Econ 702
Spring 2020
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Homework 5

1. Consider a two-period model where a household chooses consumption C_t and C_{t+1}, and saving S_t, given the endowment Y_t and Y_{t+1} and interest rate r_t to maximize following lifetime utility $U = u(C_t) + \beta u(C_{t+1})$ with $u(C) = -\exp(-\alpha C)$, $\beta \in (0,1)$, $\alpha > 0$.

   (a) Note that $u(C) < 0$ for all consumption levels C. Why don't we need to worry about negative values of utility?

   (b) Show that the utility function $u(C)$ features a positive marginal utility and a diminishing marginal utility.

   (c) Write down the household problem for finding the optimal C_t and C_{t+1}. Derive the first-order conditions. Then solve for C_t as a function of Y_t, Y_{t+1}, r_t (and, of course, the parameters β, α).

2. Consider a consumer with a lifetime utility function

$$U = u(C_t) + \beta u(C_{t+1})$$

   that satisfies all the standard assumption listed in lecture. The lifetime budget constraint is

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t} \quad .$$

   (a) Suppose the first period optimal consumption C_t is less than Y_t, i.e. the consumer is saving at the first period. Graphically depict the optimality condition. Carefully label the intercepts of the budget constraint and optimal consumption points.

   (b) Suppose there is a decrease in future income Y_{t+1}. Graphically depict the effects of a decrease in Y_{t+1}. Carefully label the intercepts of the budget constraint and optimal consumption points.

3. Prove that $\widetilde{C}_t < \frac{Y_t + E_t Y_{t+1}}{2}$ at page 11 in the lecture slide: Optimal Consumption and Saving, part 2. (Hint: In proving this, it is helpful to bring the term involving the square root to one side of the inequality, and everything else to the other side, then square both sides.)