Econ 690
Spring 2019
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Answers to Homework 6

1. Suppose r_P is the return on a portfolio of risky assets. The portfolio consists of two assets that pay return r_1 and r_2 respectively. The expected returns on the two assets are m_1 and m_2 (that is, $m_1 \equiv E(r_1)$ and $m_2 \equiv E(r_2)$.) The share of the portfolio in asset 1 is *a*, and the share in asset 2 is $1-a$. So

$$r_p = ar_1 + (1-a)r_2 .$$

Let v_1 be the variance of the return on asset 1 and v_2 be the variance of the return on asset 2. Let c_{12} be their covariance. (That is, $v_1 = \text{var}(r_1)$, $v_2 = \text{var}(r_2)$, and $c_{12} = \text{cov}(r_1, r_2)$.)

We know from the properties of expectations and variances that:

$$E\left(r_p\right) = am_1 + (1-a)m_2$$

$$\text{var}\left(r_P\right) = a^2 v_1 + (1-a)^2 v_2 + 2a(1-a)c_{12}$$

a. Find the value of *a* that maximizes $\frac{\left(E(r_P)\right)^2}{\text{var}(r_P)}$. Your answer should express *a* in terms of m_1 and m_2, and v_1, v_2, and c_{12}. Please try to simplify your answers (making cancellations) as much as possible.

<u>Answer:</u>

We want to maximize $\frac{\left(E(r_P)\right)^2}{\text{var}(r_P)} = \frac{\left(am_1 + (1-a)m_2\right)^2}{\left(a^2 v_1 + (1-a)^2 v_2 + 2a(1-a)c_{12}\right)}$. Take the derivative with respect to *a* and set to zero:

$$\frac{2(m_1 - m_2)\left(am_1 + (1-a)m_2\right)\left(a^2 v_1 + (1-a)^2 v_2 + 2a(1-a)c_{12}\right) - 2\left(am_1 + (1-a)m_2\right)^2 (a_1 v_1 - (1-a)v_2 + (1-2a)c_{12})}{\left(a^2 v_1 + (1-a)^2 v_2 + 2a(1-a)c_{12}\right)^2} = 0$$

Set the numerator to zero and cancel some terms to get:

$$(m_1 - m_2)\left(a^2 v_1 + (1-a)^2 v_2 + 2a(1-a)c_{12}\right) = \left(am_1 + (1-a)m_2\right)(a_1 v_1 - (1-a)v_2 + (1-2a)c_{12})$$

Cancel some more terms:

$$m_1\left((1-a)v_2 + ac_{12}\right) = m_2(av_1 + (1-a)c_{12})$$

Solve for *a*:

$$a = \frac{v_2 m_1 - c_{12} m_2}{m_1(v_2 - c_{12}) + m_2(v_1 - c_{12})}$$

b. For simplicity, now assume $m \equiv m_1 = m_2$ and $c_{12} = 0$. Write out the solution for *a* in this special case.

<u>Answer:</u>

$$a = \frac{v_2}{v_1 + v_2}$$

c. Continue to assume $m \equiv m_1 = m_2$ and $c_{12} = 0$. Now, let's interpret the problem in the following way. We are looking at the risky portfolio of a home investor. Asset 1 is the foreign bond that pays $i^* + s_{+1} - s$ and asset 2 is an equity that pays r_x. Here, *s* is the log of the exchange rate, and we are writing the approximate return on the foreign investment. At the time the portfolio choice is made, i^* and *s* are known, but the random variables are s_{+1} and r_x. The moments of these random variables are $E(s_{+1})$, $E(r_x)$, $\text{var}(s_{+1})$, $\text{var}(r_x)$, $\text{cov}(s_{+1}, r_x)$. Write your solution to part b, now using the specific interpretation given here.

$$a = \frac{\text{var}(r_x)}{\text{var}(r_x) + \text{var}(s_{+1})}$$

d. Now, use the general formula derived in part a. That is, do not assume $m \equiv m_1 = m_2$ and $c_{12} = 0$. Write that formula using the specific assumptions about what the risky asset are from part c. What variables determine the investors demand for foreign bonds?

$$a = \frac{\text{var}(r_x)\left[E(s_{+1}) - s + i^*\right] - \text{cov}(s_{+1}, r_x)E(r_x)}{\left[E(s_{+1}) - s + i^*\right]\left[\text{var}(r_x) - \text{cov}(s_{+1}, r_x)\right] + E(r_x)\left[\text{var}(s_{+1}) - \text{cov}(s_{+1}, r_x)\right]}$$

The investor will tend to hold more of the foreign bond when it has a higher return relative to the return on the equity, when the variance of the exchange rate is low relative to the variance of the return on the equity, and when the covariance of the exchange rate with the return on the equity is low.