Our discussion of aggregate output has alternated between the GDP identity and the description of equilibrium GDP as determined by the income expenditure model. The income expenditure model represents a way of understanding equilibrium in the goods market when certain factors are fixed. In particular prices are assumed fixed above the equilibrium level.

I want to review this concept before proceeding onto a more general discussion of the IS curve.
The distinction between identities and equilibrium notions of output is important and hard.

When one says that GDP fulfills the identity

\[ Y = C + I + G + NX \]

what this means is that an observed level of GDP can always be broken up into these four components. Each represents a “type” of demand in the economy. So, if I know what was produced in the US in 2002, I can say what parts went to consumption, investment, etc.

Put differently, the GDP identity holds for any \( Y \)

An identity is always true; this means it is a tautology.
The GDP identity, while useful in formulating macroeconomic models, does not by itself contain much information about the economy. Specifically, it does not say anything about how a change in one variable influences the value of another in equilibrium, e.g. how a change in government purchases will alter equilibrium output.

To answer such questions, one must work with an equilibrium model, one that accounts for the interdependences between the variables.

To see why identities cannot be informative about issues of interest, I will start with a simple example.
Prices and Expenditures on Compact Disks

An individual buys a certain number of compact disks. There is an identity that describes the amount of money that is spent. Let $V$ denote the amount of money spent. Let $P$ denote the price of each compact disk. Let $Q$ denote the number of disks purchased. Then,

$$V = P \times Q$$

This is an identity because it “follows by definition.” Put differently, the relationship is a tautology.

Suppose that compact disks initially cost $\bar{P}$ and that the individual purchases $\bar{Q}$ at that price. This means, of course, that $V$ is equal to $\bar{P} \times \bar{Q}$. 
Now, suppose that the price of compact disks had not been $\bar{P}$; rather the price was some lower value $\bar{P}$. How much would the consumer have spent on compact disks?

Clearly, we would not want to say that the answer is

$$\bar{P} \times \bar{Q}$$

which would amount to putting the new price into the identity. Why is this wrong? This is wrong because we have failed to account for how the number of compact disks that are purchased depends on the price level.
The question I have posed cannot be answered unless we know the number of compact disks that would be purchased if the price were $\bar{P}$.

In other words, we need a theory to explain how one component of the identity, price, affects the other component, quantity.

Suppose that we have such a theory, i.e. we can specify a demand function $Q(P)$ that describes the number of compact disks the individual will purchase given a price. Suppose that $Q(\bar{P}) = \bar{Q}$, then the expenditures at $\bar{P}$ will equal

$$\bar{P} \times \bar{Q}$$

which is of course consistent with the identity.
Let’s apply this to our macroeconomics context.

To see how the GDP identity says *nothing* about the equilibrium effect of a change in taxes, government spending, etc., suppose that output were fixed because all productive resources in the economy (capital and labor) are employed.

Then a change in $G$ would have to come at the expense of one of the other components.

Hence a change in $G$ would not affect $Y$ in equilibrium, even though the GDP identity holds for any level of government purchases!
Alternatively, suppose that $C$, $I$, and $NX$ were fixed and that output is determined by demand. Then a change in $G$ of one 1 unit will raise $Y$ by 1 since the other components are unchanged (by assumption!)

Finally, suppose that the level of government spending $G$ determines the level of investment (for example, because higher $G$ is devoted to public infrastructure). Suppose, in particular, that a 1 unit increase in $G$ always induces a $\frac{1}{2}$ unit increase in $I$. Assume that none of the other components depend on $G$ and that none of the components depend on $Y$.

Then if demand determines output, then a change in $G$ of 1 unit will raise $Y$ by $1\frac{1}{2}$ units.

Why? The change in $G$ induces an additional $\frac{1}{2}$ change in $I$. Nothing else happens.
Hence one must specify how output is determined in equilibrium in order to discuss policy effects, etc.

Our first model of equilibrium output is based on the idea that the level of output affects consumption since output is equal to income for the economy as a whole. In other words, we move from the identity to the equation

\[ Y = C(Y) + I + G + NX \]

This no longer holds for any \( Y \).
When one specifies the values of $I$, $G$, and $NX$, there will be a single value of $Y$ such that this equation holds.

This equation in turn lets us answer questions about the equilibrium effect of a change in $G$ on aggregate output.

These answers lead us to the idea of a multiplier.
IS Curve

In the last lecture, the basic demand determined model of output was generalized to allow for interest rate effects. This was done in a very particular way. Investment and only investment was allowed to depend on the real interest rate. Hence investment was no longer treated as an exogenous variable (i.e. determined outside the system) but rather as a function of the interest rate. This led to the equilibrium equation

\[ Y = C(Y) + I(r) + G + NX \]

which explains how, for a given interest rate, output is determined.
If the real interest rate is exogenously fixed, then this equation represents the equilibrium for the original income expenditure model, except we now say that investment is exogenous because the interest rate level is exogenous.

But this leads to a new idea: each real interest rate level is associated with a different equilibrium output level in the income expenditure model. If we graph the equilibrium output levels against the interest rate level, we produce the IS curve, as illustrated in Figure 1.

The downward slope reflects the fact that lower interest rates raise investment and thereby raise equilibrium output.
Figure 1

$Y = C(X) + I(t) + G + NK$
Notice that the values that are graphed are pairs of $r$ and $Y$ such that

$$Y = C(Y) + I(r) + G + NX$$

holds.

The IS curve is implicitly defined by this equation. Plug in a value of $r$ and you will find the corresponding $Y$ on the curve.
Basic Algebra

Let’s consider a linear version of the IS curve to get a sense of how behavioral parameters affect its slope.

1. Start with the GDP identity.

\[ Y = C + I + G + NX \]

2. We will assume that \( G \) and \( NX \) are exogenous. This means we treat them as determined outside the model.

Note:
- exogenous variables are those we take as fixed.
- endogenous variables are those that we solve for, i.e. those that the model attempts to explain as a function of the exogenous variables.
3. The level of consumption chosen by consumers is assumed to depend linearly on income

\[ C = a + cY \]

4. The level of investment chosen by firms is assumed to depend linearly on the real interest rate

\[ I = I_0 + br \]

Note: \( b \) is a behavioral parameter. Our discussion last Wednesday implies that it is negative.

We have all the ingredients to turn the GDP identity into a behavioral model, i.e. the IS equation.
To do this, we substitute the consumption function and investment function into the GDP identity. This yields

\[ Y = a + cY + I_0 + br + G + NX \]

I am going solve for \( r \) as a function of the other variables. (By tradition, the axes for the IS curve place output on the horizontal axis)

To do this, I rewrite the equation as

\[ (1-c)Y - a - I_0 - G - NX = br \]

Dividing both sides by \( b \),

\[ r = \frac{(1-c)Y - a - I_0 - G - NX}{b} \]
Let’s rewrite this as

\[ r = \frac{-a - I_0 - G - NX}{b} + \frac{(1-c)}{b} Y \]

This expression, although it contains many terms, is a linear equation. The term

\[ \frac{-a - I_0 - G - NX}{b} \]

is the intercept and the term

\[ \frac{(1-c)}{b} \]

is the slope.

If \( b < 0 \), it must be the case that the intercept is positive and the slope is negative.
Figure 2

Linear IS Curve

intercept = \frac{-a - I_0 - G - NX}{b}

\frac{(1-c)}{b} = \text{slope}
Changes in Exogenous Variables and the IS Curve

How does the location of the IS curve in Figure 2 depend on the different exogenous variables? Specifically, how does $G$ affect the location of the curve?

From the equation that describes the IS curve, it is clear that $G$ only affects the intercept term; the slope term depends on the behavioral parameters $c$ and $b$.

We know that the intercept is larger when $G$ is larger; this follows from the intercept equation

$$-a - I_0 - G - NX$$

$$b$$
If one fixes $Y$ and raises $G$, it must be the case that the associated $r$ on the IS curve is increased. In other words, an increase in $G$ shifts the IS curve up, as illustrated in Figure 3.

Intuitively, if $Y$ is fixed and $G$ increases, the only way to preserve the output equilibrium is for interest rates to rise in order to reduce investment to cancel out the demand increase induced by higher government spending.

Key Idea: If output is constant, increases in government purchases will induce interest rate changes that reduce investment. This is known as crowding out.

Since investment affects the productive capacity of the economy, this is an important possibility.
Figure 3

IS curves for alternative levels of government purchases.