

Answers to Homework 7

Uncovered interest parity implies $\frac{E_t S_{t+1} - S_t}{S_t} = \frac{F_t - S_t}{S_t}$. Here, S_t is the spot exchange rate (expressed as home currency price of foreign currency), and F_t is the forward price for foreign currency one-period hence. We have seen that a common test of this hypothesis is to estimate the parameters a and b in the regression:

$$\frac{S_{t+1} - S_t}{S_t} = a + b \left(\frac{F_t - S_t}{S_t} \right) + u_{t+1}.$$

Under the null hypothesis that uncovered interest parity holds, we have $a = 0$ and $b = 1$.

In fact, while the tests do not reject $a = 0$, they tend to find $b < 1$. In fact, they often find $b < 0$. We might conclude that, contrary to uncovered interest parity, we have:

$$\frac{E_t S_{t+1} - S_t}{S_t} = b \left(\frac{F_t - S_t}{S_t} \right), \text{ with } b < 1.$$

Now, recall that the excess return on the foreign bond, er_{t+1} is defined as:

$$er_{t+1} = \frac{S_{t+1}}{S_t} (1 + i_t^*) - (1 + i_t).$$

Here, i_t^* is the foreign interest rate, and i_t is the home interest rate. The purpose of this problem is to understand what the regression results tell us about the expected excess return, $E_t er_{t+1}$.

We will derive this relationship in steps:

- i. Starting with the relationship $\frac{E_t S_{t+1} - S_t}{S_t} = b \left(\frac{F_t - S_t}{S_t} \right)$, with $b < 1$, subtract $\frac{F_t - S_t}{S_t}$ from both sides of the equation.

Answer:
$$\frac{E_t S_{t+1} - F_t}{S_t} = (b - 1) \left(\frac{F_t - S_t}{S_t} \right)$$

- ii. Using the fact that from covered interest rate parity, $\frac{F_t}{S_t} = \frac{1+i_t}{1+i_t^*}$, replace the expression for the forward premium on the right hand side of the equation with an expression involving only i_t and i_t^* .

Answer:
$$\frac{F_t - S_t}{S_t} = \frac{1+i_t}{1+i_t^*} - 1 = \frac{i_t - i_t^*}{1+i_t^*}.$$

- iii. On the left-hand side of the equation you derived in part (i), show how you can use $\frac{F_t}{S_t} = \frac{1+i_t}{1+i_t^*}$ to write the left hand side as $\frac{1}{1+i_t^*} E_t e r_{t+1}$.

Answer: On the left-hand side we have:

$$\frac{E_t S_{t+1} - F_t}{S_t} = \frac{E_t S_{t+1}}{S_t} - \left(\frac{1+i_t}{1+i_t^*} \right) = \frac{1}{1+i_t^*} \left[\frac{E_t S_{t+1}}{S_t} (1+i_t^*) - (1+i_t) \right] = \frac{1}{1+i_t^*} E_t e r_{t+1}$$

- iv. Now, using your answers to parts (ii) and (iii), rewrite the expression from part (i) so that you can express $E_t e r_{t+1}$ as a linear function of $i_t^* - i_t$. (Cancel the $\frac{1}{1+i_t^*}$ term that would appear on both sides of the equation.)

Answer: We have $\frac{1}{1+i_t^*} E_t e r_{t+1} = (b-1) \frac{i_t - i_t^*}{1+i_t^*}$, so $E_t e r_{t+1} = (1-b)(i_t^* - i_t)$

- v. From part (iv) we see that $E_t e r_{t+1}$, the expected excess return on the foreign bond varies positively with $i_t^* - i_t$ since $b < 1$. But we also know from the CAPM that

$$E_t e r_{t+1} = \beta_t (E_t r_{t+1}^m - i_t).$$

Here, β_t is the “beta” for the risky investment in foreign bonds, and this equation says that the expected excess return on the foreign bond investment is its beta times the expected excess return on the market portfolio. Note that we have written a time t subscript on the beta, allowing for the possibility that the beta could change over time.

In fact, taking the above equation, and the empirical result that we have rewritten in part (iv), we can derive an expression for β_t as a function of the interest rate differential, $i_t^* - i_t$, and the expected excess return on the market, $E_t r_{t+1}^m - i_t$. Derive that expression here.

Answer: From (iv), we have $E_t e r_{t+1} = (1-b)(i_t^* - i_t)$ and from the equation directly above, we have $E_t e r_{t+1} = \beta_t (E_t r_{t+1}^m - i_t)$. Together, these imply:

$$(1-b)(i_t^* - i_t) = \beta_t (E_t r_{t+1}^m - i_t).$$

Together, these give us: $\beta_t = (1-b) \frac{i_t^* - i_t}{E_t r_{t+1}^m - i_t}$.

The interesting thing we have learned from this problem is that, if CAPM is the correct model for risk premiums, then the way we can account for the empirical findings on foreign exchange risk is that the beta of the foreign exchange risk changes over time. In particular, since $E_t r_{t+1}^m - i_t > 0$, the beta on foreign exchange risk is positive when $i_t^* - i_t > 0$ and negative when

$i_t^* - i_t < 0$. Since CAPM tells us that $\beta_t = \text{cov}_t \left(\frac{S_{t+1} - S_t}{S_t}, r_{t+1}^m \right) / \text{var}_t(r_{t+1}^m)$ in this case, then

CAPM posits that the covariance of the rate of depreciation of the currency itself changes over time as $i_t^* - i_t$ changes over time. The covariance is positive when $i_t^* - i_t > 0$ and negative when $i_t^* - i_t < 0$. It is an undecided empirical matter whether this relationship between the covariance and $i_t^* - i_t$ actually does hold in the data, because there is not agreement on how to measure r_{t+1}^m .