

### Answers to Second Test

1. Consider the following balance sheet for a central bank such as the U.S. Federal Reserve:

Assets	Liabilities
Official international reserves	Deposits of private financial institutions (bank reserves)
Domestic credit <ul style="list-style-type: none"> <li>• Government bonds</li> <li>• Loans to domestic financial institutions</li> </ul>	Currency in circulation
Other	Other

In the transactions below, assume that when the Federal Reserve buys assets from the public, it pays with a wire transfer to the account of a private citizen at a commercial bank. And, if it sells assets to the public, the private citizen pays the Federal Reserve with a transfer from her account.

- a. (4pts)** Suppose the Federal Reserve undertakes a domestic open market operation to increase the U.S. money supply. How are the assets and liabilities of the balance sheet affected? Be specific about which components are affected, and in what way (increase or decrease.)

*Answer:* Asset side: Government bonds increase.

Liability side: Deposits of private financial institutions increases (or, possibly, currency in circulation increases.)

- b. (4pts)** Suppose the Federal Reserve undertakes unsterilized foreign exchange market intervention in support of the U.S. dollar (that is, in order to try to make the U.S. dollar appreciate.) How are the assets and liabilities of the balance sheet affected? Be specific about which components are affected, and in what way (increase or decrease.)

*Answer:* Asset side: Official international reserves decrease.

Liability side: Deposits of private financial institutions decreases (or, possibly, currency in circulation decreases.)

- c. (5pts)** Suppose the Federal Reserve undertakes sterilized foreign exchange market intervention to weaken the U.S. dollar (that is, in order to try to make the U.S. dollar depreciate.) How are the

assets and liabilities of the balance sheet affected? Be specific about which components are affected, and in what way (increase or decrease.)

*Answer:* Asset side: Official international reserves increase, Government bonds decreases.  
Liability side: No change.

2. Let  $F_t$  be the one-period ahead forward rate, and  $S_t$  be the spot exchange rate, both expressed as the home currency price of foreign currency. Let  $i_{h,t}$  be the interest rate on a one-month money-market deposit in the home country, and  $i_{f,t}$  be the one-month money-market deposit in the foreign country.

a. (4tps) Assuming no market frictions, if  $\frac{F_t}{S_t} < \frac{1+i_{h,t}}{1+i_{f,t}}$ , how might an investor in the home country take advantage of this deviation from covered interest parity to make money?

*Answer:* The investor in home country can make profit by borrowing from foreign money market and investing in domestic money market. To be specific, if the investor borrows one unit of foreign currency, she will owe  $1 + i_{f,t}$  units of foreign currency after one-period. She can make a contract today to sell  $F_t(1 + i_{f,t})$  units of home currency for foreign currency one period from now, which she can use to pay off the loan. Meanwhile, she takes the one unit of foreign currency she has borrowed and buys  $S_t$  units of domestic currency, which she invests in domestic money market. At the end of the period, she has earned  $S_t(1 + i_{h,t})$  units of home currency. Since the amount she has earned,  $S_t(1 + i_{h,t})$  exceeds the amount she has to pay out to repay the loan,  $F_t(1 + i_{f,t})$  (because  $\frac{F_t}{S_t} < \frac{1+i_{h,t}}{1+i_{f,t}}$ ), she comes out ahead.

b. (4tps) If we take the log approximation (or use continuous compounding), how can we express covered interest parity?

*Answer:*  $i_{h,t} - i_{f,t} = \ln(F_t) - \ln(S_t)$

c. (4tps) List two reasons why covered interest parity might not hold.

*Answer:* Default risk, Exchange control, Political risk. Or, as in the aftermath of the crisis, if arbitrage requires borrowing in one currency (e.g., dollars) or selling some assets denominated in that currency, perhaps banks do not want to do so because they value the liquidity of assets in that currency.

**3. (6pts)** Suppose that one can buy a three-year bond that has a price now of  $P$ . At the end of each year (at the end of the first, second, and third years), the bond will pay a coupon of  $C$ . The bond has a maturity value or face value of  $M$ . Write down the formula that will determine the yield to maturity,  $y$ , of this bond.

Answer: 
$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \frac{C}{(1+y)^3} + \frac{M}{(1+y)^3}$$

**4.** The following questions involve  $S_t$ , which is the exchange rate (home currency price of foreign currency);  $F_t$ , the one-period ahead forward exchange rate;  $E_t S_{t+1}$ , the expected level of the exchange rate;  $i_{h,t}$ , the interest rate on a one-month money-market deposit in the home country;  $i_{f,t}$ , the one-month money-market deposit in the foreign country; and,  $sd_t$ , the standard deviation of the one-period rate of depreciation of the currency.

**a. (4pts)** What is the formula for uncovered interest parity?

Answer: 
$$\frac{E_t S_{t+1}}{S_t} (1 + i_{f,t}) = 1 + i_{h,t}$$

**b. (4pts)** If both covered interest parity and uncovered interest parity hold, what is the relationship between  $F_t$  and  $E_t S_{t+1}$ ?

Answer: 
$$F_t = E_t S_{t+1}$$

**c. (4pts)** How do we write the *expected* excess return for a home investor on an investment in a foreign money market deposit?

Answer: 
$$\text{Expected excess return} = \frac{E_t S_{t+1}}{S_t} (1 + i_{f,t}) - (1 + i_{h,t})$$

**d. (4pts)** Suppose the rate of change of this currency has a Normal probability distribution. If we are interested in assessing the probability that next period's exchange rate is less than or equal to some value  $\tilde{S}_{t+1}$ , how do we “normalize” the random variable  $\tilde{S}_{t+1}$  so that we can express it as a random variable that has a Normal distribution with a conditional mean of zero and a variance equal to one (that is, so that it has a “standard Normal” distribution)?

Answer: 
$$\frac{\tilde{S}_{t+1} - E_t S_{t+1}}{S_t sd_t}$$

**5.** Suppose an investor has only two options for investing his wealth – a riskless asset that pays a rate of return  $r$ , and a risky asset that pays a rate of return  $y$ . Let  $Ey$  be the expected return on the risky asset, and let  $Vy$  be the variance of the return on the risky asset.

Assume the investor's objective is to maximize:

$$E(rx + y(1 - x)) - zVar(rx + y(1 - x))$$

In this expression,  $x$  is the share of the portfolio invested in the riskless asset.  $z$  is a constant that measures this investor's dislike of risk.  $E(.)$  is the expectation, and  $Var(.)$  is the variance.

**a. (5pts)** Derive the first-order condition for choosing  $x$ .

*Answer:* Since  $r$  is the return on the riskless asset, we can rewrite the investor's objective as

$$E(y) + x(r - E(y)) - zVar(y(1 - x)) = E(y) + x(r - E(y)) - z(1 - x)^2Var(y).$$

Then the first-order condition for choosing  $x$  is

$$r - E(y) + 2z(1 - x)Var(y) = 0$$

**b. (5pts)** Use the answer to part **a** to solve for  $x$ . Hint: Your solution for  $x$  should be in terms of  $r$ ,  $Ey$ ,  $Vy$  and  $z$ .

*Answer:* Rearranging the first-order condition in (a), we have

$$x = 1 - \frac{Ey - r}{2zVy}.$$

**c. (6pts)** How do expected returns, the variance of return on the risky asset, and the degree of risk aversion affect the demand for the riskless asset?

*Answer:* From (b), we conclude that  $x$  increases if the expected return on the risky asset decreases, the variance of return on risky asset increases, and the degree of risk aversion increases.

**6.** Suppose an investor has only two options for investing his risky portfolio – one pays a rate of return  $r$ , and the other risky asset pays a rate of return  $y$ . Let  $Er$  and  $Ey$  be the expected returns on the risky assets, and let  $Vr$  and  $Vy$  be the variances of the returns on the risky assets. We will assume the covariance of the returns on the assets is zero.

The investor puts a share  $x$  of their portfolio in the asset that earns a return  $r$ . Assume the investor chooses  $x$  to maximize:

$$\frac{(xEr + (1 - x)Ey)^2}{x^2Vr + (1 - x)^2Vy}.$$

**a. (5pts)** Now assume  $Er = Ey$ . Show that, or explain why, choosing  $x$  to maximize the above expression is equivalent under this assumption to choosing  $x$  to minimize the denominator,  $x^2Vr + (1 - x)^2Vy$ .

*Answer:* If  $E_r = E_y$ , then the numerator of the expression doesn't depend on  $x$ . Thus, minimizing the denominator is equivalent to maximizing the objective function.

In other words, if  $E_r = E_y$ , the assets only differ in their variances, so the objective is to choose a portfolio that minimizes the variance.

**b. (5pts)** Solve for the value of  $x$  that minimizes  $x^2 V_r + (1-x)^2 V_y$ .

*Answer:* First-order condition for  $x$  is

$$2x V_r - 2(1-x)V_y = 0$$

Solving for  $x$ , we have

$$x = \frac{V_y}{V_y + V_r}$$

**8.** The forward rate unbiasedness hypothesis has been tested by estimating the regression:

$$\frac{S(t+30) - S(t)}{S(t)} = a + b \cdot fp(t) + \varepsilon(t+30).$$

Here,  $fp(t)$  refers to the 30-day forward premium.

**a. (5pts)** Why is this a test of forward rate unbiasedness? What is the null hypothesis in this regression?

*Answer:* Under the rational expectation assumption, the realized rate of appreciation can be decomposed into the conditional expectation plus an error term which doesn't depend on current information. So we can write

$$\frac{S(t+30) - S(t)}{S(t)} = \frac{E_t S(t+30) - S(t)}{S(t)} + \varepsilon(t+30).$$

Substituting the unbiasedness hypothesis

$$F(t) = E_t S(t+30)$$

into the above equation gives

$$\frac{S(t+30) - S(t)}{S(t)} = fp(t) + \varepsilon(t+30).$$

Thus, under the null hypothesis  $a = 0$  and  $b = 1$ , we can test the unbiased hypothesis by estimating

$$\frac{S_{t+30} - S_t}{S_t} = a + b \cdot fp_t + \epsilon_{t+30}$$

**b. (3pts)** What have studies tended to find for the estimated value of  $b$  in the regression above?

*Answer:* Studies have tended to find  $b$  is significantly less than 1 and sometime even less than 0.

**9.** Let  $P_i$  be the home-currency price of good  $i$ ;  $P_i^*$  the foreign-currency price of good  $i$  sold in the foreign country;  $P$  the home-currency price of the home consumption basket;  $P^*$  the foreign-currency price of the foreign consumption basket; and  $S$  the exchange rate (home currency price of foreign currency.)

**a. (4pts)** Write the formula for the Law of One Price

*Answer:*  $P_i = SP_i^*$

**b. (4pts)** Write the formula for Absolute Purchasing Power Parity

*Answer:*  $P = SP^*$

**c. (4pts)** Write the formula for Relative Purchasing Power Parity

*Answer:*  $P = kSP^*$ , for some constant  $k > 0$ . Alternatively,  $\frac{S_{t+1}}{S_t} = \frac{1 + \pi_{t+1}}{1 + \pi_{t+1}^*}$ .

**d. (4pts)** Write the formula for the PPP exchange rate,  $S^{PPP}$ .

*Answer:*  $S^{PPP} = \frac{P}{P^*}$