Econ 702
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C. Engel

Homework 2

1. Suppose that the production function is the following:

$$Y_t = F(K_t, N_t) = K_t^{\alpha} N_t^{1-\alpha} + \gamma(K_t + cN_t)$$

It is assumed that $\gamma, c > 0$ and $0 < \alpha < 1$.

   a. Assume the price of the good is one. Show the profit of a firm with the above production function equals zero.

   b. What are the degrees of homogeneity for the first partial derivatives of output with respect to K_t and N_t ($F_K(K_t, N_t)$ and $F_N(K_t, N_t)$)?

   c. What is the sign (positive, negative or zero) of the cross partial derivative of output with respect to K_t and N_t ($F_{KN}(K_t, N_t)$)?

   d. Take the first partial derivative of the output per worker (Y_t/N_t) with respect to capital per worker (K_t/N_t). What value does the derivative converge to as $K_t/N_t \to \infty$?

2. Suppose that the production function is the following:

$$Y_t = AF(K_t, N_t) = A\left[\alpha K_t^{\frac{\nu-1}{\nu}} + (1-\alpha) N_t^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$$

It is assumed that $\nu \geq 0$ and $0 < \alpha < 1$.

   a. Prove that this production function features constant returns to scale.

   b. What are signs (positive, negative or zero) of the first partial derivatives of output with respect to K_t and N_t ($AF_K(K_t, N_t)$ and $AF_N(K_t, N_t)$)?

   c. What are signs (positive, negative or zero) of own second partial derivatives of output with respect to K_t and N_t ($AF_{KK}(K_t, N_t)$ and $AF_{NN}(K_t, N_t)$)?

   d. Express the capital per worker as a function of factor price ratio (W_t/R_t).

e. Under what conditions on the parameters is the labor share of output ($W_t N_t / Y_t$) is increasing or decreasing with the factor price ratio?