Econ 690
Spring, 2019
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Homework #3

1. If the spot exchange rate of the yen relative to the dollar is ¥109.75, and the 90-day forward rate is ¥107.25/$, is the dollar at a forward premium or discount? Express the premium or discount as a percentage per annum for a 360-day year.

   *Answer:* When the forward rate of yen per dollar is less than the spot rate of yen per dollar, the dollar is said to be at a discount in the forward market. The magnitude of the discount is expressed in percentage per annum by dividing the difference between the forward rate and the spot rate by the spot rate and multiplying by reciprocal of the fraction of the year corresponding to the maturity of the forward contract (360/N days) and by 100. Thus, the annualized forward discount is 9.11% because

   $$\frac{(¥107.25/\$ \;-\; ¥109.75/\$)}{¥109.75/\$} \times \frac{360}{90} \times 100 = -9.11\ \%$$

   Notice that the word "discount" implies that the forward rate is less than the spot rate.

2. As a foreign exchange trader for JPMorgan Chase, you have just called a trader at UBS to get quotes for the British pound for the spot, 30-day, 60-day, and 90-day forward rates. Your UBS counterpart stated, "We trade sterling at $1.2945-50, 47/44, 88/81, 125/115." What cash flows would you pay and receive if you do a forward foreign exchange swap in which you swap into £5,000,000 at the 30-day rate and out of £5,000,000 at the 90-day rate? What must be the relationship between dollar interest rates and pound sterling interest rates?

   *Answer:* The fact that you are swapping into £5,000,000 at the 30-day rate forward rate means that you are paying dollars and buying pounds. You would do this transaction at the bank's 30-day forward ask rate. To find the forward ask rate, you must realize that the 30-day forward points of 47/44 indicate the amounts that must be subtracted from the spot bid and ask quotes to get the forward rates. We know to subtract the points because the first forward point is greater than the second. Hence, the first part of the swap would be done at $1.2950/£ - $0.0044/£ = $1.2906/£. Therefore, to buy £5,000,000 you would pay

   $$\$1.2906/£ \times £5{,}000{,}000 = \$6{,}453{,}000$$

   In the second leg of the swap, you would sell £5,000,000 for dollars in the 90-day forward market. Because you are selling pound for dollars, you transact at the 90-day forward bid rate of $1.2945/£ - $0.0125/£ = $1.2820/£. Therefore, you would receive

   $$\$1.2820/£ \times £5{,}000{,}000 = \$6{,}410{,}000$$

   Notice that you get back fewer dollars than you paid, but you had use of £5,000,000 for 60 days. Thus, the pound must be the higher interest rate currency.

3. Consider the following spot and forward rates for the yen–euro exchange rates:

| Spot | 30 days | 60 days | 90 days | 180 days | 360 days |
|---|---|---|---|---|---|
| 109.30 | 108.75 | 108.15 | 106.75 | 106.37 | 100.85 |

Is the euro at a forward premium or discount? What are the magnitudes of the forward premiums or discounts when quoted in percentage per annum for a 360-day year?

*Answer:* The forward rates of yen per euro are lower than the spot rates. Therefore, the euro is at a discount in the forward market. The annualized forward premium or discount for the N day forward contract is

$$\frac{F-S}{S} \times \frac{360}{N\ days} \times 100$$

If the value of this calculation is negative, say -2%, we say there is a 2% discount. Therefore, the discounts are 6.04% for 30 days, 6.31% for 60 days, 9.33% for 90 days, 5.36% for 180 days, and 7.73% for 360 days.

4. Download the monthly U.S./U.K. exchange rate for January 1975 – December 2018 from FRED, the Federal Reserve Bank of St. Louis data base: https://fred.stlouisfed.org/

   a. Plot the data on a graph.

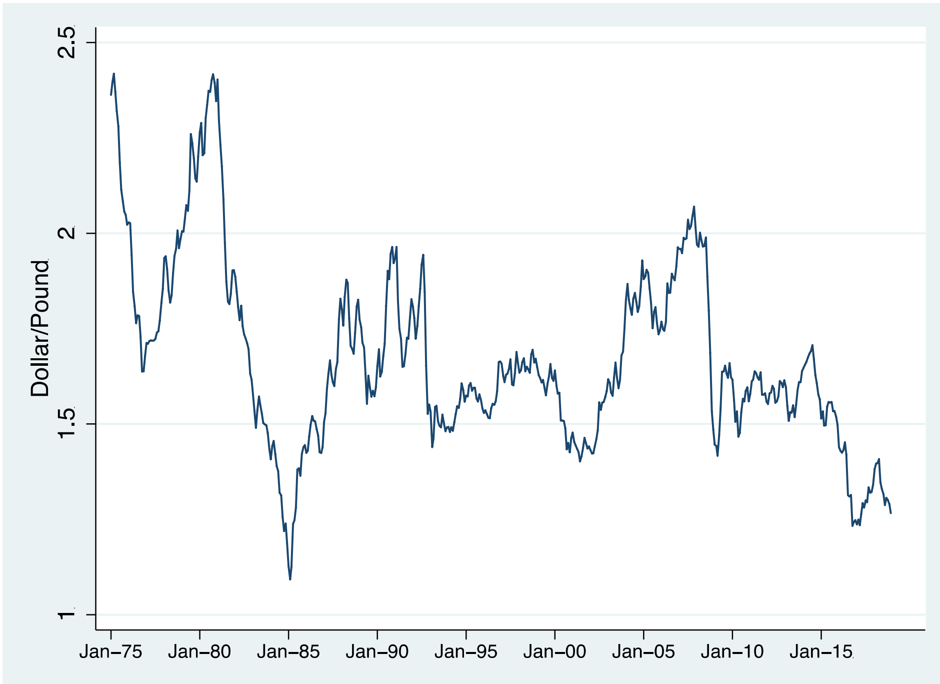
   b. Now take the natural log of the exchange rate. Then for each month, starting in February 1975, calculate the change in the log of the exchange rate: $\ln(S_t)-\ln(S_{t-1})$. Then report the mean and the standard deviation of your values for $\ln(S_t)-\ln(S_{t-1})$.

   c. If the exchange rate today is $S_t = \$1.30$ per pound, what is $\ln(S_t)$? Using your calculation for the mean of $\ln(S_t)-\ln(S_{t-1})$, what value do you expect for the log of the exchange rate in one month, given that $S_t = \$1.30$ ? (That is, what is your expectation of $\ln(S_{t+1})$?) What value do you expect for the level (that is, $S_{t+1}$, not $\ln(S_{t+1})$) of the exchange rate?

   d. If the change log of the exchange rate has a Normal distribution, then 95.45% of the time, the actual value of the change in the log of the exchange rate will be in a range of ± two standard deviations of its mean value. Given this knowledge, what is the 95.45% range of your predictions for $\ln(S_{t+1})$? What is the 95.45% range of your prediction for $S_{t+1}$?

*Answer:*

a.




b. Using the data sample, the mean of monthly change in the log of exchange rate is -0.0012 and the standard deviation of $\ln(S_t) - \ln(S_{t-1})$ is 0.0241.

c. If today's exchange rate is \$1.30 per U.K. pound, then the log value of 1.30 is ln(1.30) = 0.2624. Since $\ln(S_{t+1}) = \ln(S_t) + \ln(S_{t+1}) - \ln(S_t)$, the conditional mean of $\ln(S_{t+1})$ given today's exchange rate is

$$E_t[\ln(S_{t+1})] = \ln(S_t) + E_t\left[\ln(S_{t+1}) - \ln(S_t)\right].$$

From the data sample, we know the mean of $\ln(S_t) - \ln(S_{t-1})$ is -0.0012 so that we could expect $\ln(S_{t+1})$ - $\ln(S_t)$ will be the same as -0.0012. Therefore, the expected value of $\ln(S_{t+1})$ is

$$0.2624 - 0.0012 = 0.2612.$$

Since

$$E_t\left[\ln(S_{t+1}) - \ln(S_t)\right] \approx E_t\left[\frac{S_{t+1} - S_t}{S_t}\right]$$

the expected value of $S_{t+1}$ would be

$$E_t[S_{t+1}] = S_t \times \left(1 + E_t\left[\frac{S_{t+1} - S_t}{S_t}\right]\right) \approx S_t \times \left(1 + E_t\left[\ln(S_{t+1}) - \ln(S_t)\right]\right)$$

Thus, the expected value of $S_{t+1}$ is

$$1.30 \times (1 - 0.0012) = 1.2984$$

The other way to compute the expected value of $S_{t+1}$ is just to take exponential, e, raised to the power of the log. Specifically, taking $E_t(ln\, S_{t+1})$, you could simply do

$$\exp\big(E_t(ln\, S_{t+1})\big) \;\approx\; E_t[S_{t+1}].$$

This gives the expected value of $S_{t+1}$

$$\exp(0.2612) = 1.2984,$$

which is the same answer as the first way.

d. Given the expectation of $\ln\left(S_{t+1}\right)$ in hand, we could calculate the range from

0.2612 – 2 * 0.0241 = 0.213 to 0.2612 + 2 * 0.0241 = 0.3094.

Thus, the range of the log future exchange rates that encompasses all but 4.55% of the future possible values of exchange rates is 0.213 to 0.3094.

For the level of the exchange rate, the conditional standard deviation is

$$\sigma_t[S_{t+1}] = S_t \times \sigma_t\left[1 + \frac{S_{t+1}-S_t}{S_t}\right] = S_t \times \sigma_t[\frac{S_{t+1}-S_t}{S_t}]$$

Again, using the relation $\ln\left(S_{t+1}\right) - \ln\left(S_t\right) \approx \frac{S_{t+1}-S_t}{S_t}$, we have

$$\sigma_t[S_{t+1}] = 1.30 \times 0.0241 = 0.0313.$$

Therefore, the corresponding range around its mean is from 1.2984 – 2*0.0313 = 1.2358 to 1.2984 + 2 * 0.0313 = 1.361.

As in part c), the other simpler way is to take exponential. Then we have the corresponding range from exp(0.213) =1.237 to exp(0.3094) = 1.363, which is similar to the answer from the first way.