Answers to Problem Set 2

1. From the description of the problem,

\[ x_t^* = \rho x_{t-1}^* = \epsilon_t + \eta_t - \rho \epsilon_{t-1} \]

The process \( \epsilon_t + \eta_t - \rho \epsilon_{t-1} \) is MA(1), therefore \( x_t^* \) is ARMA (1,1).

2. \( x_{t+k} = \rho^k x_t \). Therefore, \( x_{t+k} - x_{t+k-1} = \rho^k x_{t-k} - \rho^{k+1} x_{t-k-1} \), however, since \( x_{t-k} = \rho x_{t-k-1} + \epsilon_{t-k} \), \( x_{t+k} - x_{t+k-1} = \rho^k \epsilon_{t-k} \), which is orthogonal to \( x_{t-k-1} \).

3. The cases \( x_t = \epsilon \) and \( x_t = k \) are observationally equivalent, i.e. a realization of the \( x \) process cannot distinguish them. Any draw of \( \epsilon \) can be asserted to be the value of \( k \). The case \( x_t = \epsilon \) has the property that it does not obey the law of large numbers, i.e. it is an example of a nonergodic process.

4. Since the two processes possess identical autocovariance functions, they must possess identical Wold moving average representations. This means \( \text{var} \left( x_t - x_{t+k} \right) = \text{var} \left( y_t - y_{t+k} \right) \) since the terms in the variance expressions are functions of the respective Wold MA representations.