We are now in a position to link the IS and LM equations and produce an integrated view of the money market and output market.

Recall that the IS equation is

\[ Y = C(Y) + I(r) + G + NX \]

and the LM equation is

\[ \frac{M}{P} = L(r + \pi, Y) \]
This gives us 2 equations in 2 unknowns: $r$ and $Y$. Hence, the two equations will (subject to uninteresting mathematical conditions) provide an output level and an interest rate level such that both the goods market and the money market are in equilibrium.

To see this, let’s graph both the IS curve and the LM curve using $r$ and $Y$ axes.

This is done in Figure 1.

What is important about this Figure?

We have a joint interest rate/output equilibrium!
Figure 1

IS/LM Equilibrium

\[ \frac{M}{P} = L(r + \pi, Y) \]

\[ IS: Y = C(Y) + I(r) + G + NX \]
The IS/LM framework allows us to discuss the effects of different combinations of monetary and fiscal policy. You will be expected to be able to do this for midterm 2.

**Example 1. Increase in Government Spending**

Suppose that the level of $G$ increases whereas the level of $M$ is held constant.

According to our discussion of how to shift the IS and LM curves, this means that the LM curve does not move whereas the IS curve is shifted upwards, as illustrated in Figure 2.
Figure 2

Illustration of how an increase in G raises equilibrium output and interest rates.
Example 2. Increase in the Money Supply

For our next exercise, suppose that $M$ increases whereas $G$ is held fixed.

This means that the IS curve does not shift. The LM curve shifts to the right: for each level of interest rates, a higher level of income is needed for money market equilibrium.

Equivalently, the LM curve shifts down; for each level of income, a lower interest is needed for money market equilibrium.

We can conclude that $Y$ increases and $r$ decreases.
Example 3. Increase in $G$ and $M$

Suppose that $M$ and $G$ both increase. In this case, the equilibrium effects on $r$ cannot be determined a priori, whereas the effect on $Y$ can be determined.

The increase in $M$ shifts the LM curve down, lowering $r$ and raising $Y$.

The shift in $G$ shifts the IS curve up, raising $r$ and raising $Y$.

We can conclude that $Y$ will increase. However, we cannot tell whether $r$ will increase, decrease, or remain unchanged, unless we have additional information about the size of the policy shifts and the parameters of the IS/LM model.
Example 4 Increasing $M$ and lowering $G$

Suppose that $M$ increases and $G$ decreases. In this case, the equilibrium effects on $Y$ cannot be determined a priori.

The increase in $M$ shifts the LM curve down, lowering $r$ and raising $Y$.

The shift in $G$ shifts the IS curve down, lowering $r$ and lowering $Y$.

We can conclude that $r$ will decrease. However, we cannot tell whether $Y$ will increase, decrease, or remain unchanged, unless we have additional information about the size of the policy shifts and the parameters of the IS/LM model.
Enriching the IS/LM Model

Given the basic framework, we can provide a qualitative analysis of how different factors we have so far ignored affect equilibrium output and interest rates.

Example: Future taxes.

How might an increase in future taxes affect the IS/LM equilibrium? At this point, we cannot really say, since we have not explicitly incorporated future taxes into the model. Let’s see how we might do this.
Comment: I am assuming that future taxes are known with certainty. In general, consumption today depends on expectations of future taxes.

A natural place to introduce future taxes is the autonomous consumption term \( a \). This is a component of the autonomous part of expenditure, \( A \).

Higher future taxes are likely lower current expenditures via a reduction in \( a \).

A reduction in \( a \) shifts the IS curve down (make sure you understand why!), as illustrated in Figure 3. As the figure indicates, an increase in future taxes will lower current output and lower current interest rates.

To be more precise about this would require an explicit theory of intertemporal consumption.
Figure 3
Effect of future taxes on IS/LM equilibrium
Financial Markets and Risk

So far, we have discussed risk free assets, i.e. there is no uncertainty in their returns. However, the returns on virtually all assets are risky.

How does one think of risk? Here is one way. Suppose we have an assets $a$ and $b$ with associated returns $r_{a,t}$ and $r_{b,t}$ at time $t$. What is the relationship between them?
To answer such a question, it is necessary to distinguish between the expected return on an asset and its actual return.

By expected, I refer to a particular mathematical property, but we need not worry about what it means at this point. Suffice it to say the expected return is related to what one expects to happen on average.

What does matter is that we will want to think about the return on an asset \( a \) as consisting of two parts: the expected return and a residual that reflects the fact that it is risky.

In other words:

\[
 r_{a,t} = E_t \left( r_{a,t} \right) + \varepsilon_{a,t}
\]
Comment: $E_t(\varepsilon_{a,t})$ always equals 0.

To say that asset $a$ is risk free is to say that $\varepsilon_{a,t}$ always equals zero.

The riskiness of an asset will depend upon the properties of $\varepsilon_{a,t}$. Further, we would intuitively expect that riskier assets are associated with higher expected returns? Why? How is an individual compensated for holding a riskier asset? By higher expected returns!

But how does one compare the riskiness of different assets?
One way to evaluate riskiness is in terms of the variance of $\varepsilon_{a,t}$ (denoted as $\sigma_{a,t}^2$). The term variance has a precise mathematical definition, but for our purposes it is sufficient to understand it as measuring how much $\varepsilon_{a,t}$ moves around 0 (its expected value.)

Example. Suppose that $\varepsilon_{a,t} = 100$ with probability 1/2 and $\varepsilon_{a,t} = -100$ with probability 1/2. Suppose we compare it to an $\varepsilon_{b,t}$ which has the property that $\varepsilon_{b,t} = 10$ with probability 1/2 and $\varepsilon_{b,t} = -10$ with probability 1/2.

$\varepsilon_{a,t}$ has a higher variance than $\varepsilon_{b,t}$, even though each has an expected value of 0.

But does this imply that $a$ is a riskier asset than $b$?
In general, the answer is no.

Recall the following question from Problem Set 1:

3. Suppose you are worker at Ford motor company. Assume that stock in the Ford Motor company has an expected return of 5%; however, with probability .5 the return is 4% and with probability .5 the return is 6%. In contrast, Honda stock has an expected return of 5%; with probability .5 the return is 3.8% and with probability .5 and 6.2% with probability .5. Which stock would you buy? Would you answer differ if you were a college professor? Relate your answer to the issue of risk versus return in personal investments.

The purpose of the question was to indicate that risk cannot be properly measured independently of the other factors.
Key idea: Risk of one asset is determined by the covariance of the asset with the rest of your portfolio. Note, covariance is denoted $\sigma_{a,b,t}$.
The covariance of assets $a$ and $b$ measures whether the returns on the assets move together or not. When the covariance is positive, then $\varepsilon_{a,t}$ tends to be high when $\varepsilon_{b,t}$ is high, etc.

So, for the Ford worker, Honda stock may be less risky than Ford stock because Honda stock diversifies his portfolio, i.e. off at higher rates when the rest of his portfolio is doing badly. In portfolio, I am including wages!

Notice that a college professor will not assess risk in the same way.
Capital Asset Pricing Model

There is a famous formula that relates the risk of assets to their expected returns in a financial market equilibrium, known as the capital asset pricing model.

This formula does not fully (empirically) explain the return structure to different assets, but does have explanatory power. It is often used in internal investment calculations.

The formula is based on considering a financial market consisting of many different assets.
Let

\[ E_t(r_{a,t}) \]

denote the expected return of any asset \( a \) given information available at \( t \)

\[ E_t(r_{rf,t}) \]

denote the expected return of a risk free asset \( rf \) (This may not exist, but one can infer what the expected return would be if it did).

\[ E_t(r_{m,t}) \]

denote the expected return on the market as a whole. Think of a market portfolio made up of all the outstanding stocks, for example.
The Capital Asset Pricing Model (CAPM) states that the equilibrium expected asset returns will have the following relationship:

\[ E_t(\mathbf{r}_{a,t}) = E_t(\mathbf{r}_{rf,t}) + \frac{\sigma_{a,m,t}}{\sigma_{m,t}^2}(\mathbf{r}_{m,t} - \mathbf{r}_{rf,t}) \]

I will not derive this formula. What is essential is the term \( \frac{\sigma_{a,m,t}}{\sigma_{m,t}^2} \). This is also known as \( \beta_a \) and measures the riskiness of asset \( a \) relative to the market. The key idea is that assets that covary positively with the market return of those assets that are riskier!!
Term Structure of Interest Rates

We have discussed the relationship between bond prices for interest rates for a one period bonds. How do we think about interest rates and longer period bond?

Suppose that a bond will pay $100 2 periods from now. Suppose the bond has a price of $P_2$. The implicit 2 period interest rate, $i_{2,t}$, is defined by:

$$P_{2,t} = \frac{100}{(1+i_{2,t}) \times (1+i_{2,t})}$$

The implicit interest rate on the bond is what is produced a constant return for two periods
For a $K$-period bond (one that pays $100\ K$ periods in the future, the implicit interest rate is defined by

$$P_{K,t} = \frac{100}{(1+i_{K,t})^K}$$

What is the relationship between the interest rates for bonds of different maturities?

One way to think about this is to consider the alternative strategy to buying a long term bond. For a two period bond, for example, one can buy a 1 period bond and reinvest. Suppose that an agent is risk neutral, so he/she only cares about the expected return. Let $E(i_{t+1})$ denote the agent’s expectation about the one period interest rate at $t+1$. 

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Then interest rates much have the property that all investment strategies produce the same expected value.

This means

\[(1 + i_t) \times (1 + E(i_{t+1})) = (1 + i_{2,t}) \times (1 + i_{2,t})\]

This equality implies that approximately

\[i_{2,t} = \frac{1}{2} \left( i_t + E_t(i_{t+1}) \right)\]

This holds for bonds of different maturities.

\[i_{K,t} = \frac{1}{K} \left( \sum_{j=0}^{K-1} E_t(i_{t+j}) \right)\]

Note that \(E_t(i_t) = i_t\) always holds.
This is known as the term structure of interest rates.

Key idea: long term interest rates reflect beliefs about short term interest rates.

If one graphs $i_{K,t}$ against $K$, one has the yield curve. An upward sloping yield curve implies short term rates are expected to rise in the future.

Comment: one can generalize the basic term structure to include risk premia. Suppose that $\phi_{K,t}$ denotes the risk premium for $K$ period bonds, then a risk adjusted term structure equation is

$$i_{K,t} = \phi_{K,t} + \frac{1}{K} \left( \sum_{j=0}^{K-1} i_{t+j}^e \right)$$