Exercise 1: Demand Elasticity.

Consider a monopoly facing the following demand curve:

\[ y(p) = 10 - p \]

- (a) Find the inverse demand \( p(y) \) and marginal revenue \( MR(y) \) functions and graph them. Why, intuitively, is the marginal revenue curve below the demand curve?
  
  * Inverse demand is \( p(y) = 10 - y \)
  * Marginal revenue is \( TR'(y) \): Since \( TR(y) = p(y) \cdot y = (10 - y) \cdot y = 10y - y^2 \), we have \( TR'(y) = MR(y) = 10 - 2y \)
  * Why MR is below demand: To sell an additional unit, since demand is downward sloping, the firm must lower the price on that unit and all the previous units. So the overall addition to revenue can't be as high as the price of that additional unit \( p \).

- (b) Find the elasticity of demand \( \varepsilon(y) \) and calculate it for \( y = 0, 5, \) and 10.

  **Elasticity of demand is the percent change in quantity demanded resulting from a percent change in price:**

  \[ \varepsilon(y) = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = y(p) \cdot \frac{p}{y} \]

  \[ \Rightarrow \varepsilon(y) = (-1) \cdot \frac{10 - y}{y} \]

  So \( \varepsilon(0) = \frac{-10-0}{0} = -\infty \)

  \( \varepsilon(5) = -\frac{10-5}{5} = -1 \)

  \( \varepsilon(10) = -\frac{10-10}{10} = 0 \)

- (c) What is elasticity \( \varepsilon(y) \) when \( MR(y) = 0 \)? What is elasticity \( \varepsilon(y) \) when \( MR(y) > 0 \)?

  * First \( MR(y) = 0 \Rightarrow 10 - 2y = 0 \Rightarrow y = 5 \)
  
  * We found above that \( \varepsilon(5) = -1 \)

  Finding \( \varepsilon(y) \) when \( MR(y) > 0 \) takes a few steps:

  **In the end, we want an equation that relates \( \varepsilon \) and \( MR \)**

  We know that \( MR = TR' \) and that \( TR = p(y) \cdot y \) so \( MR(y) = TR'(y) = p(y) + y \cdot p'(y) \) (product rule). \( \Rightarrow MR(y) = p(y) + y \cdot p'(y) \)

  Why? This is one of those problems where the only reason we're expressing something a certain way is to eventually get it to look like something else. Here, it's MR in terms of \( \varepsilon \). Don't be too discouraged if that step isn't obvious—it's sort of a trick.

  So \( MR(y) > 0 \Rightarrow p(1 + \frac{1}{\varepsilon}) > 0 \Rightarrow \varepsilon > 1 \) and demand is elastic whenever \( MR(y) > 0 \).
• (d) What will be the markup over $MC$ at the profit maximizing level of output $y$ in terms of elasticity $\varepsilon(p)$?

We'll produce where $MR = MC$. From part (c) we have

\[ MR = p [1 + \frac{1}{\varepsilon(p)}], \text{ so } MR = MC \implies p [1 + \frac{1}{\varepsilon(p)}] = MC \]

\[ \implies p = MC \left[ 1 + \frac{1}{\varepsilon(p)} \right] \text{ mark up in terms of } \varepsilon(p) \]

Exercise 2: First-Degree Price Discrimination.

Suppose the firm above has marginal cost $MC = 1$.

• (a) What is its profit level with no price discrimination?

Producing where $M\Pi(y) = MC(y) \implies 10 - 2y = 1 \implies y = 4.5 \implies p(y) = 10 - y = 5.5$

\[ \Pi = TR-TC = 4.5(5.5) - 4.5 = 20.25 \]

• (b) What is its profit if it can perfectly price discriminate (first-degree)?

Now $MR$ is $p$, so producing where $MR = MC \implies p(y) = MC(y) \implies 10 - y = 1 \implies y = 9$. Profit is revenue-cost for every unit, which is the triangular area $\Pi$:

\[ \Pi = \frac{1}{2} (10-1)(9) = 40.5 \]

• (c) Is the allocation Pareto efficient (show graphically)?

Yes - there's no deadweight loss. Here, producer surplus = total surplus. No other mutually beneficial trades could be made.

Exercise 3: Third-Degree Price Discrimination.

Cupcake monopolist Marc's Minicakes can produce an additional cupcake for $MC = 1$. There are two types of potential buyers in the cupcake market with the following inverse demand functions:

(i) $p^F(y) = 8 - \frac{7}{10}y$ (people who know the whole cupcake fad is over)

(ii) $p^B(y) = 10 - \frac{3}{10}y$ (the believers/addicts)
• (a) Find the equilibrium prices and profit when Marc’s Minicakes’ can distinguish between the two types of buyers.

We can treat this as two different markets:

(1) Fad is over:

\[ MR_F(y) = 8 - \frac{14}{10} y \]

*Producing at the quantity where \( MR_F(y) = MC(y) \) \( \Rightarrow \) \( 8 - \frac{14}{10} y = 1 \)

\[ p^F(y) = 8 - \frac{7}{10} (5) = 4.5 = p^F \]

\[ \Pi = 5(4.5) + 5(1) = 17.5 = \Pi^F \]

(2) Believers:

\[ MR_B(y) = 10 - \frac{6}{10} y, \text{ and } MR_B(y) = MC(y) \Rightarrow 10 - \frac{6}{10} y = 1 \]

\[ p^B(y) = 10 - \frac{3}{5} (15) = 5.5 = p^B \]

\[ \Pi^B = 15(5.5) - 15 = 67.5 = \Pi^B \]

Total profit is 17.5 + 67.5 = 85.

• (b) How do you go about finding profit when the two types of buyers are indistinguishable? Would you expect profit to be lower or higher than in part (a)?

- Add demand curves to get market demand, \( y^{\text{total}}(p) \) (not inverse demand curves p(y))

- Produce where \( MR(y) = MC(y) \)