Lecture Notes 7

The IS Curve

In this lecture, the basic demand determined model of output will be generalized to allow for interest rate effects.

This extends the income expenditure model in a way that allows us to understand the role of interest rates in aggregate demand and output determination.

Our analysis will be done in a very particular way. Investment (and only investment) is allowed to depend on the real interest rate.
In other words, investment will no longer be treated as an exogenous variable (i.e. determined outside the system) but rather as a function of the interest rate.

\[ I = I(r) \]

So, in parallel to our discussion of consumption, we will be replacing the exogenous investment term with an investment function.

This dependence will allow us to integrate financial markets with the demand for output.
Why should an investment decision depend on the real interest rate?

Suppose that a machine costs 1 unit of output. Suppose that the machine will produce $1+x$ units of output tomorrow.

Here is my choice. Either save the unit of output (i.e. buy a bond) and receive $1+r$ units of output tomorrow, or buy the machine and receive $1+x$ tomorrow. The investment is justified only if

$$1+r \leq 1+x$$

Hence, the higher $r$, the more productive a machine has to be in order to justify purchase.

This general idea applies to much more general investment problems.
Treating investment as a function of the real interest rate leads to the equilibrium equation

\[ Y = C(Y) + I(r) + G + NX \]

This equation explains how, for a given interest rate, output is determined.

If the real interest rate is exogenously fixed, then this equation represents the equilibrium for the original income expenditure model.

Each choice of \( r \) produces a different income/expenditure model equilibrium.
But this leads to a new idea: each real interest rate level is associated with a different equilibrium output level in the income expenditure model.

If we graph the equilibrium output levels against the interest rate level, we produce what is known as the IS curve, as illustrated in Figure 1.

The downward slope reflects the fact that lower interest rates raise investment and thereby raise equilibrium output.
Figure 1

IS Curve

\[ Y = C(Y) + I(r) + G + NX \]
Notice that the values that are graphed are pairs of $r$ and $Y$ such that

$$Y = C(Y) + I(r) + G + NX$$

holds. Implicitly the income expenditure equilibrium for each possible interest rate is described.

The IS curve is defined by this equation.

Plug in a value of $r$ and you will find the corresponding $Y$ on the curve.
Basic Algebra

Let’s consider a linear version of the IS curve to get a sense of how behavioral parameters affect its slope.

1. Start with the GDP identity.

\[ Y = C + I + G + NX \]

2. We will assume that \( G \) and \( NX \) are exogenous. As before, this means we treat them as determined outside the model.

Remember:

– exogenous variables are those we take as fixed.
– endogenous variables are those that we solve for, i.e. those that the model attempts to explain as a function of the exogenous variables.
3. The level of consumption chosen by consumers is assumed to depend linearly on income

\[ C = a + cY \]

4. The level of investment chosen by firms is assumed to depend linearly on the real interest rate

\[ I = I_0 + br \]

Note: \( b \) is a behavioral parameter. We expect it to be negative.

We have all the ingredients to turn the GDP identity into a behavioral model, i.e. the IS equation.
To do this, we substitute the consumption function and investment function into the GDP identity. This yields

$$Y = a + cY + I_0 + br + G + NX$$

I am going solve for $r$ as a function of the other variables. (By tradition, the axes for the IS curve place output on the horizontal axis)

To do this, I rewrite the equation as

$$(1-c)Y - a - I_0 - G - NX = br$$

Dividing both sides by $b$,

$$r = \frac{(1-c)Y - a - I_0 - G - NX}{b}$$
Let’s rewrite this as

\[ r = \frac{-a - I_0 - G - NX}{b} + \frac{(1 - c)}{b} Y \]

This expression, although it contains many terms, is a linear equation. The term

\[ -a - I_0 - G - NX \]

is the intercept and the term

\[ \frac{(1 - c)}{b} \]

is the slope.

If \( b < 0 \), it must be the case that the intercept is positive and the slope is negative.
Figure 2

Linear IS Curve

intercept = \frac{-a - I_0 - G - NX}{b}

\frac{(1 - c)}{b} = \text{slope}
Changes in Exogenous Variables and the IS Curve

How does the location of the IS curve in Figure 2 depend on the different exogenous variables? Specifically, how does $G$ affect the location of the curve?

From the equation that describes the IS curve, it is clear that $G$ only affects the intercept term; the slope term depends on the behavioral parameters $c$ and $b$.

We know that the intercept is larger when $G$ is larger; this follows from the intercept equation

\[-a - I_0 - G - NX \]

\[\frac{b}{b}\]
If one fixes $Y$ and raises $G$, it must be the case that the associated $r$ on the IS curve is increased. In other words, an increase in $G$ shifts the IS curve up, as illustrated in Figure 3.

Intuitively, if $Y$ is fixed and $G$ increases, the only way to preserve the output equilibrium is for interest rates to rise in order to reduce investment to cancel out the demand increase induced by higher government spending.

Key Idea: If output is constant, increases in government purchases will induce interest rate changes that reduce investment. This is known as crowding out.

Since investment affects the productive capacity of the economy, this is an important possibility.
Figure 3

IS curves for alternative levels of government purchases.