Econ 690
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Answers to Third Test

1. Consider a monopolist that sets the price of its product, P. The quantity it sells is given by Q. The demand for the product is $Q = aP^{-g}$, where $a > 0$ and $g > 1$ are constant parameters . The cost per unit of producing the product is W. The firms profits are given by $PQ - WQ$.

a. Find the price level that maximizes the firm's profits. Express your answer in terms of the cost per unit (and, of course, the parameters of the demand function.)

Answer: Profits are given by $PQ - WQ = aP^{1-g} - aWP^{-g}$. The first-order condition for maximizing profits gives us: $(1-g)aP^{-g} + agWP^{-g-1} = 0$. Dividing through by aP^{-g}, we get $(1-g) + gWP^{-1} = 0$, which gives us $P = \frac{g}{g-1}W$.

b. Let $\tilde{P}$ be the optimal price, as solved for in part (a). Calculate $\frac{d\tilde{P}}{dW}$ and then the pass-through elasticity $\frac{d\tilde{P}}{dW}\frac{W}{\tilde{P}}$.

Answer: $\frac{d\tilde{P}}{dW} = \frac{g}{g-1}$, so $\frac{d\tilde{P}}{dW}\frac{W}{\tilde{P}} = \frac{g}{g-1}W \cdot \left(\frac{g-1}{g}\frac{1}{W}\right) = 1$

c. The elasticity of demand for this firm's product is a constant, given by the parameter g. In class we learned a relationship between the pass-through elasticity and how the elasticity of demand changes as the price changes. How must the elasticity of demand change as the firm increases its price in order to have a pass-through elasticity less than one?

Answer: We learned that if the elasticity of demand increases as the price increases, the pass-through elasticity will be less than one.

2. This question refers to the limit pricing model. Suppose a U.S. firm called Abacus was a monopolist in producing "widgets". It could produce a widget at a cost of \$2 per unit.

a. If this monopolist faced a constant elasticity of demand equal to 3, what price would it charge for the widget?

<u>Answer</u>: In general, if ε is the elasticity of demand, we have $P = \frac{\varepsilon}{\varepsilon - 1} MC$, where P is the price and MC is the marginal cost. In this case $P = \frac{3}{3-1} \times 2 = 3$.

b. Suppose there was another U.S. manufacturer, called Beluga that started up. It found that it could produce the widget for $2.50, which is less than Abacus had been charging (in part a.) What is the optimal price that Abacus would charge?

<u>Answer</u> Abacus would charge $2.50 or just a slight bit less.

c. Suppose the cost of producing a widget rose to $2.25 for Abacus, but the cost remained at $2.50 for Beluga. What price would Abacus charge?

<u>Answer</u>: Abacus would still charge $2.50.

d. Suppose that Abacus was actually producing its product in Mexico, and the increase in the cost per unit from $2 per unit to $2.25 arose from an increase in the dollar price of a Mexican peso (rather than from a change in the cost in pesos of producing the good.) Can this model explain why final goods prices might not be sensitive to exchange rate changes? Explain.

<u>Answer</u>: Yes. The exchange rate changes but, as in part c, Abacus does not change its price. In the limit pricing model, the firm is not influenced in pricing decisions in some cases by its own costs, but rather the costs of its competitors.

3. In the following questions, we can use the following definitions:

s_t is the log of the exchange rate (home currency price of foreign currency.)

q_t is the log of the real exchange rate.

p_t is the log of the home price level.

p_t^* is the log of the foreign price level.

i_t is the one-period home nominal interest rate.

i_t^* is the one-period foreign nominal interest rate.

$\pi_{t+1} \equiv p_{t+1} - p_t$ is the home inflation rate.

$\pi_{t+1}^* \equiv p_{t+1}^* - p_t^*$ is the foreign inflation rate.

$E_t r_{t+1} = i_t - E_t \pi_{t+1}$ is the home expected real interest rate.

$E_t r_{t+1}^* = i_t^* - E_t \pi_{t+1}^*$

a. Express the log of the real exchange rate, q_t, in terms of s_t, p_t and p_t^*.

Answer: $q_t = s_t + p_t^* - p_t$

b. Assume that uncovered interest parity holds. What is the equation that relates $E_t r_{t+1}$, $E_t r_{t+1}^*$, and $E_t q_{t+1} - q_t$ when uncovered interest parity holds?

Answer: Uncovered interest parity tells us:

$$i_t = E_t s_{t+1} - s_t + i_t^*$$

Subtract $E_t \pi_{t+1}$ from both sides, and add and subtract $E_t \pi_{t+1}^*$ on the right-hand-side:

$$\begin{aligned} i_t - E_t \pi_{t+1} &= -E_t \pi_{t+1} + E_t \pi_{t+1}^* + E_t s_{t+1} - s_t + i_t^* - E_t \pi_{t+1}^* \\ &= -E_t \left(p_{t+1} - p_t \right) + E_t p_{t+1}^* - p_t^* + E_t s_{t+1} - s_t + i_t^* - E_t \pi_{t+1}^* \\ &= E_t \left(s_{t+1} + p_{t+1}^* - p_{t+1} \right) - \left(s_t + p_t^* - p_t \right) + i_t^* - E_t \pi_{t+1}^* \end{aligned}$$

This tells us $E_t r_{t+1} = E_t q_{t+1} - q_t + E_t r_{t+1}^*$.

c. Suppose that $E_t q_{t+1} - q_t = \kappa \cdot q_t$, where $-1 < \kappa < 0$. This equation implies that when the real exchange rate is positive, we expect the real exchange rate to fall, and vice-versa. Using this relationship and the one derived in part b, what is the equation that relates q_t to $E_t r_{t+1}$ and $E_t r_{t+1}^*$? Is there a home real appreciation or depreciation (which is it?) when $E_t r_{t+1}$ rises?

Answer: Substituting $E_t q_{t+1} - q_t = \kappa \cdot q_t$ into the expression from part b, which was $E_t r_{t+1} = E_t q_{t+1} - q_t + E_t r_{t+1}^*$, we get $E_t r_{t+1} = \kappa q_t + E_t r_{t+1}^*$. Therefore:

$$q_t = \frac{1}{\kappa}\left(E_t r_{t+1} - E_t r_{t+1}^* \right).$$

Since $\kappa < 0$, an increase in $E_t r_{t+1}$ leads to a drop in q_t, a real appreciation.

d. Suppose $q_t > 0$, and absolute purchasing power parity holds in the long run (so $q = 0$ in the long run.) How can we use this information to help forecast the change in the exchange rate, $s_{t+k} - s_t$, for some longer horizon $t + k$?

Answer: If $q_t > 0$, we can forecast a decline in the real exchange rate, so $E_t q_{t+k} - q_t < 0$. Since prices only adjust slowly, most of the decline in the real exchange rate will occur through a decline in the nominal exchange rate, so we expect $s_{t+k} - s_t < 0$.

4. Here are some additional questions about exchange rates:

a. Suppose we learn today that the Federal Reserve will tighten monetary policy and raise real interest rates six months from today. Does that have any effect on the exchange rate today? If so, does the U.S. dollar appreciate or depreciate?

Answer: Yes, it will have an effect today. At the time that the monetary tightening occurs, the dollar should appreciate. If we expect a future appreciation, that should feed back into an appreciation today.

b. In the Engel and Wu paper, the home liquidity premium on government bonds, $i_t^m - i_t$ played a role in explaining exchange rates. Here, i_t^m is the rate of interest on a short-term market instrument, and i_t is the interest rate on a short-term government bond. Does the government bond pay a liquidity premium if $i_t^m - i_t > 0$ or $i_t^m - i_t < 0$? Explain briefly.

Answer: If the government bond pays a liquidity premium, that means it can pay an interest rate lower than the market rate, because investors earn not only i_t but also the liquidity return. Hence $i_t^m - i_t > 0$ if the government bond pays a liquidity premium.

c. Does an increase in $i_t^m - i_t$ in the U.S. lead to an appreciation or depreciation of the dollar, holding other things constant?

Answer: An increase in $i_t^m - i_t$ leads to an appreciation of the dollar because the government bond is paying a higher liquidity premium.

5. Assume (in a 2-period model) that a sovereign borrower gets income in first period of $Q_1 = 5$ and income in the second period of $Q_2 = 10$. If he could borrow and then commit to repaying, he would pay no interest (his gross interest rate would be $R = 1$). Assume that the sovereign's preferences are such that he wants to smooth consumption as much as possible over the two periods. That is, he wants to make C_1 and C_2 equal if he could, so that under commitment he would choose $C_1 = 7.5$ and $C_2 = 7.5$. His first-period debt, *D*, would be the amount he borrowed so that he could achieve $C_1 = 7.5$, which would be $D = 2.5$.

a. Now suppose that the sovereign could not commit to repay, so he has an incentive to default on any first period loan he gets. But suppose he would suffer a loss of output of 2.0 if he defaulted. Would he be able to borrow at all at a rate of $R = 1$? If he could borrow, would he want to borrow? If he does want to borrow, how much would he borrow?

Answer: He would be able to borrow up to 2.0. He could commit to repaying that amount. He would want to borrow 2.0, because that would help to smooth his consumption.

b. Now suppose that the situation is identical to part (a), but the loss of output if he defaults is greater. Suppose he would lose 3.0. What are your answers to the three questions posed in part (a) now?

Answer: In this situation, the punishment is so large, that the borrower could effectively commit to borrowing his most desired amount of debt, 2.5, and repaying it all. So he would borrow 2.5 and repay 2.5.

c. Is greater punishment for default always better for sovereign borrowers? Explain.

Answer: Ex ante, at the time of borrowing in period 1before the output is realized in period 2, a greater punishment is always better for the borrower (or at least, it makes him no worse off), because it increases the amount he can borrow and lowers the interest rate. But if it turns out that the situation is such that the borrower finds it optimal to default in period 2, a greater punishment hurts the borrower more.

d. Is loss of collateral a significant punishment that can be imposed on sovereign borrowers?

Answer: No, in general, sovereign loans are not collateralized.

e. What does "time consistency" mean?

Answer: Time consistency means that the decisions that one would make at time t for different possible realizations of the state of the world at some future date $t+k$ are the same decisions that one would make if the decision were made at date $t+k$ in the given state of the world.

6. Suppose the current value of the spot exchange rate for euros is \$1.14. That is, the dollar price of the euro is \$1.14. You are considering buying an American put option to sell euros, at a strike price of \$1.15, with an expiration date of June 30, 2019.

a. Is this put option "in the money"? For what values of the exchange rate is the put option in the money?

Answer: Yes, this option is in the money. It is currently valuable if it were exercised today. It is in the money for all spot rates less than or equal to \$1.15.

b. Would a put option with a strike price of \$1.16 cost more or less than the put option described above?

Answer: The option to sell euros at a price of \$1.16 is more valuable than the option to sell them at \$1.15, so that option has a higher price.

c. Suppose the volatility of the dollar/euro exchange rate increases, without any change in the mean of the probability distribution. How would that affect the value of the option?

Answer: An increase in volatility will increase the value of the option. When there is higher volatility, the option has a greater possibility of large payoffs because it is more likely that the spot exchange rate will be far below $1.15.

d. Would the price of an American put option with a strike price of $1.15 and expiration date of September 30, 2019, be more expensive or less expensive than the one that expires June 30, 2019?

Answer: The option with the later expiration date has a higher value. The later expiration date has a greater uncertainty about the future spot rate and therefore is like question c.