ECON 302: Intermediate Macroeconomics

Midterm Examination 2

Instructions

1. This is a 75 minutes examination worth 100 total points
2. Notes and books will not be permitted
3. Please write your answers as clearly and concisely as possible.

Exercise 1 (37 Points)

Consider the following Solow model. Total output is given by

\[ Y = zK^{\frac{1}{3}}N^{\frac{2}{3}} \]

where \( K \) is the aggregate capital stock, \( N \) denotes the workforce, and \( z \) denotes total factor productivity. Assume that \( z = 1 \). Saving/investment is equal to 10 percent of output. Capital depreciates at a rate of 8 percent per year, while the workforce grows at a rate of 2 percent a year.

1. (5 points) Derive the per worker production function, \( y = zf(k) \), where \( y \) and \( k \) are output and capital per worker, respectively.

2. (12 points) In the Solow model, the steady-state capital stock, \( k^* \), will satisfy

\[ s\cdot zf(k^*) = (n + d)k^* \]

where \( s \) denotes the saving rate, \( n \) denotes the growth rate of the labor force, and \( d \) denotes the depreciation rate. Calculate the steady state quantities \( k^* \), \( y^* \) and \( c^* \).

3. (20 points) Suppose the saving rate increases and is now equal to 20 percent of output
   (a) Use a graph with \( k \) on the x-axis to illustrate how \( k^* \) changes in the long-run. Calculate \( k^* \), \( y^* \) and \( c^* \).
   (b) Explain with words (without calculation) what is the impact on the growth rate of capital per capita in the short run and in the long run. Illustrate with a graph with time on the x-axis and \( k \) on the y-axis.

Exercise 2 (33 Points)

Consider the two-period problem of the representative consumer and assume the consumer has current-period net income \( y = 0 \), and future-period net income \( y' = 110 \), and faces a market real interest rate of \( r = 0.1 \). The consumer’s preferences over \( c \) and \( c' \) are represented by the following utility function:

\[ U(c, c') = \log c + \log c' \]
The associated \( MRS \) is

\[
MRS = \frac{c'}{c}
\]

1. Assume there are no taxes in the present nor the future: \( t' = t = 0 \).
   
   (a) (5 points) Calculate the consumer’s lifetime wealth.
   
   (b) (5 points) What is the endowment point of this consumer?
   
   (c) (10 points) Calculate the consumer’s optimal current consumption, future consumption and savings. Is the consumer a lender or a borrower?

2. The government increases the interest rate to 0.2. Calculate life-time wealth, optimal current consumption, future consumption and savings. Explain the two forces at play here. (13 points)

Question 1 (10 Points)

There is empirical evidence that income per worker is converging in the richest countries. Interpret this fact using the solow model.

Question 2 (10 Points)

What is the impact of an improvement in technology on population and consumption per capita in the Malthusian model?

Question 3 (10 Points)

What produces a larger increase in consumer’s current consumption, a transitory tax cut or a permanent tax cut? Briefly explain.