Chapter 20

Foreign Currency Futures and Options
20.1 The Basics of Futures Contracts

• Futures (versus forwards)
  • Allow individuals and firms to buy and sell specific amounts of foreign currency at an agreed-upon price determined on a given future day
  • Traded on an exchange (e.g., CME Group, NYSE Euronex’s LIFFE CONNECT, and Tokyo Financial Exchange)
  • Standardized, smaller amounts (e.g., ¥12.5M, €125,000, C$100,000)
  • Fixed maturity dates
• Credit risk
  • Futures brokerage firms register with the Commodity futures trading commission (CFTC) as a futures commission merchant (FCM)
  • Clearing member / clearinghouse
20.1 The Basics of Futures Contracts

- **Margins**
  - Credit risk is handled by setting up an account called a margin account, wherein they deposit an asset as collateral
    - The first asset is called the initial margin
    - Asset can be cash, US government obligations, securities listed on NYSE and American Stock Exchange, gold warehouse receipts or letters of credit
    - Depends on size of contract and variability of currency involved
    - Margin call – when the value of the margin account reaches the maintenance margin, the account must be brought back up to its initial value

- **Marking to market – deposit of daily losses/profits**
  - Maintenance margins
    - Minimum amount that must be kept in the account to guard against severe fluctuations in the futures prices (for CME, about $1,500 for USD/GBP and $4,500 for JPY/USD)
Exhibit 20.1 An Example of Marking to Market in the Futures Market

- Euro contract (€125,000)
  - On September 16, you “go long in December Euro”
    - In other words, you buy a Euro contract that is deliverable in December
  - Maintenance margin: $1500

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<tr>
<th>Day</th>
<th>Futures price ($/€)</th>
<th>Change in futures price ($/€)</th>
<th>Gain or loss</th>
<th>Cumulative gain or loss</th>
<th>Margin account</th>
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<td>$t$</td>
<td>1.3321</td>
<td>0</td>
<td>0</td>
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<td>$t+1$</td>
<td>1.3315</td>
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<td>$2,462.50</td>
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Initial Margin – for both buyer and seller

Settle Price

Contract Size $\times \Delta F$

Margin call

Exhibit 20.1 An example of marking to market in the futures market
20.1 The Basics of Futures Contracts

• The pricing of futures contracts
  • The payoff on a forward contract:
    • $S(T) - F(t)$
    • where $S(T)$ is the future spot rate at maturity time $T$ and $F(t)$ is the forward price at time $t$
  • The payoff on a futures contract
    • $f(T) - f(t)$
    • Where $f(T)$ is the futures price at maturity time $T$ and $f(t)$ is the futures price at time $t$
  • Payoffs for futures can differ than those from forwards because the interest that is earned on future profits or that must be paid on future losses in a futures contract
### Exhibit 20.2 Futures Quotes from August 5, 2015

<table>
<thead>
<tr>
<th>Contract size</th>
<th>JPY12,500,000</th>
<th>CAD100,000</th>
<th>GBP62,500</th>
<th>EUR125,000</th>
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<td>Exchange rate</td>
<td>USD per 100 JPY</td>
<td>USD per CAD</td>
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<td>USD per EUR</td>
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<td>Maturity</td>
<td>SEP</td>
<td>DEC</td>
<td>MAR</td>
<td>SEP</td>
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<td>Open price</td>
<td>0.80435</td>
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<td>High price</td>
<td>0.80665</td>
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<td>Low price</td>
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<td>Settle price</td>
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<td>Open interest</td>
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<th>MXN500,000</th>
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<td>USD per 100,000 MXN</td>
<td>JPY per EUR</td>
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<td>431</td>
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<td>167,683</td>
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20.2 Hedging Transaction Risk with Futures

• It is mid-February and Nancy Foods expects a receivable of €250,000 in one month
  • Will need 2 contracts (since contracts are €125,000)
  • Wants to receive $ when the € weakens to protect against a loss in receivable
    • Thereby selling €
  • If contract delivery date coincides with receivable date, maturity is matched perfectly
• Example:
  • February: Spot ($1.24/€); Future ($1.23/€)
  • March: Spot ($1.35/€); Future ($1.35/€); 30-day i(€) = 3% p.a.
  • Receivable in 30 days
20.2 Hedging Transaction Risk with Futures

- Value upon receipt of money (mid-March)
  - Sell receivable in spot market in March
    - $250,000 × $1.35/€ = $337,500
  - Loss on futures contract
    - \[ ([\$1.23/€] - [\$1.35/€]) \times €250,000 = -\$30,000 \]
- Combination of CFs
  - $337,500 − $30,000 = $307,500
- Effective exchange rate
  - \( \frac{\$307,500}{€250,000} = $1.23/€ \)
20.2 Hedging Transaction Risk with Futures

• Potential problems with a futures hedge
  • What if you need to hedge an odd amount?
  • What if the contract delivery date does not match your receivable/payable date?
20.3 Basics of Foreign Currency Option Contracts

• Gives the buyer the right but not the obligation to buy (call) or sell (put) a specific amount of foreign currency for domestic currency at a specific forex rate
  • Price is called the premium
  • Traded by money center banks and exchanges (e.g., NASDAQ, OMX, PHLX)
  • European vs. American options:
    • European options can only be exercised on maturity date; Americans can be exercised anytime (i.e., “early exercise” is permitted)
• Strike / exercise price (“K”) – forex rate in the contract
• Intrinsic value – revenue from exercising an option
  • In the money / out of the money / at-the-money
    • Call option: \( \max[S - K, 0] \)
    • Put option: \( \max[K - S, 0] \)
20.3 Basics of Foreign Currency Option Contracts

Example: A Euro Call Option Against Dollars

• A particular euro call option offers the buyer the right (but not the obligation) to purchase €1M @ $1.20/€
  • If the price of the € > K, owner will exercise the option at expiration date
    • To exercise: the buyer pays ($1.20/€) × €1M = $1.2M to the seller and the seller delivers the €1M
  • The buyer can then turn around and sell the € on the spot market at a higher price!
  • For example, if the spot is $1.25/€, the revenue is:
    • [($1.25/€) − ($1.20/€)] × €1M = $50,000
      • This is the intrinsic value of the option, not the profit
  • Buyer could therefore simply accept $50,000 from the seller if both parties prefer to do so
20.3 Basics of Foreign Currency Option Contracts

Example: A Yen Put Option Against the Pound

• A particular yen put option offers the buyer the right (but not the obligation) to sell ¥100M @ £0.6494/¥100
  • If the price of the ¥100 < K, owner will exercise
    • To exercise: the buyer delivers ¥100M to the seller
      • The seller must pay (£0.6494/¥100) × ¥100M = £649,400
  • For example, say the spot at exercise is £0.6000/¥100
    • The revenue then is:
      • [(£0.6494/¥100) − (£0.6000/¥100)] × ¥100M = £49,400
      • Intrinsic value of option, not the profit
      • Buyer could therefore accept £49,400 from seller if both of the parties prefer to do so
20.3 Basics of Foreign Currency Option Contracts

• Options trading
  • Mostly traded by banks in the interbank market or the OTC market
    • Typically European convention in OTC market
    • CFs either exchanged or cash settlement
    • Considerable counterparty risk, managed by exposure limits
  • Currency options on the NASDAQ OMX PHLX
    • Mostly options on spot currencies vs U.S. Dollar
    • Expiration months:
      • March, June, September and December
      • Two nearest future months
    • Last trading day is the third Friday of expiring month
    • European-exercise type but settlement is in dollars
    • Options Clearing Corporation serves as clearinghouse
## Exhibit 20.4 Prices of options on futures contracts

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<th>Currency</th>
<th>Type</th>
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<td>2.49</td>
<td>2.77</td>
<td>3.07</td>
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20.3 Basics of Foreign Currency Option Contracts

• Currency options at the CME group
  • Contract sizes and expiration months follow those of futures contracts
  • Trading closes on Friday immediately preceding the third Wednesday of the contract month
20.3 Basics of Foreign Currency Option Contracts

- Exchange-listed currency warrants
  - Longer-maturity foreign currency options (> 1 year)
  - Issued by major corporations
  - Actively traded on exchanges such as the American Stock Exchange, London Stock Exchange, and Australian Stock Exchange
- American-style option contracts
- Issuers include AT&T, Deutsche Bank, Ford, Goldman Sachs
  - Not taking on currency risk – likely hedged in OTC market
  - Buying an option at wholesale price and selling at retail price
- Allow retail investors and small corporations that are too small to participate in OTC market to purchase L/T currency options
20.4 The Use of Options in Risk Management

• A bidding situation at Bagwell Construction
  • U.S. company wants to bid on a building in Tokyo (in ¥)
  • Transaction risk since bid is in ¥
  • Cannot use forward hedge because if they do not win, it will be a liability
  • Option allows flexibility in case they do not win

• Using options to hedge transaction risk
  • Forward / futures contracts do not allow you to benefit from the “up” side
  • Allows a hedge but maintains the upside potential from favorable exchange rate changes
20.4 The Use of Options in Risk Management

- Pfimerc
  - Today is Friday, 1st October 2010
  - Receivable of £500,000 on Friday, 19th March 2011
    - S: $1.5834/£
    - 170-day F: $1.5805/£
    - $ 170-day interest rate: 0.20% p.a.
    - £ 34-day interest rate: 0.40% p.a.
  - Option data for March contracts in $/£:

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<th>Strike</th>
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<th>Put Price</th>
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<td>160</td>
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20.4 The Use of Options in Risk Management

- How should Pfimerc hedge?
  - £ put option: right (but not obligation) to sell £ at a specific price if the value of the £ falls
  - In order to sell £500,000, Pfimerc must pay:
    - £500,000 × ($0.0481/£) = $24,050
  - Exercise option if £ falls below $1.58/£:
    - £500,000 × \( \frac{1.58}{\text{£}} \) = $790,000 if \( S(t + 170) \leq 1.58/\text{£} \)
  - Sell £ in spot market if £ is worth $1.58 in 170 days:
    - £500,000 × \( S(t + 170) > 790,000 \) if \( S(t + 170) > 1.58/\text{£} \)
  - Either way, cost of the put is:
    - \([24,050 \times (1 + (0.002 \times 170/360))] = 24,073\)
  - Minimum revenue is therefore:
    - $790,000 − $24,073 = $765,927
20.4 The Use of Options in Risk Management

• Options as insurance contracts
  • Hedging foreign currency risk with forwards and options
  • Options as insurance contracts
    • As amount of coverage increases so does the cost (premium) to insure
  • Changing the quality of the insurance policy
    • Make ceiling on our cost of the foreign currency as low as possible
### Exhibit 20.7 Hedging and Speculating Strategies

<table>
<thead>
<tr>
<th>Underlying transaction</th>
<th>Foreign currency receivable</th>
<th>Foreign currency payable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward hedge (or futures hedge)</td>
<td>Sell forward (Go short)</td>
<td>Buy forward (Go long)</td>
</tr>
<tr>
<td></td>
<td>Buy a put</td>
<td>Buy a call</td>
</tr>
<tr>
<td></td>
<td>Establishes a revenue floor of $K - (1+i)P$</td>
<td>Establishes a cost ceiling of $K + (1+i)C$</td>
</tr>
<tr>
<td>Option hedge</td>
<td>Sell a call</td>
<td>Sell a put</td>
</tr>
<tr>
<td></td>
<td>Imposes a revenue ceiling of $K + (1+i)C$</td>
<td>Imposes a liability floor of $K - (1+i)P$</td>
</tr>
<tr>
<td></td>
<td>but allows unlimited risk</td>
<td>but allows unlimited risk</td>
</tr>
<tr>
<td>Option speculation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
20.4 The Use of Options in Risk Management

• Option valuation – Black and Scholes (1973)
  • The intrinsic value of an option
    • If the owner exercises it, will it make money (in / at / out of the money)?
  • The time value of an option
    • The part of the option’s value that is attributed to the time left to expiry
    • Time value = Option price – intrinsic value
  • Increasing the exercise price (call)
    • Reduced the probability that the option will be exercised so it decreases the option’s value
20.4 The Use of Options in Risk Management

• An increase in the variance
  • The distribution with the larger variance yields possibly larger payoffs so it increase the value of the option

• Increasing the time to expiration
  • American – increases uncertainty of spot rate at maturity so it increases the option’s value
  • European – generally increases the option’s value but it depends because in-the-money European options can lose value as time evolves
Exhibit 20.10 Different Probability Distributions of Future USD/EUR
Exhibit 20.11 Different Probability Distributions of Future USD/EUR
20.5 Combinations of Options and Exotic Options

• Exotic options
  • Options with different payoff patterns than basic options
  • Range forward contract
    • Allows a company to specify a range of future spot rates over which the firm can sell or buy forex at the future spot rate
    • No money up front
  • Cylinder options
    • Allows buyers to specify a desired trading range and either pay money or potentially receive money up front for entering into the contracts
  • Both can be synthesized
    • Buying a call and selling a put (at a lower K)
    • For range forward contract:
      • K must be set such that $P(K_p) = C(K_c)$
20.5 Combinations of Options and Exotic Options

- Average-rate options (or “Asian” option)
  - Most common exotic option
  - Payoff is $\max[0, \hat{S} - K]$
    - $\hat{S}$ defines the average forex rate between the initiation of the contract and the expiration date (source and time interval are agreed upon)
- Barrier options
  - Regular option with additional requirement that either activates or extinguishes the option if a barrier forex rate is reached
- Lookback options
  - Option that allows you to buy/sell at least/most expensive prices over a year (more expensive than regular options)
- Digital options (“binary” options)
  - Pays off principal if $K$ is reached and 0 otherwise
An example of option pricing

• Suppose that we want to buy a call option that allows us to buy euros three months from now at a price of $1.14.

• Suppose there are only two possible values of the euro three months from now – either $1.16 or $1.13.

• Suppose the 3-month interest rate in the U.S. is 0.01 (not annualized), and in Europe is 0.005

• Suppose the current spot exchange rates is 1.15

• I am going to build a portfolio that replicates the payoffs to the option, and then figure out what that portfolio costs.
Replicating portfolio

• I will buy €X, and borrow $Y. The idea is that I am going to find an X and Y that will give me a payoff equal to that of the call option that lets me buys euros for $1.14
• This portfolio I buy today has a cost C given by $C = (1.15 \times X) - Y$
• Now, if I bought the call option, it has two possible payoffs. Suppose the call option allows me to buy €100 at the price $1.14.
  • If the spot price of euros in 3 months turns out to be $1.13, the call option is worthless.
  • If the spot price of euros in 3 months is $1.16, the value of the call option is $(1.16 - 1.14) \times 100 = 2.00$
• Now we want to see what values of X and Y will give us payoff of $2.00 when the spot exchange rate is $1.16, and $0 when the spot exchange rate is $1.13
Pricing the option

• The value of my portfolio in one month is
  \((S \times \欧元 X)(1.02) - \$Y(1.01)\), where \(S\) is the spot exchange rate in one month.

• We are looking for the values of \(X\) and \(Y\) that satisfy these two equations:
  \(\((1.13 \times \欧元 X)(1.005) - \$Y(1.01) = 0\)
  \(\((1.16 \times \欧元 X)(1.005) - \$Y(1.01) = 2.00\)

• These are two linear equations in two variables, \(X\) and \(Y\), which we can solve
• We find \(\欧元 X = \欧元 66.335\) and \(\$Y = \$74.59\)
• Then the cost today of the portfolio that has the same payoff as the option is
  • \(C = (1.135 \times \欧元 X) - \$Y = (1.15 \times \欧元 66.335) - \$74.59 = \$1.695\)
  • \$1.695 would be the price of the call option
Greater variance

• Suppose instead of $1.13 and $1.16 as possible future spot exchange rates, the possibilities had a greater variance but the same mean: $1.12 and $1.17
• The option’s value when $S = 1.17 is $(1.17 - 1.14) \times \€100 = $3.00
• We are looking for the values of $X$ and $Y$ that satisfy these two equations:
  \[(1.12 \times \€X)(1.005) - Y(1.01) = 0\]
  \[(1.17 \times \€X)(1.005) - Y(1.01) = 3.00\]
• These are two linear equations in two variables, $X$ and $Y$, which we can solve
• We find $\€X = \€59.70$ and $Y = $66.53
• Then the cost today of the portfolio that has the same payoff as the option is
  • $C = (1.135 \times \€X) - Y = (1.15 \times \€59.70) - 66.53 = $2.125
  • $2.125 would be the price of the call option instead of $1.695