Lecture 8

Interest Rates and the
Income/Expenditure Model

In this lecture I will discuss how to introduce interest rates in the income expenditure model.

So far, we have discussed how a demand-determined equilibrium level of output reflects feedbacks between the level of aggregate output and the level of the components of output, primarily consumption. Today, our focus will be on the role of interest rates in determining these components.

We will focus on real interest rates.
Effects of Real Interest Rates on Components of Demand

Real interest rates primarily affect demand through their effect on consumption and investment.

We have discussed the effects of real interest rates on consumption and concluded the effects are, as a matter of theory, ambiguous. Income effects would suggest that higher real interest rates raise current consumption whereas substitution effects suggest that higher real interest rates lower consumption.
Empirical evidence does not suggest that consumption is particularly sensitive to interest rates in the short run.

For expositional purposes, I will assume that the effects “cancel out”. I do this so we can focus on the effects of consumption on investment.

The effects of interest rates on investment are possibly of greater empirical import, and so this will be the focus of the following discussion.
Investment: A Stylized Example

To see how investment is affected by interest rates, let’s consider a very simplified investment problem for a firm.

I will assume that the price of goods is constant across time. Capital goods cost the same as output.

The firm is assumed to want to maximize the present discounted value of profits. What this means is that it makes decisions at time $t$ on the basis of their effects on the profits generated in current and future periods. However, it discounts these profits.
Why should profits be discounted?

The reason is the same as applied to the two period consumption problem: a dollar used today to produce investment goods can alternatively be used to invest in a government bond. Either use of a dollar produces dollars in the future. The tradeoff between dollars today and tomorrow is determined by interest rates.

Initially, I will assume that the firm only exists for two periods. The firm is deciding whether to purchase a unit of capital (think of this as a machine) that costs $P$ today. The machine will produce $MPK$ units of output tomorrow. ($MPK$ is chosen to mean marginal product of capital.)

Should the investment be made?
To determine the answer, it is necessary to calculate the effect of the investment on the present discounted value of profits.

The cost of the investment is $P$; recall that this is paid today.

The return to the investment is $P \times MPK$; recall that this is received tomorrow.

The effect on the present discounted value of profits is

$$-P + \frac{1}{1+r} (P \times MPK)$$

If this is positive, then the firm should make the investment.
Why is the payoff to the investment discounted?

The reason is that the payoff occurs in the future. As argued for the two period consumption problem, when prices are constant, the tradeoff between dollars today and tomorrow is determined by the $1 + r$.

This is an application of the idea we discussed early in the course. Would you rather have 1 dollar today or 1 dollar tomorrow? The answer we derived was that the dollar today may be invested at the risk free rate, hence the question is equivalent to asking whether I would rather have 1 dollar tomorrow or $1 + r$ dollars tomorrow.
In fact, I would be indifferent between \( \frac{1}{1+r} \) dollars today and 1 dollar tomorrow, since

\[
(1+r) \frac{1}{1+r} = 1
\]

Here is another way to think about the problem. Instead of purchasing a new machine, the firm can always purchase a government bond. If the firm were to invest \( P \) in government bonds, then it would receive

\[
(1+r)P
\]
If the firm instead purchases a machine, it receives

\[ P \times MPK \]

tomorrow. Hence the decision to purchase a machine rather than government bonds requires that

\[ (1 + r)P < P \times MPK \]

Dividing both sides by \(1 + r\), purchasing the machine requires that

\[ P < \frac{1}{1 + r} P \times MPK \]

which is the same condition we found before.
Role of Interest Rates

From the perspective of aggregate demand, what is important about the equation

\[-P + \frac{1}{1+r}(P \times MPK)\]

is that it depends inversely on the level of \( r \). This makes intuitive sense. High interest rates imply that the value of the future product of the machine is discounted at a higher rate.

The example we have discussed is quite stylized. In general, investment decisions will depend not only on the level of interest rates today but in future periods.
To see this, let’s suppose that the firm exists for 3 periods: 0, 1, 2. Denote the interest rate between times 0 and 1 as $r_0$ and the interest rate between times 1 and 2 as $r_1$.

As before, assume that the price of output does not change across periods and that the price of capital goods is the same as output.

The only difference between this problem and the earlier one is that I will assume that the machine produces for two periods. Let $MPK_1$ denote its output in period 1 and $MPK_2$ denote its output in period 2.

Q: Why might the $MPK$’s differ across time?
As before, the cost of the machine is

\[ P \]

What are the properly discounted returns?

For the output produced during period 1, the present discounted value is

\[ \frac{1}{1+r_1} P \times MPK_1 \]

For the output produced during period 2, the present discounted value is

\[ \frac{1}{(1+r_1)(1+r_2)} P \times MPK_2 \]
Where does this discount factor come from?

The discount factor reflects the fact that we need to discount dollars 2 periods in the future. To do this, let’s follow our trick of asking how to trade off 1 dollar today versus 1 dollar 2 periods from now.

If I take 1 dollar today and invest it, I will receive $1 + r_1$ dollars tomorrow. If I then take the $1 + r_1$ dollars and re-invest them tomorrow, I will receive

\[
(1 + r_1)(1 + r_2)
\]

dollars two periods from now.
Therefore, 1 dollar received 2 periods from now is worth

\[
\frac{1}{(1+r_1)(1+r_2)}
\]
dollars today.

Important message: investment levels will in general depend on current and future 1-period interest rates.

For more realistic investment problems, this will involve expectations of future 1-period rates over longer horizons. Such calculations are facilitated by comparisons that use long term interest rates.
Notice that if we multiply out the discount rate,

\[(1 + r_1)(1 + r_2) = 1 + r_1 + r_2 + r_1r_2\]

This can be well approximated by

\[1 + r_1 + r_2\]

since \(r_1r_2\) is typically quite small compared to \(r_1\) and \(r_2\). For example, if the real short term rates are each equal to .05, then their product is .0025, hence the average future rates are important for understanding discount rates over different horizons.

Exercise: (not to hand in) Repeat these calculations for 3 periods.
Implications for Aggregate Equilibrium

When investment depends on interest rates, then changes in interest rates will change aggregate output. To see this, consider the GDP identity when I have substituted in both the consumption function $C(Y)$ and an investment function $I(r)$.

$$Y = C(Y) + I(r) + G + NX$$

Each level of the interest rate $r$ determines an equilibrium level of output. In turn, a change in the interest rate will induce a change in equilibrium output level. Figure 1 illustrates this for two interest rate levels, $\bar{r}$ and $\overline{r}$.
Change in equilibrium level of output when interest rate declines from $\bar{r}$ to $\bar{\bar{r}}$

Figure 1
This allows to one think about the following question:

What pairs of interest rates and output levels are consistent with an aggregate demand equilibrium?

To interpret this question, we can think about a graph whose axes are $r$ and $Y$. For each interest rate level, we can associate an equilibrium output level. Figure 2 illustrates this for the two interest rates $\bar{r}$ and $\overline{r}$. 
Equilibrium pairs of interest rates and output from Figure 1 graphed using \((r, Y)\) axes.

Figure 2
If we do this for all possible values, we will end up with a curve such as the curve illustrated in Figure 3.
IS Curve

Combination of interest rates and output that are consistent with equilibrium in the income expenditure model.

Figure 3
The interest rate/output combinations in Figure 3 constitute the set of possible equilibrium interest rate/output levels for the economy. This curve is known as the IS curve, where IS stands for Investment/Savings. The idea is that investment equals savings in equilibrium.

The IS curve makes clear how a complete theory of aggregate output, when output is demand determined, must explain how interest rates are set. This leads us to the analysis of financial markets, which is the subject of the next lecture(s).