Econ 690
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Answers to Homework 10

1. In the following questions, assume that the one-year spot rate, $r(1)$ is 0.06. For the two-year rate, assume $(1+r(2))^2 = 1.11$; and, the three-year spot-rate satisfies $(1+r(3))^3 = 1.17$.

In the following problems, calculate the price of a bond that promises a coupon of 10 in periods 1, 2, and 3, and has a face value of 100 (so that it pays 100 at the end of the third period, in addition to the coupon of 10.)

a. What is the price of this bond if the probability of default is zero?

Answer: $V = \frac{10}{1.06} + \frac{10}{1.11} + \frac{110}{1.17} = 112.46$

b. Now suppose that there is a constant probability 0.15 that the borrower will default and not repay at the end of each period. If he defaults, he will repay nothing in that period or any future period. What is the price of this bond?

Answer: The probability of repayment in period *j* is $(0.85)^j$. The price of the bond is given by:

$$V = .85\frac{10}{1.06} + (.85)^2\frac{10}{1.11} + (.85)^3\frac{110}{1.17} = 72.266$$

c. Here the question is the same as in part b, but in the event of default, the recovery value is 30. What is the price of this bond?

Answer: The probability of receiving the recovery value in period 1 is 0.15. The probability of receiving the recovery value in period 2 is $0.15 \cdot 0.85$. The probability of receiving the recovery value in period 3 is $0.15 \cdot (0.85)^2$.

The price of the bond is given by:

$$V = .85\frac{10}{1.06} + .15\frac{30}{1.06} + (.85)^2\frac{10}{1.11} + (.15\times.85)\frac{30}{1.11} + (.85)^3\frac{110}{1.17} + (.15\times.85^2)\frac{30}{1.17} = 82.7362$$

d. Now suppose we have the same payoffs, probability of default, and recovery value as in part c, but exactly half of the coupon and face value is collateralized by a bond that has a zero probability of default. That means the other half (coupons of 5 each period, and face value of 50) is not collateralized, has a probability of default of 0.15, and has a recovery value of 30. What is the stripped value of the bond? What is the price of the bond (the sum of the stripped value and the value of the collateral)?

Answer:

The stripped value is calculated as:

$$StV = .85\frac{5}{1.06} + .15\frac{30}{1.06} + (.85)^2\frac{5}{1.11} + (.15\times.85)\frac{30}{1.11} + (.85)^3\frac{55}{1.17} + (.15\times.85^2)\frac{30}{1.17} = 46.603$$

The value of the collateral is exactly half the answer to part a, equal to 56.230.
So the price of the collateralized bond is 102.833.

2. This question simply asks you to do some numerical examples of the model of sovereign default from the lecture slides. It uses the notation and model as those lecture slides.

Assume $L = 1.5$ and $H = 4$. Assume $p = 0.5$

a. Using the model where the borrower can commit, what are D, C_1, C_{2L} and C_{2H}?

Answer: We found in the chapter that $D = 1$ and therefore $C_1 = 1$. Then
$C_{2L} = L - D = 1.5 - 1 = 0.5$ and $C_{2H} = H - D = 4 - 1 = 3$

b. Suppose $k = 0.9$. What is kL?
In that case, in the model of no commitment, what are D, C_1, C_{2L} and C_{2H}?

Answer: $kL = 1.35$. Because $kL > 1$, the punishment is sufficiently high that the borrower will not ever default. He will borrow at a rate $R^p = 1$ and $D = 1$, $C_1 = 1$, $C_{2L} = 0.5$, $C_{2H} = 3$