Econ 690
Spring, 2019
C. Engel

Answers to Homework 8

**1.** Suppose that firm *i* has one domestic production facility that supplies both the domestic and foreign markets. Assume that the demand for the product in the domestic market is $D_i = 2{,}000 - 3P_i$ and in the foreign market, demand is given by $D_i^* = 2{,}000 - 2P_i^*$. Assume that the domestic cost per unit of production is a constant 600. If the initial real exchange rate is 1 (i.e., set the national price levels equal to one, so $P = P^* = 1$, and assume initially S = 1), what are the optimal prices and quantities sold in the two markets? By how much will the firm change the relative prices of the product if the foreign currency appreciates in real terms by 10%? What happens to production?

<u>*Answer*</u>

For the domestic market, the firm sets the price $P_i$ to maximize

$$P_i(2000 - 3P_i) - 600(2000 - 3P_i)$$

FOC gives

$$2000 - 6P_i + 1800 = 0 \;\Rightarrow\; P_i = \frac{3800}{6} = \frac{1900}{3} \text{ and } Q_i = 2000 - 3 \times \frac{1900}{3} = 100$$

For the foreign market, the firm sets the price $P_i^*$ to maximize

$$SP_i^*(2000 - 2P_i^*) - 600(2000 - 2P_i^*)$$

FOC gives

$$(2000 - 4P_i^*)S + 1200 = 0 \;\Rightarrow\; P_i^* = \frac{2000S + 1200}{4S} = \frac{500S + 300}{S}$$

$$\text{and} \quad Q_i^* = 2000 - 2 \times \frac{500S+300}{S} = 1000 - \frac{600}{S}$$

Thus, when S=1, $P_i^* = 800$, $Q_i^* = 400$.

The relative price is

$$R_i = \frac{SP_i^*}{P_i} = \frac{3}{1900}\left(S \times \frac{500S + 300}{S}\right) = \frac{15}{19}S + \frac{9}{19}$$

Since the elasticity of $R_i$ with respect to S at S=1 is

$$\frac{\partial R_i}{\partial S}\frac{S}{R_i} = \frac{15}{19} \times \frac{S}{\frac{15}{19}S + \frac{9}{19}} = \frac{15}{19} \times \frac{1}{\frac{15}{19} + \frac{9}{19}} = \frac{15}{24} = \frac{5}{8},$$

if S increases by 10%, then $R_i$ goes up by $\frac{5}{8} \times 10 = 6.25\%$.
Similarly, since the elasticity of ${Q_i}^*$ to S at S=1 is

$$\frac{\partial {Q_i}^*}{\partial S}\frac{S}{{Q_i}^*} = \frac{3}{2} \ ,$$

, the goods sold in foreign market increases by $\frac{3}{2} \times 10 = 15\%$.

**2.** Suppose the log of the exchange rate is determined as in the model we have presented:

$$s_t = (1-a)x_t + aE_t s_{t+1} \text{ , where } 0 < a < 1$$

so we can determine: $s_t = (1-a)\left(x_t + aE_t x_{t+1} + a^2 E_t x_{t+2} + a^3 E_t x_{t+3} + \ldots\right) = (1-a)\sum_{j=0}^{\infty} a^j E_t x_{t+j}$

**a**. Suppose $E_t x_{t+j} = x_t$ for all $j \geq 0$. That is, the fundamental variable is not expected to change. Using the model formula above, calculate $s_t$. Your answer should express $s_t$ as a function of $x_t$.

<u>*Answer*</u>

$$s_t = (1-a)\sum_{j=0}^{\infty} a^j x_t = (1-a) \times \frac{x_t}{(1-a)} = x_t.$$

b. Now suppose $E_t x_{t+j} = \rho^j x_t$, for all $j \geq 0$, where $0 < \rho < 1$. That is, the fundamental variable is expected to converge to zero over time, with its persistence measured by the constant $\rho$. Using the model formula above, calculate $s_t$. Your answer should express $s_t$ as a function of $x_t$.

<u>*Answer*</u>

$$s_t = (1-a)\sum_{j=0}^{\infty} (a\rho)^j x_t = (1-a) \times \frac{x_t}{(1-\rho a)} = \frac{1-a}{1-a\rho} x_t.$$

**3.** Use the same model as in question 2, but now assume that $x_{t+j} = \overline{x}$ for $j = 0,1,2$, Starting in period 3, $x$ is permanently higher than $\overline{x}$. It takes on a value of $\hat{x}$ where $\hat{x} > \overline{x}$. That is, assume $x_{t+j} = \hat{x}$ for $j \geq 3$. Assume at all times, even at time *t* that investors know with certainty the path of *x*: that it will equal $\overline{x}$ in periods $t, t+1, t+2$ and $\hat{x}$ every period after that.

Solve for $s_t$, $s_{t+1}$, $s_{t+2}$ and $s_{t+3}$. Your solutions should be functions only of $\overline{x}$ and $\hat{x}$ (and, of course, the parameter *a*.)

Why does the exchange rate change in periods $t+1$ and $t+2$ even though the economic fundamental does not change?

*Answer*

$$s_t = (1-a) \times \left( \bar{x} + a\bar{x} + a^2\bar{x} + \sum_{j=3}^{\infty} a^j \hat{x} \right) = (1-a)\left(\frac{1-a^3}{1-a}\bar{x} + \frac{a^3}{1-a}\hat{x}\right)$$

$$= (1-a^3)\bar{x} + a^3\hat{x}$$

$$s_{t+1} = (1-a) \times \left( \bar{x} + a\bar{x} + \sum_{j=2}^{\infty} a^j \hat{x} \right) = (1-a)\left(\frac{1-a^2}{1-a}\bar{x} + \frac{a^2}{1-a}\hat{x}\right)$$

$$= (1-a^2)\bar{x} + a^2\hat{x}$$

$$s_{t+2} = (1-a) \times \left( \bar{x} + \sum_{j=1}^{\infty} a^j \hat{x} \right) = (1-a)\left(\bar{x} + \frac{a}{1-a}\hat{x}\right) = (1-a)\bar{x} + a\hat{x}$$

$$s_{t+3} = (1-a) \times \left( \sum_{j=0}^{\infty} a^j \hat{x} \right) = (1-a)\left(\frac{1}{1-a}\hat{x}\right) = \hat{x}$$

The exchange rate is not only affected by the current economic fundamental but also affected by the future fundamentals. Even though the economic fundamentals at $t+1$ and $t+2$ are the same, since the stream of future values are different at $t+1$ and $t+2$, the exchange rate changes in periods $t+1$ and $t+2$.