Econ 702
Spring 2020
C. Engel

Answers to First Test

1. List three of the six "Kaldor facts" about growth over time.

Answers:

Output per worker grows at a constant rate
Capital per worker grows at a constant rate
Capital/output ratio is constant over long periods
Labor's share is constant over time
Rate of return to capital is roughly constant
Real wages grow at a constant rate

2. Suppose output, $Y_t$ is produced using capital, $K_t$, and labor, $N_t$, according to the production function:

$$Y_t = A\left(K_t^{\alpha} N_t^{1-\alpha} + K_t^{\varepsilon} N_t^{1-\varepsilon}\right)$$

where the parameters satisfy $0 < \alpha < 1$, $0 < \varepsilon < 1$, and $A > 0$.

a. Write the production function in "per worker" terms. That is, if we define $y_t \equiv \frac{Y_t}{N_t}$, and $k_t = \frac{K_t}{N_t}$, write $y_t$ as a function of $k_t$.

Answer: Divide both sides of the production function by $N_t$:

$$\begin{aligned}\frac{Y_t}{N_t} &= A\frac{\left(K_t^{\alpha} N_t^{1-\alpha} + K_t^{\varepsilon} N_t^{1-\varepsilon}\right)}{N_t} \\ &= A\left(\frac{K_t^{\alpha} N_t^{1-\alpha}}{N_t} + \frac{K_t^{\varepsilon} N_t^{1-\varepsilon}}{N_t}\right) \\ &= A\left(K_t^{\alpha} N_t^{-\alpha} + K_t^{\varepsilon} N_t^{-\varepsilon}\right)\end{aligned}$$

So, we have $y_t = A\left(k_t^{\alpha} + k_t^{\varepsilon}\right)$

b. In the Solow model, the rental rate on capital, $r_t$, equals the marginal product of capital. Use your answer to part (a) to find the expression for $r_t$ in terms of $k_t$.

Answer:

$$r_t = \frac{dy_t}{dk_t} = A\left(\alpha k_t^{\alpha-1} + \varepsilon k_t^{\varepsilon-1}\right)$$

3. In the basic Solow model, we had an equation:

$$k_{t+1} - k_t = sAf\left(k_t\right) - \delta k_t ,$$

where $0 < s < 1$, $0 < \delta < 1$, $A > 0$, $f' > 0$, and $f'' < 0$. (Also, assume the Inada conditions hold.) $k_t$ is capital per worker.

a. On the right hand side of this equation, what do the terms $sAf\left(k_t\right)$ and $\delta k_t$ each represent?

Answer:

$sAf\left(k_t\right)$ is the saving or investment of households, and $\delta k_t$ is the amount of capital that depreciates in period *t*.

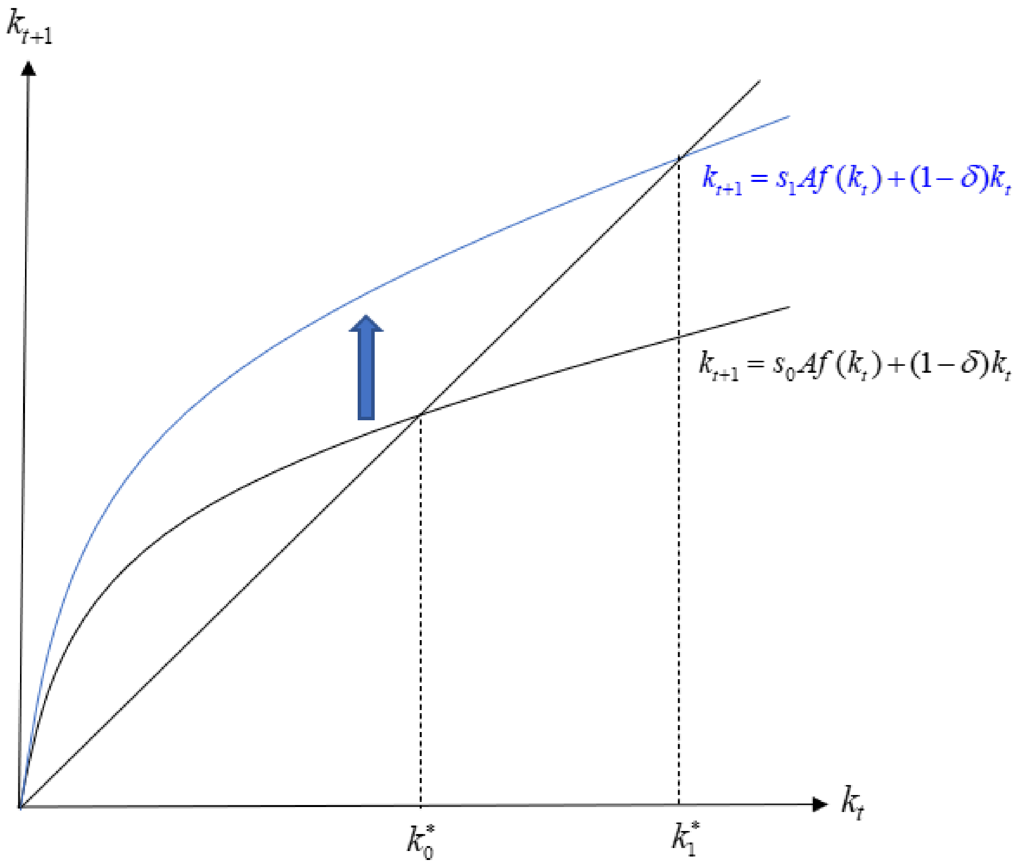
b. Let $\bar{k}$ be the steady-state capital per worker, and $\bar{y} = Af\left(\bar{k}\right)$ be the steady state output per worker. How are each of these two variables changed by an increase in the saving rate (that is, just state the direction of change for each $\bar{k}$ and $\bar{y}$)?

Answer:

Both of them increase

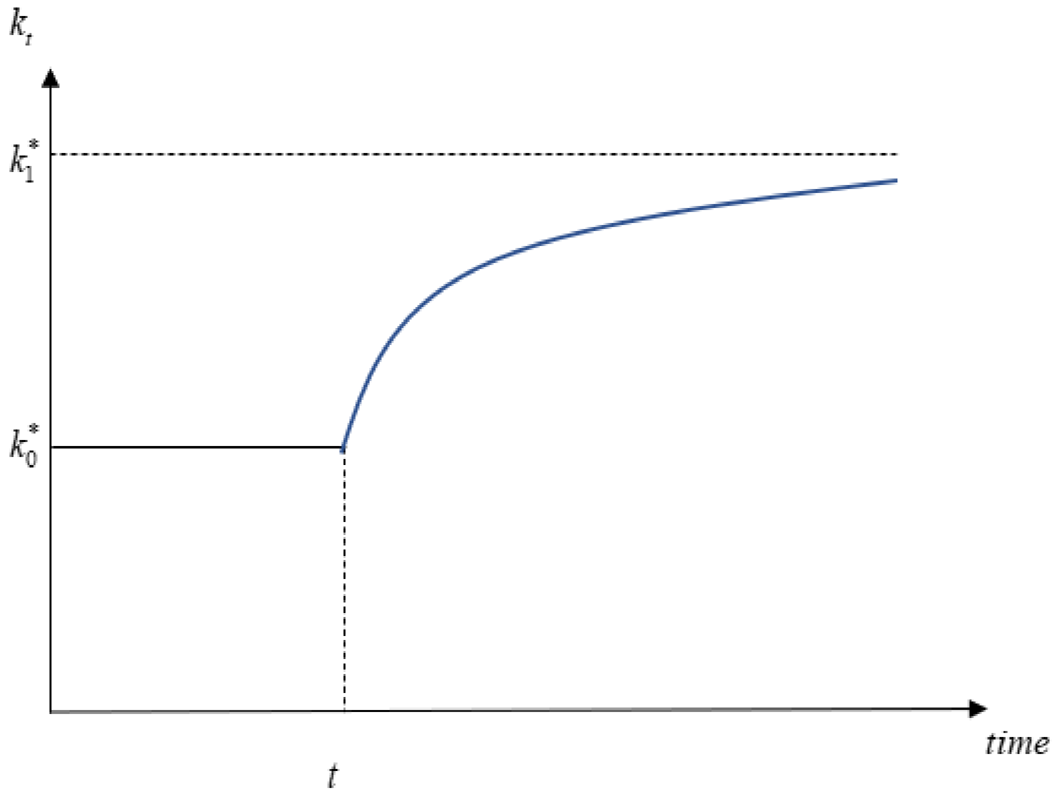
c. Draw the graph of the Solow model in which we put $k_{t+1}$ on the vertical axis, and $k_t$ on the horizontal axis, and draw the curve for $k_{t+1} = sAf\left(k_t\right) + \left(1-\delta\right)k_t$. Show how the curve changes when $s$ increases.

Answer:



d. On a graph with $k_t$ on the vertical axis and time on the horizontal axis, show how $k_t$ evolves over time if the saving rate rises at time *t*.

Answer:




e. Explain the two opposite effects on steady-state consumption per worker when the saving rate rises. How is the effect on consumption per worker of the increase in the saving rate related to the "golden rule" level of saving?

Answer:

There are two effects. The increase in the saving rate raises income per worker, which raises consumption. But the fraction of income that is consumed falls because the fraction that is saved increases. Consumption per worker will increase if the saving rate is below the golden rule rate, and will decrease if the saving rate is already above the golden rule level.

f. Suppose initially the capital stock is lower than the steady state level. Why does output converge to a steady state in the basic Solow model? Explain briefly in a sentence or two, making sure to explain the key economic mechanism.

Answer:

As capital increases, the marginal productivity of capital falls. So the new additions to output get smaller and smaller, and since saving is proportional to income, the new saving declines until it equals the amount of capital that is depreciating.

g. Suppose initially the capital stock is lower than the steady state level. Why In the growth model in which the saving decision is based on optimizing behavior by households, what role does the return to saving play in leading the economy toward the steady state? Explain in a sentence or two.

Answer:

As the capital stock grows, the marginal product of capital declines. This reduces the real return to saving, and so saving and investment decline.

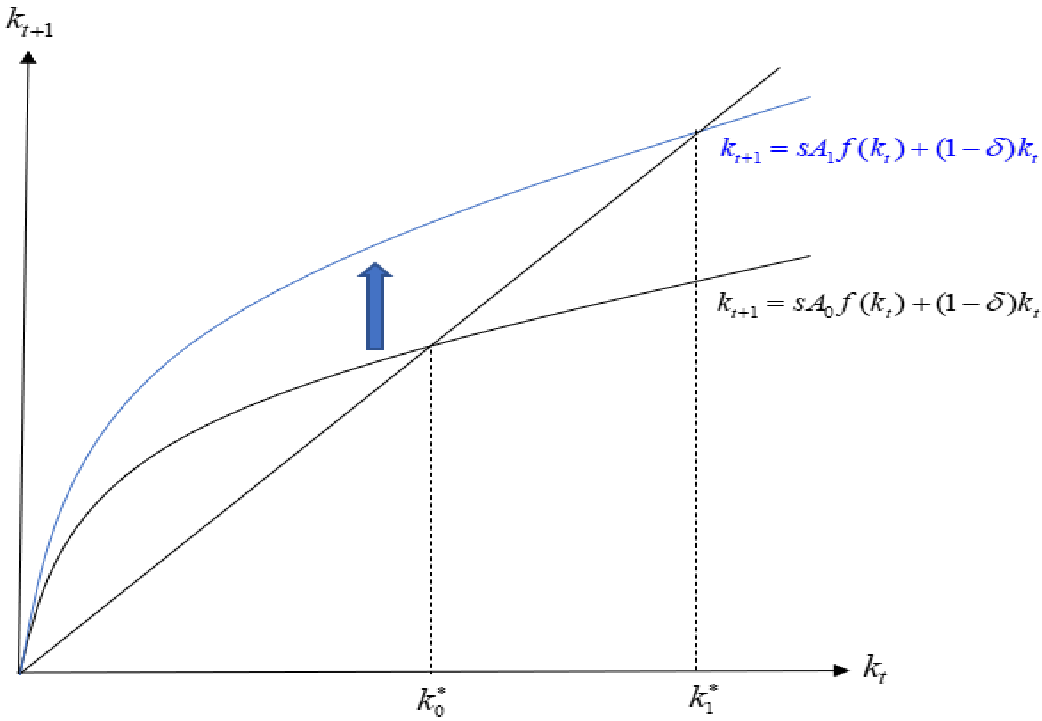
4.a. In the "augmented" Solow growth model, we found that if labor-augmenting productivity grows at a constant rate (and the labor force grows at a constant rate), then output per worker will also grow in the long run. In this model, which of these variables grow in the long-run, and which do not: Output per worker, capital per worker, the real wage, the rental rate on capital, labor's share of income?

Answer:

Output per worker, capital per worker, and the real wage grow in the long run.
The rental rate on capital and labor's share do not grow.

b. Zilibotti's paper on growth in China says that up until now, China's growth has been driven by an increase in saving and investment, and an increase in TFP coming from better allocation of resources and imitation of the leading technologies. Taking the latter case, draw the graph for the *basic* Solow model in which we put $k_{t+1}$ on the vertical axis, and $k_t$ on the horizontal axis, and draw the curve for $k_{t+1} = sAf\left(k_t\right) + \left(1-\delta\right)k_t$. Show how the curve changes when *TFP* increases. Does the economy grow, and is the growth sustainable?

Answer: The economy grows initially when TFP increases, but the growth is not sustained as the economy goes to a steady state.




5. Suppose a household that will live for two periods has the following utility function:

$$U = \ln(C_t - \tilde{C}) + \beta \ln\left(C_{t+1} - \tilde{C}\right)$$

Here, $0 < \beta < 1$. $\tilde{C}$ is considered to be a "subsistence" level of consumption, and utility is only defined for consumption levels greater than $\tilde{C}$ (that is, the person could not live with $C_t < \tilde{C}$, or $C_{t+1} < \tilde{C}$.) Assume the household receives income of $Y_t$ in period *t* and $Y_{t+1}$ in period $t+1$. The household at period *t* can borrow or save at a real interest rate of $r_t$.

a. Write the intertemporal budget constraint for the household (also known as the lifetime budget constraint.)

Answer:

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

b. Assume that the household's income is sufficient to maintain at least the subsistence level of income in both periods. Derive the household's Euler equation (the intertemporal first-order condition that comes from maximizing utility subject to the intertemporal budget constraint.)

Answer:

From the budget constraint, we get:

$$C_{t+1} = (1+r_t)(Y_t - C_t) + Y_{t+1}$$

Then we can substitute that into the utility function to get:

$$U = \ln\left(C_t - \tilde{C}\right) + \beta \ln\left((1+r_t)(Y_t - C_t) + Y_{t+1} - \tilde{C}\right)$$

The first-order condition is

$$\frac{1}{C_t - \tilde{C}} - \beta(1+r_t)\frac{1}{(1+r_t)(Y_t - C_t) + Y_{t+1} - \tilde{C}} = 0,$$

which can be written as:

$$\frac{1}{C_t - \tilde{C}} - \beta(1+r_t)\frac{1}{C_{t+1} - \tilde{C}} = 0, \text{ or } \frac{C_{t+1} - \tilde{C}}{C_t - \tilde{C}} = \beta(1+r_t)$$

c. Using the Euler equation and the budget constraint, solve for the optimal $C_t$. To simplify the algebra, for this part of the question you can assume $\tilde{C} = 0$.

Answer: Here we work out the answer in general, and then impose $\tilde{C} = 0$ at the end, though it would have been much easier to impose it right away.

From the Euler equation, we have:

$$C_{t+1} = \beta(1+r_t)\left(C_t - \tilde{C}\right) + \tilde{C}$$

Previously, we found from the budget constraint, $C_{t+1} = (1+r_t)(Y_t - C_t) + Y_{t+1}$.

Equating the right-hand sides of these two equations, we have:

$$\beta(1+r_t)\left(C_t - \tilde{C}\right) + \tilde{C} = (1+r_t)(Y_t - C_t) + Y_{t+1}$$

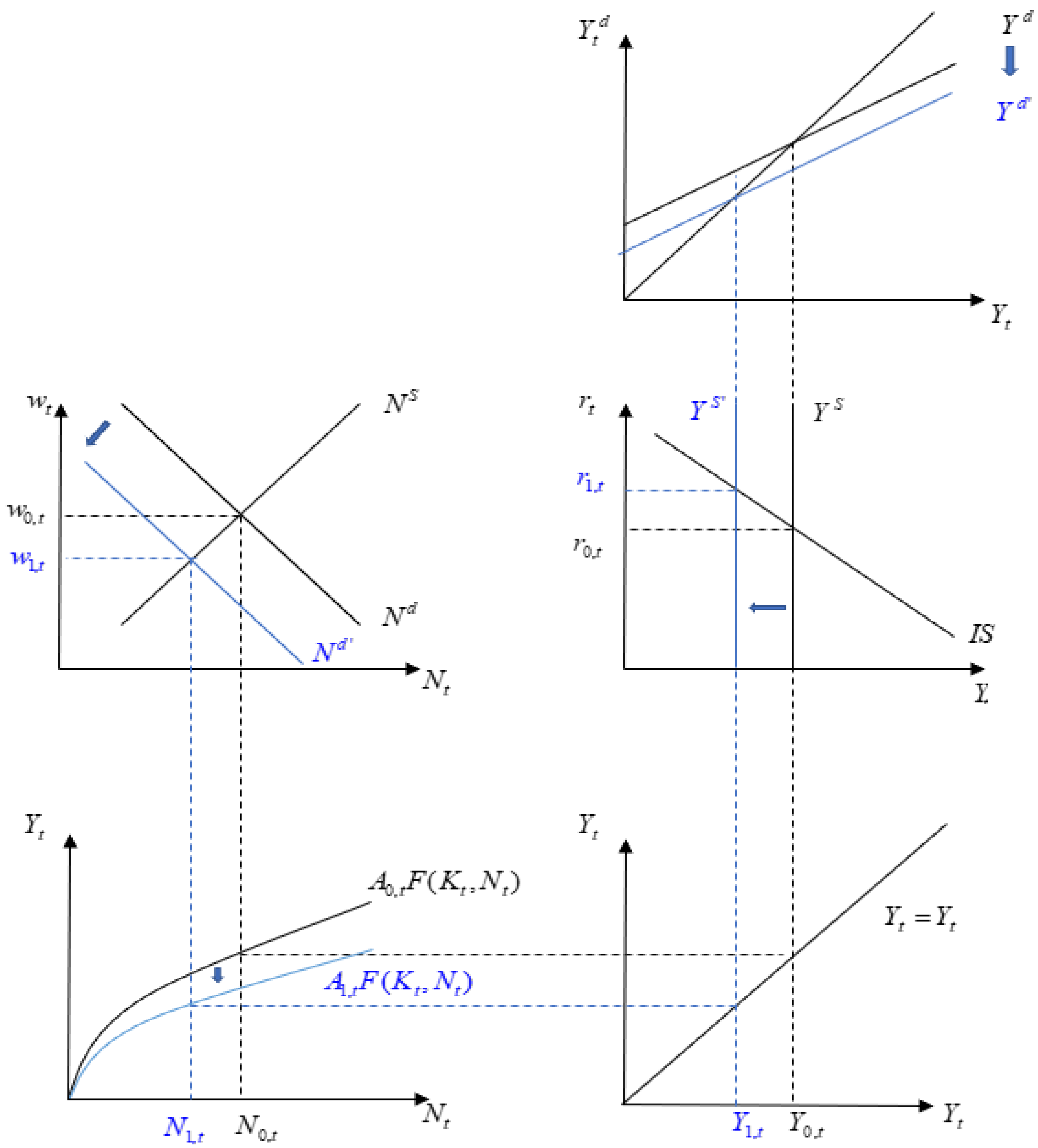
We can use those to get:

$$C_t = \frac{(1+r_t)Y_t + Y_{t+1} + \left[\beta(1+r_t) - 1\right]\tilde{C}}{(\beta+1)(1+r_t)}, \text{ which simplifies to}$$

$$C_t = \frac{(1+r_t)Y_t + Y_{t+1}}{(\beta+1)(1+r_t)} \text{ when } \tilde{C} = 0.$$

6.a. Draw the five graphs for the medium run that determine the real variables in the economy. Then show graphically how the graphs are affected by a decrease in total factor productivity.

Answer:



b. State whether each of the following variables increases, decreases, or stays the same when TFP falls: $Y_t$, $N_t$, $w_t$, $r_t$, $C_t$, $I_t$, $G_t$, $P_t$.

Answer:

$Y_t$ falls, $N_t$ falls, $w_t$ falls, $r_t$ rises, $C_t$ falls, $I_t$ falls, $G_t$ does not change, $P_t$ rises

7. Recalling that $Y_t = C_t + I_t + G_t$, explain how $C_t + I_t$ and $Y_t$ adjust, if at all, when $G_t$ increases. Explain the economic mechanism for the adjustment of $C_t + I_t$, or if it does not adjust, why not. Explain the economic mechanism for the adjustment of $Y_t$, or if it does not adjust, why not.

Answer: $Y_t$ does not change because output is only determined by the production function and the labor market, so it only changes when TFP changes or the labor supply changes.

$C_t + I_t$ must fall exactly as much as $G_t$ increases. An increase in the real interest rate makes $C_t + I_t$ fall.