Econ 690
Spring 2019
C. Engel

Homework 6

1. Suppose r_P is the return on a portfolio of risky assets. The portfolio consists of two assets that pay return r_1 and r_2 respectively. The expected returns on the two assets are m_1 and m_2 (that is, $m_1 \equiv E(r_1)$ and $m_2 \equiv E(r_2)$.) The share of the portfolio in asset 1 is *a*, and the share in asset 2 is $1-a$. So

$$r_p = ar_1 + (1-a)r_2.$$

Let v_1 be the variance of the return on asset 1 and v_2 be the variance of the return on asset 2. Let c_{12} be their covariance. (That is, $v_1 = \text{var}(r_1)$, $v_2 = \text{var}(r_2)$, and $c_{12} = \text{cov}(r_1, r_2)$.)

We know from the properties of expectations and variances that:

$$E\left(r_p\right) = am_1 + (1-a)m_2$$

$$\text{var}\left(r_P\right) = a^2 v_1 + (1-a)^2 v_2 + 2a(1-a)c_{12}$$

a. Find the value of *a* that maximizes $\dfrac{\left(E(r_P)\right)^2}{\text{var}(r_P)}$. Your answer should express *a* in terms of m_1 and m_2, and v_1, v_2, and c_{12}. Please try to simplify your answers (making cancellations) as much as possible.

b. For simplicity, now assume $m \equiv m_1 = m_2$ and $c_{12} = 0$. Write out the solution for *a* in this special case.

c. Continue to assume $m \equiv m_1 = m_2$ and $c_{12} = 0$. Now, let's interpret the problem in the following way. We are looking at the risky portfolio of a home investor. Asset 1 is the foreign bond that pays $i^* + s_{+1} - s$ and asset 2 is an equity that pays r_x. Here, *s* is the log of the exchange rate, and we are writing the approximate return on the foreign investment. At the time the portfolio choice is made, i^* and *s* are known, but the random variables are s_{+1} and r_x. The moments of these random variables are $E\left(s_{+1}\right)$, $E\left(r_x\right)$, $\text{var}\left(s_{+1}\right)$, $\text{var}\left(r_x\right)$, $\text{cov}\left(s_{+1}, r_x\right)$. Write your solution to part b, now using the specific interpretation given here.

d. Now, use the general formula derived in part a. That is, do not assume $m \equiv m_1 = m_2$ and $c_{12} = 0$. Write that formula using the specific assumptions about what the risky asset are from part c. What variables determine the investors demand for foreign bonds?