In the remaining part of Economic 102, I will focus on issues related to long run economic growth. These issues will typically not be related to issues of macroeconomic volatility, although connections will occasionally arise.

Economic growth is often thought of as a long run phenomenon; i.e. we are interested in understanding how per capita output growth over horizons of 50 to 100 years as opposed to 3 to 6 months!
Initial Issues

Our analysis of economic growth will in fact explicitly focus on questions of the long term behavior of economies. This involves two sorts of questions:

First, are there long run steady state features of economies, i.e. states towards which an economy evolves?

Second, what are the properties of the transition path of an economy towards a steady state?
Economic growth analysis is typically conducted under the assumption that the productive capacity of the economy is utilized. Hence, it is done under the assumption of wage and price flexibility.

Our analysis will thus focus on how the levels of labor, capital, and technology determine aggregate output and how changes in these determine growth.
More formally, we will work with the aggregate production function

\[ Y = AF(K, L) \]

The supply of labor will be assumed to evolve exogenously. Our interest will be in the evolution of the capital stock \( K \) and the state of technology \( A \).

In order to understand growth, it is convenient to focus on the level of output per worker, denoted as \( y \). Formally,

\[ y = \frac{Y}{L} \]
Properties of $F(K,L)$

The function $F(K,L)$ is an example of the sort of production function you studied in Economics 101. In our analysis, we will assume that this function exhibits constant returns to scale. This means that if we multiply capital and labor by some factor $\lambda$, this raises output by $\lambda$, i.e.

$$F(\lambda K, \lambda L) = \lambda F(K,L)$$

So doubling the inputs to production doubles the output, etc.
Increasing returns to scale is a useful benchmark for thinking about aggregate production functions, even it does not always hold empirically. It also provides a valuable algebraic simplification for us.

Suppose we start with the aggregate production function

\[ Y = AF(K, L) \]

and divide both sides by \( L \), producing

\[ \frac{Y}{L} = y = \frac{AF(K, L)}{L} \]
Constant returns to scale implies that

\[
\frac{AF(K, L)}{L} = AF\left(\frac{K}{L}, \frac{L}{L}\right) = AF\left(\frac{K}{L}, 1\right)
\]

This tells us that output for worker will depend on \(\frac{K}{L}\).

The capital labor ratio. This ratio is denoted as \(k\). If we rewrite the function \(F(k, 1)\) as \(f(k)\), then we can think of output per worker as

\[
y = Af(k)
\]
Important message: growth in output per worker depends on growth of $k$ and $A$.

What is the shape of the function $f(k)$? Under the assumption constant returns to scale, the general shape is that illustrated in Figure 1.

The key feature of the shape is that the slope of the function is decreasing. Why does this happen? Intuitively, the effect of an increase in capital per worker on output per worker is higher when the initial capital per worker is lower.
Figure 1

“Typical” shape of $A_f(k)$
The Solow Growth Model

Our discussion of economic growth will center on a growth model due to Nobel Laureate Robert Solow. The model has been of enormous importance in both theoretical and empirical work on growth.

The Solow growth model is especially useful in illustrating the role of capital accumulation in economic growth.
To analyze the evolution of per capita output, it is necessary to allow the various elements in the aggregate production function to vary across time. We therefore work with

\[ Y_t = \text{level of output at time } t \]
\[ y_t = \text{output per worker at time } t \]
\[ A_t = \text{level of technology at time } t \]
\[ K_t = \text{level of capital stock at time } t \]
\[ L_t = \text{level of labor at time } t \]
\[ k_t = \text{level of the capital labor ratio at time } t \]
Let’s consider a very simple version of the Solow growth model to see how capital accumulation affects growth. We make the following assumptions

1. Output is divided between consumption and investment,

\[ Y_t = C_t + I_t \]

2. Technology and labor supply are constant

\[ A_t = A_{t-1} = A; \quad L_t = L_{t-1} = L \]

Notice we are ignoring population growth.
3. A constant fraction $s$ of output is saved each period and used for investment

$$I_t = sY_t$$

(The Solow model can be generalized to account model consumption/investment decisions in a way that explicitly accounts for utility maximization)

4. The capital stock evolves according to

$$K_t = I_{t-1}$$

so that it takes one period for investment to become part of the capital stock and the capital stock completely depreciates after 1 period.
These assumptions are not made because they are “true”. They are made because they allow us to identify the important features of the capital accumulation/growth relationship. To do this, recall the per worker aggregate production function

\[ y_t = Af(k_t) \]

According to the saving rule, \( sy_t \) is invested today. This will equal \( k_{t+1} \) since \( L_t \) is constant. In other words,

\[ k_{t+1} = sy_t \Rightarrow y_t = \frac{k_{t+1}}{s} \]
If we combine these two equations,

\[
\frac{k_{t+1}}{s} = Af\left(k_t\right)
\]

or

\[
k_{t+1} = sAf\left(k_t\right)
\]

This is called a difference equation; it describes how the capital labor ratio at \( t \) determines the capital labor ratio at \( t + 1 \).
The Importance of the Capital Labor Ratio

Our goal is to understand the long run behavior of per worker output. We will do this in an “indirect” fashion in that the analysis will focus on the evolution of the capital labor ratio.

Since the capital labor ratio determines per worker output, understanding one means that one understands the other, so there is no “problem” in proceeding this way. Also, notice that the only source of growth in the model is capital accumulation, so the evolution of the capital labor ratio is a natural object for us to study.
**Steady States**

Suppose that there is a level of the capital labor ratio, call it $k^*$, such that

$$k^* = sAf(k^*)$$

This is a level of the capital /labor ratio such that if it is attained, the economy will remain at the same level. It is known as a *steady state*. 
Is there a steady state in the Solow growth model? The answer is yes, assuming some technical conditions that need not interest us. We can see why a steady state exists if we consider a graph of $k_{t+1} = sAf(k_t)$. Let’s consider this graph where $k_t$ corresponds to the x-axis and $k_{t+1}$ corresponds to the y-axis. This is done in Figure 2

Notice that the 45 degree line represents pairs $k_t, k_{t+1}$ where the two elements are equal. Hence they are potential steady states. To find a steady state in the graph, we need to find points where $sAf(k_t)$ intersects the 45 degree line.
Figure 2

Steady state capital/labor in Solow Growth Model
Transition to the Steady State

Steady states are of interest only if they are “reachable” from a nontrivial set of initial conditions.

Hence, we need to see what happens over time when the initial capital labor ratio, call it $k_0$, is not equal to $k^*$. These are known as transition dynamics.

In order to do this, we will employ the diagram used in Figure 2. Remember that the 45 degree line denotes points that relate $k_t$ to $k_{t+1}$.
We first consider the case \( k_t < k^* \), Figure 3 indicates that

\[
k_{t+1} > k_t
\]

since \( k_{t+1} = sAf\left( k_t \right) \) and \( k_t < sAf\left( k_t \right) \).

In other words, when the capital labor ratio is below \( k^* \), the higher marginal product of additional capital per worker means that the capital labor ratio will grow.
On the other hand, suppose that $k_t < k^*$. Figure 3 indicates that

$$k_{t+1} < k_t$$

since $k_{t+1} = sA f(k_t)$ and $k_t > sA f(k_t)$.

In other words, when the capital labor ratio is below $k^*$, the lower marginal product of additional capital per worker means that the capital labor ratio will decrease.
Figure 3
Dynamic of Capital/Output Ratio
These arguments mean that the economy will evolve towards $k^*$. The steady state is therefore \textit{stable}, which means that the economy will move towards it under alternative initial conditions.

Some steady state are not stable. For example, in Figure 4, there are two steady states; the steady state $k^* = 0$ is not stable. If the economy starts at any place other than $k_0 = 0$, then the only steady state that is reached is the nonzero one.
Figure 4
Multiple Steady States
**Back to Growth**

Our discussion has focused on the capital labor ratio, not output per worker. However, since the capital labor ratio determines output per worker, we can use this model to draw some conclusions about the growth process.

These conclusions are all ceteris paribus, which means “other things equal” and are all statements about the transition path.
1. Countries with lower initial capital labor ratios will grow faster than countries with higher ratios. Why? This is a consequence of the decreasing marginal product of capital.

This is important as it suggests a mechanism by which poorer countries converge to richer ones. To the extent we fail to observe this, it may indicate something is missing from the model.

2. Countries with higher rates of savings will grow faster than countries with lower rates of savings. This could allow rich countries to grow faster than poor ones.
However, there is one key feature missing. At the steady state there is no growth!!

Once the capital labor ratio reaches its steady state, growth of output per worker is zero. All growth in this model is associated with adjustment towards the steady state.

In order to understand steady state growth, we will need to consider factors such as technical change or increasing returns to scale.