In today’s lecture, we will begin the discussion of the determination of interest rates. We will do this by modeling the demand for and supply of money.

Our goal is to *endogenize* interest rates so that we can understand how interest rates and aggregate output are simultaneously determined.

Recall that the IS curve described possible interest rate, equilibrium output combinations. Our goal, therefore, is to determine which combination is in fact the equilibrium.
Review of Interest Rate Definitions

In discussing interest rates, recall that

\( r \) denotes real interest rates, i.e. the rate at which a unit of goods today may be translated into goods tomorrow.

\( i \) denotes nominal interest rate. This describes how dollars today are translated into dollars tomorrow.

\( \pi \) denotes inflation. This is the percentage change in prices between today and tomorrow.

An identity relates these

\[ r = i - \pi \]
Money Demand: Initial Ideas

Why is money a universal aspect of modern economies? Traditionally, economists have identified three reasons.

1. **Means of exchange.** Money provides a way for individuals to make indirect exchanges. What do I mean by indirect? Money facilitates economic exchange between individuals who do not produce something each wishes to directly exchange with the other. Avoids the need for “double coincidence of wants.”

2. **Unit of account.** Money provides a way of measuring prices in comparable units.
3. **Store of value.** Money represents a way of holding wealth.

At first glance, money (as we know it in the US) would seem to be an inefficient way to hold wealth. After all, the nominal interest rate on a dollar bill is 0.

The key virtue of money versus other assets is that money is (by definition) *liquid*. The term *liquidity* refers to the “ease” with which an asset may be converted into the medium of exchange.

Illiquid assets create what are known as “shoe leather” costs when one wants to convert the assets in order to consume.
Types of Money

The attributes I have used to describe why economies need money do not specify the form money will take. There are two basic types of money in different economies:

**Commodity money:** If the objects that are defined as money also have an intrinsic value, then they are forms of commodity money. Gold and silver are traditional forms of commodity money.

Another nice example: cigarettes in US prisons and in Soviet labor camps.
Money in the US is not commodity money. The physical currency we use would provide no use to us if one could not exchange it. The second type of money is of this type.

**Fiat money:** If the objects that are defined as money have no intrinsic value, then they are forms of fiat money. Currency is an example of fiat money.

Fiat means that the government “by fiat” has declared certain intrinsically worthless objects to be money

“This note is legal tender for all debts public and private”
Why should it be possible for 1000 pieces of paper with green marking and words such as “one hundred dollars” to be exchanged for a brand new Mercedes Benz?

Answer, because the recipient of the 1000 pieces of paper believes that these pieces of paper can in turn be exchanged for other items that do have intrinsic value!
Money and Beliefs About the Future

This discussion indicates how the value of money depends on expectations.

Here is a morbid example. Suppose that everyone were suddenly to come to believe that, with probability 1, the world will end in two days (at some fixed time $T_{doomsday}$, for the sake of argument. What would happen to the value of US currency today?

Answer: we would expect US currency to become valueless immediately. Why? No one will accept US currency immediately before $T_{doomsday}$ because they will not be able to trade the currency. Working backwards, no one will accept money in a transaction now.
Defining Money in the United States

Defining money is actually somewhat trickier than one might think.

Clearly, currency is a form of money.

How about traveler’s checks? Checking accounts?

In general, in thinking about money, one wants to isolate assets that are sufficiently liquid that one can immediately use them in exchanges for commodities.

Given the many complicated assets that exist, there will always be marginal cases.
Standard Measures of Money

M1: this measure is equal to

Currency + demand deposits + traveler’s checks + some additional minor types of checkable accounts

Demand deposits are simply bank accounts on which one can write personal checks.

Other checkable accounts refers to certain other accounts that provide checkwriting privileges

In 2002, M1 equalled approximately $1.2 trillion
M2: This is a broader measure that equals

M1 + savings accounts + money market mutual funds + other additional accounts with partial checkwriting capacity.

For our purposes, the definition of money (M1 versus M2) is not essential.
However, the definitions indicate that even though, in basic macroeconomic models, we assume that the money supply is determined by the government, in fact the private sector has first order effects on the actual money supply.

The government agency that “controls” (to repeat, only partially!) the money supply is the Federal Reserve System (FRS).
Some Background on the Federal Reserve System

The FRS consists of two parts:

**Federal Reserve Board (FRB).** This is located in Washington DC and is run by a 7 member Board of Governors. The chairman is very influential in determining the policies of the Board. The current chairman is Alan Greenspan, who has served since 1987. Governors serve 14 year terms; the chairman serves a 4 year term. They are appointed by the President.

**Regional Banks.** There are 12 regional Federal Reserve Banks. Each regional bank has a president selected by the board of directors of the bank.
Federal Open Market Committee

Decisions by the Federal Reserve concerning the supply of money are made by the Federal Open Market Committee (FOMC).

The FOMC consists of 12 voting members: the 7 members of the FRB and 5 regional bank presidents. Voting power rotates across the bank presidents. The one exception is the president of the NY regional bank, who is always eligible to vote, presumably because the NY bank actually implements the financial market trades necessary to implement FOMC policy.
How Does the Federal Reserve Alter the Money Supply

Details of the money supply process are well discussed in Parkin.

For now, it is sufficient to understand that the main mechanism by which this is done is called open market operations (OMO). Open market operations refer to the buying and selling of government bonds by the Federal Reserve. (The NY bank actually conducts this, as noted above). The Federal Reserve injects money (which it has the authority to create) by purchasing government bonds that are held by the public.
Relationship to the Rest of the Government

Fiscal policy (at the Federal level) is determined by the Congress and President. While the governors of the Federal Reserve Board are appointed by the President and must be ratified by the Senate, the Federal Reserve Board is in fact an example of an independent agency, which means that its decisions on the money supply (for example) cannot be overturned by another branch of the government (outside of the courts, of course, although this is really not an issue when it comes to monetary policy).

This means that there is no necessary coordination of monetary and fiscal policy in the United States; different authorities control the two and may not agree on what needs to be done!
Stock Versus Flows

How is it that $1.2$ trillion of M1 can allow for the purchase of $10.5$ trillion in output (using 2002 figures)?

The answer is that the money supply is a stock whereas the level of output is a flow. What this means is that while the output is constantly being produced and purchased throughout the year, the stock of money is fixed but is circulated across individuals as transactions are made.

The velocity of money $V$ refers to the rate at which it turns over, i.e. is circulated through the economy.
There is a famous identity in macroeconomics that formalizes the notion of velocity

\[ MV = PY \]

or

\[ V = \frac{PY}{M} \]

All this means is that velocity tells us how frequently money needs to “turn over” to sustain a certain level of transactions. Notice the transactions are measured in terms of their nominal value.
Money Anomalies

There are some interesting anomalies about money demand in the US.

In 2001, there was $580 billion in outstanding US currency, which would translate to approximately $2700 for each adult in the US that year. This seems unreasonably high.

Possible explanations

1. demand for currency by criminals, primarily drugs

2. holding of US currency in other countries. Why would this occur? Because of greater confidence in US dollars.
Money Market Equilibrium

Let’s now consider a first pass at understanding how the supply and demand for money can allow us to understand the joint determination of output and interest rates.

I will assume

1. inflation rate and current price level are fixed at values $\pi$ and $P$. This is because we are considering the short run.

2. the nominal supply of money is fixed at some value $M$. By nominal, I mean the number of dollars that are outstanding.
We can now engage in supply and demand analysis for money.

The nominal supply of money is fixed at $M$ and prices are fixed so therefore the real supply of money (the value of money in terms of what it can purchase) is fixed at

$$\frac{M}{P}$$

Why do we think in real terms, when money is a nominal asset? Because our willingness to hold money will depend on how much it can purchase, which depends on prices.

This leads us to money demand.
Money Demand

The real demand for money will depend, as a first approximation, on the level of transactions that individuals will engage in (in terms of what they buy) and on the return on money versus alternative stores of value.

One factor we would expect matters is the level of real income. High levels of real income induce higher money demand. Hence money demand will depend on $Y$.

Second, one would expect the demand for money to depend on the opportunity cost of holding money.
What is this opportunity cost? We discussed this earlier; it is the nominal interest rate $i$. I will repeat the argument made earlier.

Suppose that an individual is comparing two assets, money and bonds. The individual cares about real returns. Bonds pay a nominal return of $i$. In comparing the two assets, one will look at the differential in the real returns.
What is the nominal interest rate on money?
Answer: 0

What is the real interest rate on money?
Answer: $-\pi$

What is the nominal interest rate on bonds?
Answer: $i$

What is the real return on bonds?
Answer: $i - \pi = r$

What is the real return differential?
Answer: $i_t - \pi_t - (-\pi_t) = i_t$

Hence the nominal interest rate is the relevant object for comparing assets.
Money Market Equilibrium

Let the demand for money be denoted as $L$ (for liquidity). What variables will $L$ depend on? We have identified two variables: nominal interest rates and output. Therefore, we can think of a money demand function

$$L(i,Y)$$

Notice that I can write this money demand function as

$$L(r + \pi, Y)$$
Therefore, money market equilibrium requires

\[ \frac{M}{P} = L(r + \pi, Y) \]

This identifies pairs of real interest rates and levels of income that equate money demand and money supply.

Since money demand is decreasing in \( i \), it must be decreasing in \( r \) when \( \pi \) is fixed. (This is because nominal and real rates vary 1 for 1 when \( \pi \) is fixed.)

Further, we have assumed that money demand is increasing in \( Y \).

This means that the pairs of real interest rate and output levels that satisfy money market equilibrium will be upward sloping.
Let’s see why this is so.

Suppose that there is an initial interest rate $\bar{r}$ and an initial income level $\bar{Y}$ such that money supply equals money demand.

Suppose that the real rate is lowered to $\bar{r}$. At the original output level $\bar{Y}$, money demand will be higher than before, since the nominal rate is now lower.

In other words,

$$L(\bar{r} + \pi, \bar{Y}) > L(\bar{r} + \pi, \bar{Y})$$
But this means that the money market is no longer in equilibrium, since

\[ L(\bar{r} + \pi, \bar{Y}) > L(\bar{r} + \pi, \bar{Y}) = \frac{M}{P} \]

i.e.

\[ L(\bar{r} + \pi, \bar{Y}) > \frac{M}{P} \]

What must hold in order for equilibrium to occur? Income must be lower! In other words, associated with \( \bar{r} \) is an income level \( \bar{Y} \) such that

\[ L(\bar{r} + \pi, \bar{Y}) = \frac{M}{P} \]

with \( \bar{Y} < \bar{Y} \). The pairs are illustrated in Figure 1.
Pairs of real interest rates and output levels such that money market is in equilibrium.
Just as we did in the derivation of the IS curve, we can trace out equilibrium pairs of real interest rates and output levels for all interest rate values. This is illustrated in Figure 2.

This is called the LM curve. (Think of it as demand for Liquidity equals Money supply).

Notice that the LM curve is upward sloping. This simply generalizes our earlier argument.
Figure 2

$\frac{M}{P} = L(r + \pi, Y)$

LM curve describing pairs of interest rates and output levels that equilibrate money market.
Algebra of the LM curve

We can develop some algebra for a linear version of the LM curve, just as we did for the IS curve in Lecture 9.

Suppose that money demand may be described by the linear relationship

\[ m + l(r + \pi) + dY \]

In this linear equation, \( m \) is a constant term, \( l \) is a behavioral parameter that measures the sensitivity of money demand to the nominal interest rate (why?) and \( d \) is a behavioral parameter that measures the sensitivity of money demand to the level of aggregate output. We have argued that \( l < 0 \) and \( d > 0 \).
If we set money demand and money supply equal to each other, we have the equation

\[ \frac{M}{P} = m + l(r + \pi) + dY \]

This is the linear form of the LM equation.

As was done with the IS schedule, I will rewrite this equation in order to produce a linear equation that relates income and interest rates.

Notice that for this equation, we are treating the inflation rate \( \pi \) and the money supply \( \frac{M}{P} \) as exogenous variables.
As before, I will want to think about the graph of this equation where \( r \) corresponds to the y-axis and \( Y \) corresponds to the x-axis.

Step 1: rewrite the LM equation by multiplying out the nominal interest rate term, with a bit of rearranging terms, this gives us

\[
\frac{M}{P} = m + l\pi + lr + dY
\]

If we subtract \( \frac{M}{P} \) and \( lr \) from both sides, we have

\[
-lr = m - \frac{M}{P} + l\pi + dY
\]
Finally, divide both sides of the equation by \(-l\).

This produces the equation

\[ r = \frac{-m}{l} + \frac{M}{lP} - \pi - \frac{d}{l}Y \]

As occurred for the IS equation, this representation of the LM equation is complicated in that it contains many terms, but is in fact a linear equation. The intercept is

\[-\frac{m}{l} + \frac{M}{lP} - \pi\]

and the slope is

\[-\frac{d}{l}\]
Can we determine the signs of the intercept and slope? To do this, it is critical to remember that $l$ has been assumed to be negative. For the intercept

$$\frac{-m}{l} + \frac{M}{lP} - \pi$$

the first term is positive (assuming $m$ is positive), the second term is negative, and the third term is negative (I rule out deflation). Hence we cannot say whether the sum is positive or negative. In fact, the sign of the intercept is of little importance by itself.

The slope term

$$\frac{-d}{l}$$

is, on the other hand, unambiguously positive.
Figure 3

Linear LM Curve
Changes in Exogenous Variables and the LM Curve

How does the location of the LM curve in Figure 3 depend on the different exogenous variables? Specifically, how does $M$ affect the location of the curve?

From the equation that describes the LM curve, it is clear that $M$ only affects the intercept term; the slope term depends only on the behavioral parameters $l$ and $d$.

By inspecting the intercept,

$$\frac{-m}{l} + \frac{M}{lP} - \pi$$

it is clear that the intercept is decreasing in $M$. 
Therefore, if one fixes $Y$ and raises $M$, it must be the case that the associated $r$ on the LM curve is reduced. In other words, an increase in $M$ shifts the LM curve down, as illustrated in Figure 4.
Figure 4

LM Curves for alternative levels of money supply
What is the intuition for this? At a fixed level of $Y$, the Federal Reserve increases the money supply by purchasing government bonds. This increase in the demand for bonds raises bond prices and drives down interest rates. In order to induce the public to hold a higher level of money and a lower level of bonds, the attractiveness of bonds relative to money must diminish.

I discussed the relationship between bond prices and interest rates in the addendum to Lecture 4; I repeat this discussion given its relevance.
Suppose that a bond exists that pays $1 at time $t+1$. It sells for price $P$ at time $t$. What is the nominal interest rate on the bond?

We can solve for the interest rate by recognizing that we can think of the bond as taking $P$ dollars at $t$ and paying $(1+i)P$ dollars at $t+1$. Further, we know that the bond by assumption pays $1$. Therefore the nominal interest rate is implicitly defined by

$$(1+i)P = 1$$
This equation may be rearranged, i.e.

\[
1 + i = \frac{1}{P}
\]

so that

\[
i = \frac{1}{P} - 1 = \frac{1}{P} - \frac{P}{P} = \frac{1 - P}{P}
\]

This makes intuitive sense; \(1 - P\) is what one receives beyond the price of the bond at \(t + 1\).

Notice that the price of bonds is inversely related to the nominal interest rate.

This basic idea holds for more complicated bonds.
Important Clarification

The LM curve has been discussed in terms of relating real interest rates and output.

The discussion of the bond price interest rate equilibrium has referred to nominal interest rates.

Is there a problem relating the two discussions?

Since we are treating inflation as fixed, any change in nominal interest rates is equivalent to a change in real interest rates, which is why the two are actually dealing with the same thing.