1. Suppose that the one period ahead interest rate $r_{1,t}$ is an AR(1) process with AR coefficient less than 1 in magnitude. Suppose that the two period ahead interest rate, $r_{2,t}$, obeys the pure expectations-based term structure model

$$r_{2,t} = \frac{1}{2}(r_{1,t} + E_t(r_{1,t+1}))$$

Determine whether each of the following must hold, can hold, or cannot hold.

a. $r_{1,t} > r_{2,t}$

b. $\text{var}(r_{1,t}) > \text{var}(r_{2,t})$

c. $\text{var}(r_{1,t+1} - E_t(r_{1,t+1})) > \text{var}(r_{2,t+1} - E_t(r_{2,t+1}))$

2. Denote labor income as $y_t$ and labor wealth as $W_t = E_t\left(\sum_{j=0}^{\infty} \beta^j y_{t+j}\right)$. Provide a formula for $W_t - W_{t-1}$ in terms of current and past labor income.

3. To assess the hypothesis that stock prices are a random walk, propose a test based on Hilbert space projections.

4. Let $x_t = \alpha(L)\epsilon_t$ denote the fundamental MA representation of a process and $x_t = \beta(L)\eta_t$ any non-fundamental MA representation. What must be true about the relationship between $\sigma^2_\epsilon$ and $\sigma^2_\eta$? Interpret.
5. Suppose that output $y_t$ is determined entirely by the history of technology shocks, $\theta_t$, i.e. $y_t = \pi(L)\theta_t$. Can these be recovered from $H_t(y)$? Explain.