1. Suppose that the output does not Granger cause the money supply, so that in a bivariate vector autoregression

\[ m_t = \pi(L)m_{t-1} + \eta_t \]

Suppose that one wants to distinguish between two theories concerning the evolution of output. In the first theory, the behavioral equation for output is

\[ y_t = \alpha(L)y_{t-1} + \beta(L)E\left(m_{t-1}|F_{t-1}\right) + \varepsilon_t \]

so that expected money supply movements affect output; note that \( E\left(m_{t-1}|F_{t-1}\right) = m_{t-1} \). In the second theory, the behavioral equation is

\[ y_t = \alpha(L)y_{t-1} + \beta(L)(m_{t-1} - E\left(m_{t-1}|F_{t-2}\right)) + \varepsilon_t \]

so that output only affected by the unexpected movements in the money supply.

a. Given the joint history of output and money, can one determine which of these behavioral equations correctly describes the money supply? (Comment, this is a variant of a classic paper by Sargent.)

b. Suppose the money supply process is changed at time \( T \) to

\[ m_t = \chi(L)m_{t-1} + \eta_t \]

could one then determine which behavioral equation is correct given observations on money and output?

2. Suppose that behavioral equation for output is
\[ y_t = \alpha(L)y_{t-1} + \beta(L)E(m_{t-1}|F_{t-1}) + \varepsilon_t \]

a. If the central bank is choosing money supply rules of the form

\[ m_t = \chi(L)m_{t-1} + \theta(L)y_{t-1} \]

which choice of lag polynomials in the money supply rule will minimize \( \text{var}(y_t) \)?

b. what will a bivariate vector autoregression reveal about the interrelationship of money and output?

3. Suppose that \( x_t \) is an AR(1) process. Suppose that \( x_t \) is measured with error, so that the observable \( x_t^* = x_t + \nu_t \); \( \nu_t \) is MA\((k)\) and \( E(x_t \nu_{t+j}) = 0 \ \forall j \). Characterize the spectral density of \( x_t^* \) and in terms of the AR and MA coefficients of the underlying components of \( x_t^* \). What type of ARMA process is \( x_t^* \)?

4. The hyperinflation model due to Philip Cagan consists of two parts. First, equilibrium the log of demand for money is determined by the expected inflation rate, i.e. the expected inflation rate, i.e. the expected change in log prices. Letting \( m_t \) denote the log of the nominal money supply and \( p_t \) the log of the price level, the model is thus

\[ m_t - p_t = \beta E_{\pi,t} (p_{t+1} - p_t) \]

\( \beta < 0 \). Note \( E_{\pi,t} \) denotes subjective expectation.

Assuming expectations are rational, i.e. \( E_{\pi,t} \) may be replaced with a mathematical expectation, how are prices determined by the money supply? Explain.

b. Suppose expectations are not rational, in that inflationary expectations are autoregressive, i.e.

\[ E_{\pi,t} (p_{t+1} - p_t) = \pi (p_t - p_{t-1}) \]

How are prices determined by the money supply. Contrast with your previous answer.