{\varepsilon_D} \right) \hat{c}_{t+1,J} \]

\[ \hat{p}_{t,J} = \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_{t,J} + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{q}_{t,J} + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{m}_{t,J} \]

\[ \hat{p}_{t,J} = \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_{t,J} + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{q}_{t,J} + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{m}_{t,J} \]

To derive Equation 48, take the market clearing condition for good J,

\[ \log [\exp \hat{q}_{t,J}] = \log \left[ - (1 - \delta_{C,J}) S_{C,J}^{Q} \exp \hat{c}_{t,J} + S_{C,J}^{Q} \exp \hat{c}_{t+1,J} + \sum_{l=1}^{N} S_{M,J-l}^{Q} \exp \hat{m}_{t,J-l} + S_{X,J-l}^{Q} \exp \hat{x}_{t,J-l} \right] \]

\[ \approx S_{C,J}^{Q} \delta_{C,J}^{-1} \hat{c}_{t+1,J} + S_{C,J}^{Q} (1 - \delta_{C,J}^{-1}) \hat{c}_{t,J} + \sum_{l=1}^{N} S_{M,J-l}^{Q} \hat{m}_{t,J-l} + S_{X,J-l}^{Q} \hat{x}_{t,J-l} \]
The following set of calculations yield Equation 49:

\[
P_J \left[ \frac{1}{\beta} P_{t-1,J} - \frac{P_J}{P_J} (1 - \delta_{C,J}) \right] = (\delta_{C,J})^{\frac{\varepsilon_D-1}{\varepsilon_D}} (\delta_{C,J} C_J)^{-\frac{1}{\varepsilon_D}} (\exp \{ \hat{c}_{t,J} \})^{-\frac{1}{\varepsilon_D}} \times 
\left( \sum_{l=1}^{N} (\xi_l)^{\frac{1}{\varepsilon_D}} (\delta_{C,l} C_{tt})^{\frac{\varepsilon_D-1}{\varepsilon_D}} \right)^{-1}
\]

\[
\frac{1}{1 - \beta (1 - \delta_{C,J})} \exp \hat{p}_{t-1,J} - \frac{\beta (1 - \delta_{C,J})}{1 - \beta (1 - \delta_{C,J})} \exp \hat{p}_{t,J} = (\exp \hat{c}_{t,J})^{-\frac{1}{\varepsilon_D}} \times 
\left( \frac{1}{1 - \beta (1 - \delta_{C,J})} \hat{p}_{t-1,J} - \frac{\beta (1 - \delta_{C,J})}{1 - \beta (1 - \delta_{C,J})} \hat{p}_{t,J} \approx -\frac{1}{\varepsilon_D} \hat{c}_{t,J}
\right)
\]

Write the log-linearized equations, as given in the beginning of the subsection, in matrix form.

\[
\hat{k}_{t+1} = \delta_K \hat{X}_t + (1 - \delta_K) \hat{k}_t
\]

\[
\hat{q}_t = \delta C^{-1} S^Q \hat{c}_{t+1} + (I - \delta C^{-1}) S^Q \hat{c}_t + S^Q \hat{x}_t + S^Q \hat{m}_t
\]

\[
\hat{p}_t = \beta (I - \delta C) \hat{p}_{t+1} - \frac{1}{\varepsilon_D} (I - \beta (I - \delta C)) \left[ I + S^Q (\varepsilon_D - 1) \right] \hat{c}_{t+1}
\]

\[
\hat{m}_t = \frac{\varepsilon_M}{\varepsilon_Q} (1 - \varepsilon_M) T_1 \hat{a}_t + \frac{\varepsilon_M}{\varepsilon_Q} T_1 \hat{q}_t + \left( 1 - \frac{\varepsilon_M}{\varepsilon_Q} \right) T_1 \hat{M}_t + \varepsilon_M \left[ T_1 - T_2 \right] \hat{p}_t
\]

\[
\hat{x}_t = T_1 \hat{X}_t + \varepsilon X T_1 \hat{p}^{inv} - \varepsilon X T_2 \hat{p}_t
\]

\[
\frac{1}{\varepsilon_{LS}} S^L \hat{t}_t = \hat{p}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_t + \frac{(\varepsilon_Q - 1) (I - \alpha)}{\varepsilon_Q} \hat{b}_t + \frac{1}{\varepsilon_Q} \hat{q}_t + \frac{\varepsilon_Q}{\varepsilon_Q} \frac{1}{\varepsilon_Q} \frac{\alpha \hat{c}_t + \alpha - I - \alpha \varepsilon_Q \hat{t}_t}{\varepsilon_Q}
\]

\[
\hat{p}^{inv}_t = \beta (1 - \delta_K) \hat{p}^{inv}_{t+1} + (1 - \beta (1 - \delta_K)) \left[ \hat{p}_{t+1} + \frac{1}{\varepsilon_Q} \hat{q}_{t+1} + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_{t+1} \right]
\]

\[
+ \left( -I + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \right) \hat{k}_{t+1} + (I - \alpha) \frac{\varepsilon_Q - 1}{\varepsilon_Q} \left( \hat{t}_{t+1} + \hat{b}_{t+1} \right)
\]

\[
\hat{q}_t = \hat{a}_t + (I - \alpha) (I - S_M) \hat{b}_t + \alpha (I - S_M) \hat{k}_t + (I - \alpha) (I - S_M) \hat{t}_t + S_M \hat{M}_t
\]

In these equations \( T_1 \) refers to the \( N^2 \times N \) matrix equal to \( 1 \otimes I \), where \( 1 \) is an \( N \times 1 \) vector of 1s and \( \otimes \) is the Kronecker product. Similarly, \( T_2 \) equals \( I \otimes 1_N \). Also, \( S_M \) is a diagonal matrix with the steady-state intermediate cost shares along the diagonal; \( \delta_C \) is a matrix with \( \delta_{C,J} \) along the diagonal; and \( \alpha \) is a diagonal matrix with the \( \alpha_{J,J} \) along the diagonal. Finally, \( \hat{M}_t \) and \( \hat{X}_t \) are the \( N \times 1 \) vectors which contain the intermediate input.
bundles and investment input bundles employed by each industry, whereas \( \hat{m}_t \) and \( \hat{x}_t \) refer to the \( N^2 \times 1 \) vectors which contain the flows of intermediate and investment inputs across pairs of industries.

**F.2 System reduction**

**Step 1:** Substitute out \( \hat{x}_t \) and \( \hat{m}_t \):

\[
\hat{m}_t = \frac{\varepsilon_M}{\varepsilon_Q} (\varepsilon_Q - 1) T_1 \hat{a}_t + \frac{\varepsilon_M}{\varepsilon_Q} T_1 \hat{q}_t + \left( 1 - \frac{\varepsilon_M}{\varepsilon_Q} \right) T_1 \hat{M}_t + \varepsilon_M [T_1 - T_2] \hat{p}_t
\]

\[
\hat{x}_t = T_1 \hat{X}_t + \varepsilon_T T_1 \hat{p}^{inv}_{t} - \varepsilon_T T_2 \hat{p}_t
\]

to get:

\[
\hat{k}_{t+1} = \delta_K \hat{X}_t + (1 - \delta_K) \hat{k}_t
\]

\[
\left( I - \frac{\varepsilon_M}{\varepsilon_Q} \bar{S}^Q T_1 \right) \hat{q}_t = \delta_C^{-1} \bar{S}^Q \hat{c}_{t+1} + (I - \delta_C^{-1}) \bar{S}^Q \hat{c}_t + \bar{S}^Q T_1 \hat{X}_t + \frac{\varepsilon_M}{\varepsilon_Q} (\varepsilon_Q - 1) \bar{S}^Q T_1 \hat{a}_t
\]

\[
+ \left( 1 - \frac{\varepsilon_M}{\varepsilon_Q} \right) \bar{S}^Q T_1 \hat{M}_t + \varepsilon_T \bar{S}^Q T_1 \hat{p}^{inv}_t + \left[ \varepsilon_M \bar{S}^Q [T_1 - T_2] - \varepsilon_T \bar{S}^Q T_2 \right] \hat{p}_t
\]

\[
\hat{p}_t = \beta (I - \delta_C) \hat{p}_{t+1} - \frac{1}{\varepsilon_D} (I - \beta (I - \delta_C)) \left[ I + S^C_I (\varepsilon_D - 1) \right] \hat{c}_{t+1}
\]

\[
\frac{1}{\varepsilon_{LS}} \hat{l}_t = \hat{p}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_t + \frac{(\varepsilon_Q - 1)(I - \alpha)}{\varepsilon_Q} \hat{b}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \alpha \hat{k}_t + \frac{\alpha - I - \alpha \varepsilon_Q}{\varepsilon_Q} \hat{l}_t
\]

\[
\hat{p}^{inv}_t = \beta(1 - \delta_K) \hat{p}^{inv}_{t+1} + (1 - \beta(1 - \delta_K)) \left[ \hat{p}_{t+1} + \frac{1}{\varepsilon_Q} \hat{q}_{t+1} + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_{t+1} \right]
\]

\[
+ \left( -I + \alpha \frac{\varepsilon_Q - 1}{\varepsilon_Q} \right) \hat{k}_{t+1} + (I - \alpha) \frac{\varepsilon_Q - 1}{\varepsilon_Q} \left( \hat{l}_{t+1} + \hat{b}_{t+1} \right)
\]

\[
\hat{q}_t = \hat{a}_t + (I - \alpha)(I - S_M) \hat{b}_t + \alpha (I - S_M) \hat{k}_t + (I - \alpha)(I - S_M) \hat{l}_t + S_M \hat{M}_t
\]

**Step 2:** Use \( S^X_I \hat{p}_t = \hat{p}^{inv}_t \) (\( S^X_I \) is the matrix that gives the share of different industries’ outputs in the investment input bundle) and \( \hat{X}_t = \delta_K^{-1} \hat{K}_{t+1} + (1 - \delta_K^{-1}) \hat{K}_t \); and define \( \beta \equiv
\[ 1 - \beta (1 - \delta_K). \]

\[
\left( I - \frac{\varepsilon_M}{\varepsilon_Q} S_M^Q T_1 \right) \hat{q}_t = \delta_C^{-1} S_C^Q \hat{c}_{t+1} + \left( I - \delta_C^{-1} \right) S_C^Q \hat{c}_t + \left( \varepsilon_Q - 1 \right) S_M^Q T_1 \hat{a}_t + \left( 1 - \frac{\varepsilon_M}{\varepsilon_Q} \right) S_M^Q T_1 \hat{M}_t
\]

\[
+ \left[ \varepsilon_X S_X^Q T_1 S_X^X + \varepsilon_M S_M^Q \left[ T_1 - T_2 \right] - \varepsilon_X S_X^Q T_2 \right] \hat{p}_t + S_X^Q T_1 \delta_K^{-1} \hat{k}_{t+1} + S_X^Q T_1 \left( 1 - \delta_K^{-1} \right) \hat{k}_t
\]

\[
\hat{p}_t = \beta \left( I - \delta_C \right) \hat{p}_{t+1} - \frac{1}{\varepsilon_D} \left( I - \beta \left( I - \delta_C \right) \right) \left( \hat{p}_t + \varepsilon_Q \hat{q}_t + \varepsilon_Q \right) \frac{1}{\alpha \varepsilon_Q} \hat{k}_t + \frac{\alpha - I - \alpha \varepsilon_Q}{\varepsilon_Q} \hat{l}_t
\]

\[
\frac{1}{\varepsilon_{LS}} S_{1L}^L \hat{l}_t = \hat{p}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{b}_t + \frac{\varepsilon_Q}{\varepsilon_Q} \frac{1}{\alpha \varepsilon_Q} \hat{k}_t + \frac{\alpha - I - \alpha \varepsilon_Q}{\varepsilon_Q} \hat{l}_t
\]

\[
S_1^X \hat{p}_t = \left[ \beta I + \beta (1 - \delta_K) S_1^X \right] \hat{p}_{t+1} + \frac{\beta}{\varepsilon_Q} \hat{q}_{t+1}
\]

\[
+ \beta \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_{t+1} + \beta \left( -I + \frac{\alpha \varepsilon_Q - 1}{\varepsilon_Q} \right) \hat{k}_{t+1} + \beta \left( I - \alpha \right) \frac{\varepsilon_Q - 1}{\varepsilon_Q} \left( \hat{l}_{t+1} + \hat{b}_{t+1} \right)
\]

\[
\hat{q}_t = \left( I - S_M \right)^{-1} \left( I + S_M \left( \varepsilon_Q - 1 \right) \right) \hat{a}_t + \left( I - \alpha \right) \hat{b}_t + \alpha \hat{k}_t
\]

\[\text{Step 3: Use} \]

\[\hat{M}_t = \left( \varepsilon_Q - 1 \right) \hat{a}_t + \hat{q}_t + \varepsilon_Q \left( I - S_M^T \right) \hat{p}_t\]

\[\text{where } S_M^T \hat{p}_t = \hat{p}_t^{in}\]

\[
\left( I - S_M^Q T_1 \right) \hat{q}_t = \delta_C^{-1} S_C^Q \hat{c}_{t+1} + \left( I - \delta_C^{-1} \right) S_C^Q \hat{c}_t + \left( \varepsilon_Q - 1 \right) S_M^Q T_1 \hat{a}_t
\]

\[+ S_X^Q T_1 \delta_K^{-1} \hat{k}_{t+1} + S_X^Q T_1 \left( 1 - \delta_K^{-1} \right) \hat{k}_t
\]

\[+ \left[ \varepsilon_X S_X^Q T_1 \left( I - S_M^T \right) + \varepsilon_M S_M^Q \left[ T_1 S_M^T - T_2 \right] + \varepsilon_X S_X^Q \left[ T_1 S_X^T - T_2 \right] \right] \hat{p}_t
\]

\[\hat{p}_t = \beta \left( I - \delta_C \right) \hat{p}_{t+1} - \frac{1}{\varepsilon_D} \left( I - \beta \left( I - \delta_C \right) \right) \left[ I + S_M^C \left( \varepsilon_D - 1 \right) \right] \hat{a}_{t+1}
\]

\[
\frac{1}{\varepsilon_{LS}} S_{1L}^L \hat{l}_t = \hat{p}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{b}_t + \frac{\varepsilon_Q}{\varepsilon_Q} \frac{1}{\alpha \varepsilon_Q} \hat{k}_t + \frac{\alpha - I - \alpha \varepsilon_Q}{\varepsilon_Q} \hat{l}_t
\]

\[\text{Step 4: Use the production function, given in Equation 51, to substitute } \hat{q}_t \text{ out of the first, third, and fourth equations:}
\]

\[56\]
\[
\frac{1}{\varepsilon_Q} \dot{q}_t = \frac{1}{\varepsilon_Q} (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) \dot{a}_t + \frac{1}{\varepsilon_Q} (I - \alpha) \dot{b}_t + \frac{1}{\varepsilon_Q} \alpha \dot{k}_t
\]
\[
+ \frac{1}{\varepsilon_Q} (I - \alpha) \dot{i}_t + (I - S_M)^{-1} S_M (I - S^M_1) \dot{p}_t
\]
\[
(I - \tilde{S}^Q_M T_1) \dot{q}_t = (I - \tilde{S}^Q_M T_1) (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) \dot{a}_t + (I - \tilde{S}^Q_M T_1) (I - \alpha) \dot{b}_t
\]
\[
+ (I - \tilde{S}^Q_M T_1) \alpha \dot{k}_t + (I - \tilde{S}^Q_M T_1) (I - \alpha) \dot{i}_t
\]
\[
+ (I - \tilde{S}^Q_M T_1) (I - S_M)^{-1} S_M (I - S^M_1) \varepsilon_Q \dot{p}_t
\]
to get
\[
0 = \delta_C^{-1} \tilde{S}^Q_C \dot{c}_{t+1} + (I - \delta_C^{-1}) \tilde{S}^Q_C \dot{c}_t
\]
\[
+ [\varepsilon_Q - 1] \tilde{S}^Q_M T_1 - (I - \tilde{S}^Q_M T_1) (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) \dot{a}_t
\]
\[
+ S^Q_M T_1 \delta_K^{-1} \dot{k}_{t+1} + \left[ \tilde{S}^Q_M T_1 (1 - \delta_K^{-1}) - (I - \tilde{S}^Q_M T_1) \alpha \right] \dot{k}_t
\]
\[
- (I - \tilde{S}^Q_M T_1) (I - \alpha) \dot{b}_t - (I - \tilde{S}^Q_M T_1) (I - \alpha) \dot{i}_t
\]
\[
+ [\varepsilon_X \tilde{S}^Q_M T_1 S^X_1 - T_2] - (I - \tilde{S}^Q_M T_1) \varepsilon_Q (I - S_M)^{-1} S_M (I - S^M_1) \dot{p}_t
\]
\[
+ \left[ \varepsilon_Q \tilde{S}^Q_M T_1 (I - S^M_1) + \varepsilon_M \tilde{S}^Q_M [T_1 S^M_1 - T_2] \right] \dot{p}_t
\]
\[
\dot{p}_t = \beta (I - \delta_C) \dot{p}_{t+1} - \frac{1}{\varepsilon_D} (I - \beta (I - \delta_C)) \left[ I + S^C_D (\varepsilon_D - 1) \right] \dot{c}_{t+1}
\]
\[
S^X_1 \dot{p}_t = \left[ \tilde{\beta} I + \beta (1 - \delta_K) S^X_1 + \tilde{\beta} (I - S_M)^{-1} S_M (I - S^M_1) \right] \dot{p}_{t+1}
\]
\[
+ \tilde{\beta} \left[ \frac{1}{\varepsilon_Q} (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) + \frac{\varepsilon_Q - 1}{\varepsilon_Q} I \right] \dot{a}_{t+1}
\]
\[
+ \tilde{\beta} (I - \alpha) \dot{b}_{t+1} + \tilde{\beta} (1 + \alpha) \dot{k}_{t+1} + \tilde{\beta} (I - \alpha) \dot{i}_{t+1}
\]

**Step 5:** Use the following equation:
\[
(I - \alpha) \dot{i}_t = \vartheta (I - \alpha) \dot{b}_t + \vartheta \alpha \dot{k}_t + \vartheta \left[ \frac{\varepsilon_Q - 1}{\varepsilon_Q} I + \frac{1}{\varepsilon_Q} (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) \right] \dot{a}_t
\]
\[
+ \vartheta \left[ (I - S_M)^{-1} S_M (I - S^M_1) + I \right] \dot{p}_t
\]
where \( \vartheta = (I - \alpha) \left[ \frac{1}{\varepsilon_{LS}} S^L + \alpha \right]^{-1} \)
(this equation comes from plugging Equation 51 into Equation 50 and re-arranging) to get

\[ 0 = \delta_C^{-1} \tilde{S}^Q_{q} \hat{c}_{t+1} + (I - \delta_C^{-1}) \tilde{S}^Q_{q} \hat{c}_t + \left[ (\varepsilon_Q - 1) \tilde{S}^Q_{S} T_1 - \left( I - \tilde{S}^Q_{S} T_1 \right) (I + \vartheta \varepsilon_Q^{-1}) (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) \right] \hat{a}_t \]

\[ - \left( I - \tilde{S}^Q_{S} T_1 \right) \vartheta \varepsilon_Q^{-1} \frac{1}{\varepsilon_Q} \hat{a}_t + \tilde{S}^Q_{X} T_1 \delta^K_{K} \hat{k}_{t+1} \]

\[ + \left[ \tilde{S}^Q_{X} T_1 (1 - \delta^K_{K}) - \left( I - \tilde{S}^Q_{S} T_1 \right) (I + \vartheta) \alpha \right] \hat{k}_t - \left( I - \tilde{S}^Q_{S} T_1 \right) (I + \vartheta) (I - \alpha) \hat{b}_t \]

\[ + [\varepsilon_Q \tilde{S}^Q_{S} T_1 (I - S^M) + \varepsilon_M S^Q_{S} [T_1 S^M - T_2] + \varepsilon_X \tilde{S}^Q_{X} [T^M_1 X - T_2] \right] \hat{p}_t \]

\[ + \left[ - \left( I - \tilde{S}^Q_{S} T_1 \right) \left[ \varepsilon_Q (I - S_M)^{-1} S_M (I - S^M) + \vartheta [(I - S_M)^{-1} S_M (I - S^M) + \tilde{I}] \right] \right] \hat{p}_t \]

\[ \hat{p}_t = \beta (I - \delta_C) \hat{p}_{t+1} - \frac{1}{\varepsilon_D} (I - \beta (I - \delta_C)) [I + S^C_{I} (\varepsilon_D - 1)] \hat{c}_{t+1} \]

\[ S^X_{I} \hat{p}_t = \left[ \beta (1 - \delta_K) S^X_{I} + \tilde{\beta} (I + \vartheta) (I + (I - S_M)^{-1} S_M (I - S^M)) \right] \hat{p}_{t+1} \]

\[ + \tilde{\beta} (I + \vartheta) \left[ \frac{1}{\varepsilon_Q} (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) + \varepsilon_Q^{-1} \frac{1}{\varepsilon_Q} \tilde{I} \right] \hat{a}_{t+1} + \tilde{\beta} (I + \vartheta) (I - \alpha) \hat{b}_{t+1} \]

\[ + \tilde{\beta} (-I + \alpha + \vartheta \alpha) \hat{k}_{t+1} \]

So now we are down to three equations and three sets of endogenous unknowns (\( \hat{p}_t, \hat{k}_t, \) and \( \hat{c}_t \)). How we proceed will depend on whether we allow for consumption to be durable or not.

**Case 1: No Durables**

Plug

\[ \hat{c}_t = -\varepsilon_D [I + S^C_{I} (\varepsilon_D - 1)]^{-1} \hat{p}_t \]

in to the other two equations, above, to substitute out the \( \hat{c}_t \) vector.

\[ 0 = \left[ (\varepsilon_Q - 1) \tilde{S}^Q_{S} T_1 - \left( I - \tilde{S}^Q_{S} T_1 \right) (I + \vartheta \varepsilon_Q^{-1}) (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) - \left( I - \tilde{S}^Q_{S} T_1 \right) \vartheta \varepsilon_Q^{-1} \frac{1}{\varepsilon_Q} \hat{a}_t \]

\[ + \tilde{S}^Q_{X} T_1 \delta^K_{K} \hat{k}_{t+1} \]

\[ + \left[ \tilde{S}^Q_{X} T_1 (1 - \delta^K_{K}) - \left( I - \tilde{S}^Q_{S} T_1 \right) (I + \vartheta) \alpha \right] \hat{k}_t - \left( I - \tilde{S}^Q_{S} T_1 \right) (I + \vartheta) (I - \alpha) \hat{b}_t \]

\[ + \left[ -\varepsilon_D \tilde{S}^Q_{S} T_1 + \varepsilon_Q \tilde{S}^Q_{S} T_1 (I - S^M) + \varepsilon_M S^Q_{S} [T_1 S^M - T_2] + \varepsilon_X \tilde{S}^Q_{X} [T^M_1 X - T_2] \right] \hat{p}_t \]

\[ - \left( I - \tilde{S}^Q_{S} T_1 \right) \left[ \varepsilon_Q (I - S_M)^{-1} S_M (I - S^M) + \vartheta [(I - S_M)^{-1} S_M (I - S^M) + \tilde{I}] \right] \hat{p}_t \]

\[ 0 = -S^X_{I} \hat{p}_t + \left[ \beta (1 - \delta_K) S^X_{I} + \tilde{\beta} (I + \vartheta) (I + (I - S_M)^{-1} S_M (I - S^M)) \right] \hat{p}_{t+1} \]

\[ + \tilde{\beta} (I + \vartheta) \left[ \frac{1}{\varepsilon_Q} (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) + \varepsilon_Q^{-1} \frac{1}{\varepsilon_Q} \tilde{I} \right] \hat{a}_{t+1} \]

\[ + \tilde{\beta} (I + \vartheta) (I - \alpha) \hat{b}_{t+1} + \tilde{\beta} (-I + \alpha + \vartheta \alpha) \hat{k}_{t+1} \]

**Case 2: Durables**
Combine the final two equations in the line before "So now..."

\[
-S_I^X \frac{1}{\varepsilon_D} (I - \beta (I - \delta_C))
\]

\[
\times [I + S_I^C (\varepsilon_D - 1)] \hat{c}_{t+1} = \left[ \tilde{\beta} (I + \vartheta) (I + (I - S_M)^{-1} S_M (I - S_I^M)) + S_I^X \beta (\delta_C - \delta_K I) \right] \hat{p}_{t+1}
\]

\[
+ \tilde{\beta} (I + \vartheta) \left[ \frac{1}{\varepsilon_Q} (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) + \frac{\varepsilon_Q - 1}{\varepsilon_Q} I \right] \hat{a}_{t+1}
\]

\[
+ \tilde{\beta} (I + \vartheta) (I - \alpha) \hat{b}_{t+1} + \tilde{\beta} (-I + \alpha + \vartheta \alpha) \hat{k}_{t+1}
\]

to get.

\[
\hat{c}_t = \tilde{\vartheta} \left[ \tilde{\beta} (I + \vartheta) (I + (I - S_M)^{-1} S_M (I - S_I^M)) + S_I^X \beta (\delta_C - I \delta_K) \right] \hat{p}_t
\]

\[
+ \tilde{\vartheta} \tilde{\beta} (I + \vartheta) \left[ \frac{1}{\varepsilon_Q} (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) + \frac{\varepsilon_Q - 1}{\varepsilon_Q} I \right] \hat{a}_t
\]

\[
+ \tilde{\vartheta} \tilde{\beta} (I + \vartheta) (I - \alpha) \hat{b}_t + \tilde{\beta} \tilde{\vartheta} (-I + \alpha + \vartheta \alpha) \hat{k}_t
\]

where

\[
\tilde{\vartheta} \equiv \left[ -S_I^X \frac{1}{\varepsilon_D} (I - \beta (I - \delta_C)) [I + S_I^C (\varepsilon_D - 1)] \right]^{-1}
\]

Plug this in:
\[ 0 = \tilde{S}_C \tilde{\theta} \beta (I + \vartheta) \left[ 1 \frac{1}{\varepsilon_Q} (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) + \frac{\varepsilon_Q - 1}{\varepsilon_Q} I \right] \hat{a}_t \]

\[ + \left( \varepsilon_Q - 1 \right) \tilde{S}_M^2 T_1 - \left( I - \tilde{S}_M^2 T_1 \right) (I + \vartheta \varepsilon_Q^{-1}) (I - S_M)^{-1} (I + S_M (\varepsilon_Q - 1)) - \left( I - \tilde{S}_M^2 T_1 \right) \vartheta \varepsilon_Q^{-1} \hat{a}_t \]

\[ + \delta_C^{-1} \tilde{S}_C \tilde{\theta} \beta (I + \vartheta) (I + (I - S_M)^{-1} S_M (I - S_1^M)) + S_1^X \beta (\delta_C - I \delta_K) \hat{p}_{t+1} \]

\[ + \left( I - \delta_C^{-1} \right) \tilde{S}_C \tilde{\theta} \beta (I + \vartheta) (I + (I - S_M)^{-1} S_M (I - S_1^M)) + S_1^X \beta (\delta_C - I \delta_K) \hat{b}_t \]

\( F.3 \) Blanchard-Kahn

In Equations 54 and 55, we have expressed the reduced system as

\[ \begin{bmatrix} \hat{b}_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = \Psi \begin{bmatrix} \hat{p}_t \\ \hat{k}_t \end{bmatrix} + \Phi \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix} \]

Here, \( \Psi \) has \( N \) stable and \( N \) unstable eigenvalues.

Using a Jordan decomposition write \( \Psi = VD \Psi^{-1} \) where \( D \) is diagonal and is ordered such that the \( N \) explosive eigenvalues are ordered first and the \( N \) stable eigenvalues are ordered last. Re-write:

\[ \begin{bmatrix} \hat{b}_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = V^{-1} \begin{bmatrix} \hat{b}_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = D \Psi^{-1} \begin{bmatrix} \hat{p}_t \\ \hat{k}_t \end{bmatrix} + V^{-1} \Phi_{a} \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix} \]

\[ \equiv D \Upsilon_t + \Phi_{a} \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix} \]

Partition \( \Upsilon_t \) into the first \( N \times 1 \) block, \( \Upsilon_{1t} \), and the lower \( N \times 1 \) block \( \Upsilon_{2t} \). Similarly
partition $\tilde{\Phi}^2$ and $D$.

$$\Upsilon_{1,t} = D_1^{-1}E_t[\Upsilon_{1,t+1}] - D_1^{-1}\tilde{\Phi} \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix}$$

Substitute recursively

$$\Upsilon_{1,t} = -D_1^{-1} \sum_{s=0}^{\infty} D_1^{-s} \tilde{\Phi}_1 \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix} = -D_1^{-1}(I - D_1^{-1})^{-1}\tilde{\Phi}_1 \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix}$$ \tag{56}

For $Y_{2,t}$ simply:

$$Y_{2,t} = D_2 Y_{2,t-1} + \tilde{\Phi}_2 \cdot \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix}$$

Remember that

$$\begin{bmatrix} \Upsilon_{1,t} \\ \Upsilon_{2,t} \end{bmatrix} = V^{-1} \begin{bmatrix} \hat{p}_t \\ \hat{k}_t \end{bmatrix},$$

and therefore, from Equation 56

$$\hat{p}_t = -(V_{11}^{-1})^{-1}V_{12}^{-1}\hat{k}_t + (V_{11}^{-1})^{-1}\Upsilon_{1t}$$

$$= -(V_{11}^{-1})^{-1}V_{12}^{-1}\hat{k}_t - (V_{11}^{-1})^{-1}D_1^{-1}(I - D_1^{-1})^{-1}\tilde{\Phi}_1 \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix}$$ \tag{57}

The endogenous state evolves as follows:

$$\hat{k}_{t+1} = \Psi_{22} \hat{k}_t + \Psi_{21} \hat{p}_t + \Phi_2 \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix}$$

$$= (\Psi_{22} - \Psi_{21}(V_{11}^{-1})^{-1}V_{12}^{-1})\hat{k}_t + \left(-\Psi_{21}(V_{11}^{-1})^{-1}D_1^{-1}(I - D_1^{-1})^{-1}\tilde{\Phi}_1 + \Phi_2 \right) \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix}$$ \tag{58}

For future reference:

$$\hat{p}_t = \Psi_{21}^{-1}\hat{k}_{t+1} - \Psi_{21}^{-1}\Psi_{22}\hat{k}_t - \Psi_{21}^{-1}\Phi_2 \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix}$$ \tag{59}

### F.4 Obtaining the model filter

Combine Equations 52 and 53 to write $\hat{q}_t$ as a function of the exogeneous variables, $\hat{k}$, and $\hat{p}$

$$\hat{q}_t = (I + \vartheta) (I - \alpha) \hat{b}_t + (I + \vartheta) \alpha \hat{k}_t$$

$$+ \left[ \frac{\varepsilon Q - 1}{\varepsilon Q} \vartheta + \left( \frac{\vartheta}{\varepsilon Q} + I \right) (I - S_M)^{-1} (I + S_M (\varepsilon Q - 1)) \right] \hat{a}_t$$

$$+ \left[ (\vartheta + \varepsilon Q I) (I - S_M)^{-1} S_M (I - S_M^1) + \vartheta \right] \hat{p}_t$$ \tag{60}
Plug Equation 58 and 59 in so that we may write:

$$\hat{q}_t = \Phi_{kq} \hat{k}_t + \Phi_{ba} \hat{b}_t + \Phi_{aq} \hat{a}_t,$$

(61)

where the $\Phi_{kq}$, $\Phi_{ba}$, and $\Phi_{aq}$ are matrices that collect the appropriate terms.

So long as $\Phi_{kq}$ is invertible, Equation 61 is equivalent to

$$\hat{k}_t = \Phi_{kq}^{-1} \hat{q}_t - \Phi_{kq}^{-1} \Phi_{ba} \hat{b}_t - \Phi_{kq}^{-1} \Phi_{aq} \hat{a}_t$$

Equation 61, one period ahead, is

$$\hat{q}_{t+1} = \Phi_{kq} \hat{k}_{t+1} + \Phi_{ba} \hat{b}_{t+1} + \Phi_{aq} \hat{a}_{t+1}$$

Apply Equation 58 to this previous equation

$$\hat{q}_{t+1} = \Phi_{ba} \hat{b}_{t+1} + \Phi_{aq} \hat{a}_{t+1} + \Phi_{kq} \left( M_{KK} \hat{k}_t + M_{KA} \hat{a}_t + M_{KB} \hat{b}_t \right)$$

Finally, take two adjacent periods, and use the definitions of $\omega^A_{t+1} (\equiv \hat{a}_{t+1} - \hat{a}_t)$ and $\omega^B_{t+1} (\equiv \hat{b}_{t+1} - \hat{b}_t)$ so that

$$\Delta \hat{q}_{t+1} = \Delta \Phi_{kq} M_{kk} \Phi_{kq}^{-1} \hat{q}_t + \Phi_{ba} \omega^B_{t+1} + \Phi_{aq} \omega^A_{t+1} + [\Phi_{kq} M_{ka} - \Phi_{kq} M_{kk} \Phi_{kq}^{-1} \Phi_{aq}] \hat{a}_t + [\Phi_{kq} M_{kb} - \Phi_{kq} M_{kk} \Phi_{kq}^{-1} \Phi_{ba}] \hat{b}_t$$

Parsing out the factor-neutral and labor-augmenting productivity shocks yields Equation 12 and Equation 13 of the paper.

F.5 Calculations related to Section 2.4

In this section, I solve for covariance of industries' output as functions of the model parameters and the exogeneous TFP terms. The solution involves three steps. First, I solve for the wage. Second, I solve for the relative prices and intermediate input cost shares. Third, I solve for real sales. As there is no capital or durable goods, the decisions within each period are independent of those made in other periods. As such, I will omit time subscripts in this section.

Step 1: For later use, I will first solve for the wage in each period. For this portion of the analysis, it will be sufficient to examine how much the consumer wants to work and how much she wants to consume. Since the consumer’s problems are separable across periods,
the objective function for the consumer is

$$U = \beta \log C - \frac{\varepsilon_{LS}}{\varepsilon_{LS} + 1} L^{\frac{\varepsilon_{LS} + 1}{\varepsilon_{LS}}} \text{ subject to}$$

$$P \cdot C = W \cdot L.$$ 

The solution to this constrained optimization problem is:

$$W = L^{\frac{1}{\varepsilon_{LS}}} \text{ and } C = \frac{1}{P}.$$ \hspace{1cm} (62)

Invoking the budget constraint of the representative consumer:

$$L^{\frac{\varepsilon_{LS} + 1}{\varepsilon_{LS}}} = 1,$$

implying \( W = 1 \).

**Step 2:** Now consider the cost-minimization problem of the representative firm in industry \( J \). As I argued in the text, the cost-minimization problem implies the following recursive equation for the marginal cost (equivalently, price) of industry \( J \)'s good:

$$P_J = \frac{1}{A_J} \left[ 1 - \mu + \mu \left[ \sum_{I=1}^{N} \frac{1}{N} (P_I)^{1-\varepsilon_M} \right]^{\frac{1-\varepsilon_Q}{1-\varepsilon_M}} \right]^{\frac{1}{1-\varepsilon_Q}} \text{ for } J = \{1, ... N\}. \hspace{1cm} (63)$$

The log-linear approximation to the previous equation is:

$$\log P_J \approx - \log A_J + \frac{\mu}{N} \sum_{I=1}^{N} \log P_I.$$ \hspace{1cm} (64)

for all pairs of industries, so that Equation 64 implies:

$$\log P_J \approx - \log A_J + \frac{\mu}{N} \sum_{I=1}^{N} [\log P_J + \log A_J - \log A_I].$$

Re-arranging:

$$\log P_J \approx - \log A_J - \frac{\mu}{N (1 - \mu)} \sum_{I=1}^{N} \log A_I.$$ 

Because all industries’ cost shares are identical (both in the consumer’s preferences and in the production of each industry’s intermediate input bundle):

$$\log P_J^{in} \approx \log P \approx - \frac{1}{N (1 - \mu)} \sum_{j=1}^{N} \log A_J.$$

**Step 3:** The last task is to solve for \( Q_J \). To do so, apply the market clearing condition
for good \( I \), plug in the intermediate input demand by customers of \( I \), then re-arrange:

\[
Q_I = C_I + \sum_{J=1}^{N} M_{I,J}.
\]

\[
Q_I = C_I + \frac{\mu}{N} (P_I)^{-\varepsilon_M} \sum_{J=1}^{N} Q_J (P_J)^{\varepsilon_Q-1} (P_J^{in})^{\varepsilon_M-\varepsilon_Q}
\]

Next, take the log-linear approximation around the point at which all of the \( A \)'s equal 1:

\[
\log Q_I \approx \log \left( \frac{1}{1-\mu} \right) + (1-\mu) \log C_I - \mu \varepsilon_M \log P_I + \frac{\mu}{N} \sum_{J=1}^{N} \log Q_J
\]

\[
+ \frac{\mu}{N} \sum_{J=1}^{N} (\varepsilon_Q - 1) \log P_J + (\varepsilon_M - \varepsilon_Q) \log P_J^{in}.
\]

\[
\log Q_I - \frac{\mu}{N} \sum_{J=1}^{N} \log Q_J \approx \log \left( \frac{1}{1-\mu} \right) + (1-\mu) \log C_I - \mu \varepsilon_M \log P_I + \frac{\mu}{N} \sum_{J=1}^{N} (\varepsilon_M - 1) \log P_J
\]

\[
\approx \log \left( \frac{1}{1-\mu} \right) + (1-\mu) \log C_I + \mu \varepsilon_M \log A_I + \frac{\mu [\varepsilon_M (\mu - 1) + 1]}{N (1-\mu)} \sum_{J=1}^{N} \log A_J
\]

Given the preferences of the representative consumer, the demand function for good \( I \) is:

\[
\log C_I = -\varepsilon_D \log \left( \frac{P_I}{P} \right) - \log P.
\]

\[
\approx \varepsilon_D \frac{1}{N} \sum_{J=1}^{N} \log \left( \frac{A_I}{A_J} \right) + \frac{1}{N (1-\mu)} \sum_{J=1}^{N} \log A_J
\]

\[
\approx \varepsilon_D \log A_I + \frac{1 - (1-\mu) \varepsilon_D}{N (1-\mu)} \sum_{J=1}^{N} \log A_J
\]

Plug this expression back into Equation \( 65 \) and combine terms:

\[
\log Q_I - \frac{\mu}{N} \sum_{J=1}^{N} \log Q_J \approx \log \left( \frac{1}{1-\mu} \right) + (\mu \varepsilon_M + (1-\mu) \varepsilon_D) \log A_I
\]

\[
+ \left[ \frac{(1-\mu)(1-(1-\mu)\varepsilon_D) + \mu [\varepsilon_M (\mu - 1) + 1]}{N} \right] \sum_{J=1}^{N} \log A_J
\]

\[
\approx \log \left( \frac{1}{1-\mu} \right) + (\mu \varepsilon_M + (1-\mu) \varepsilon_D) \log A_I
\]

\[
+ \frac{1 - (1-\mu) (\mu \varepsilon_M + (1-\mu) \varepsilon_D)}{N} \sum_{J=1}^{N} \log A_J
\]

\[
\text{(66)}
\]
Equation 66 is a system of $N$ linear equations. The solution to these equations are

$$\log Q_I \approx \frac{1}{1-\mu} \log \left( \frac{1}{1-\mu} \right) + (\mu\varepsilon_M + (1-\mu)\varepsilon_D) \log A_I$$

$$+ \frac{1}{N} \left[ \left( \frac{1}{1-\mu} \right)^2 - (\mu\varepsilon_M + (1-\mu)\varepsilon_D) \right] \sum_{J=1}^{N} \log A_J$$

(67)

Equation 67 is equivalent to the expression given in the body of the paper.

**Additional references**


