Human Capital Investment and College Sorting*

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January 10, 2022

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Abstract

College is traditionally viewed as one of the most important ways to promote upward mobility. However, there is a substantial over-representation of high-income families at the top of the college distribution. Difference in pre-college parental investment in human capital contributes to uneven access to the selective colleges. In this paper, I study how this investment affects sorting of students into colleges. I then estimate the efficiency of the decentralized allocation and explore the implications of pre-college investment for intergenerational mobility. To quantify the size of output losses due to the mismatch between students and colleges, I embed a student-to-college assignment model into a two-period overlapping generations model with endogenous human capital investment. Households compete for a fixed number of seats at the top-ranked colleges, and parental investment in their child's human capital promotes access to them. After controlling for human capital, I find that income and the parent's willingness to pay for a college education are major determinants of enrollment in highly selective colleges. Average human capital of enrolled students affects the quality of a college which partially mitigates the effect of income in the college admission process. The peer effects give rise to the tension between sorting students on willingness to pay and on human capital. I calibrate the model to NLSY97 cohort and find that the race to the top induces overinvestment in pre-college human capital and associated output losses relative to the first best. The effect is more pronounced for high-income families which promotes income persistence at the top of the college distribution.

*Special thanks to my advisors Simeon Alder, Ananth Seshadri, and Christopher Taber for their invaluable guidance and support on this project. I also thank Adibah Abdulhadi, Manuel Amador, Garrett Anstreicher, Job Boerma, Carter Braxton, Nisha Chikhale, Dean Corbae, Natalie Duncombe, Tim Kehoe, John Kennan, Rishabh Kirpalani, Annie Lee, Rasmus Lentz, Jingnan Liu, Paolo Martellini, Lois Miller, Minseon Park, Kim Ruhl, Jeffrey Smith, Kenneth West, Anson Zhou, and seminar and conference participants at the University of Wisconsin-Madison and Minnesota-Wisconsin Macro/International workshops for helpful comments and suggestions on this paper. All errors are my own.

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1 Introduction

College is traditionally viewed as one of the most important ways to promote upward mobility. The wage premium for college completion is high. A typical college graduate earns a premium of nearly 75 percent (Abel and Deitz (2019)). Mean earnings at the top of the college distribution are vastly different from those at the bottom. MIT graduates earn four times more than Alabama State graduates ($104,700 vs. $27,000) (Mountjoy and Hickman (2020)). Therefore, many parents think that just any college degree will not be enough for their children, and the relevant choice is which college to attend.

Sorting between family income and college quality is strong. There is a substantial overrepresentation of high-income families at the top of the college distribution. 49.9% of students at highly selective colleges come from the top quintile of income distribution, compared with 5.14% from the bottom quintile. The gap is even more pronounced at Ivy-Plus colleges[1]. 68% of students at the Ivy-Plus colleges come from the top quintile, compared with 3.8% from the bottom quintile. These shares are much closer (19.9% vs. 15.9%) in non-selective colleges (Chetty et al. (2017)).

The seats at the top-ranked colleges are scarce, which induces competition among households. Families compete on two margins: on the achievements and academic preparedness level of students; and on willingness to pay for a college and thus income itself. First, conditional on income, colleges sort students based on their human capital. The average incoming student at MIT scored 1530 (out of 1600) on the SAT, while average scores at Alabama State are around 860. Since human capital can be influenced by parental investments made during childhood (e.g., Cunha et al. (2010) and Del Boca et al. (2014)), parents compete with each other by making investments in the pre-college human capital of their children. This allows them to jump ahead of other students in this college admission tournament. Such competing may give rise to parental overinvestment.

Across income level, overinvestment made by high-income parents is larger than by low-

[1] Ivy Plus: Brown University, Harvard University, Cornell University, Princeton University, Dartmouth College, Yale University, Columbia University, University of Pennsylvania, Stanford University, MIT, University of Chicago, California Institute of Technology, Northwestern University, Duke University, Johns Hopkins University, Vanderbilt University, Rice University, and Washington University in St. Louis.

income families. High-income families spend three times more time and money on their children than low-income families. Overinvestment in response to the race to the top results in the crowding out of low-income students from highly selective colleges and thus promotes intergenerational persistence in income. This crowding out effect will be amplified by the tuition mechanism. High preparedness and high willingness to pay students take seats at the top-ranked colleges, increasing tuition level at these colleges, and this is reflected in the high tuition charged by selective-enrollment colleges. The average tuition at MIT is $53,790, while the average tuition at Alabama State is between $11,068 and $19,396.

How does pre-college parental investment shape how students sort into colleges? How efficient is it? What are the implications of pre-college investment for the intergenerational mobility? In this paper, I study how pre-college human capital investment affects the sorting of students into colleges. I estimate the effectiveness of the decentralized allocation and explore the implications of pre-college investment for intergenerational mobility. To quantify the size of output losses due to the mismatch between students and colleges, I embed a student-to-college assignment model into a two-period overlapping generations model with endogenous human capital investment. The economy is populated by heterogeneous families and colleges. Households differ in their income and their children's ability level, while colleges differ in their underlying exogenous quality. Families consist of one child and one parent. Heterogeneous colleges have a fixed number of seats and improve chances of their graduates to have high income. Since parents care about the prosperity of their children, they want their kids to attend selective enrollment colleges. Therefore, households compete for seats at these colleges, and parental investment in their child's pre-college human capital promotes access to them. Low-income parents are not able to catch up with the level of investment of their high-income peers. Hence, after controlling for human capital, income and hence the parent's willingness to pay for a college education are major determinants of enrollment in the top-ranked colleges.

The average human capital of enrolled students affects the effective, endogenous quality of a college as in Rothschild and White (1995), Epple et al. (2006), Capelle (2020), Cai and Heathcote (2021), among others. Although, in my model, colleges care only about maximizing their tuition, they have incentives to attract low-income high human capital students because it

increases the quality of the college and, thus, allows colleges to extract higher tuition. This peer effect partially mitigates the effect of income in the college admission process. It creates tension between sorting households on willingness to pay and on human capital.

The key innovation of this paper is to combine endogenous parental investment decisions and student-to-college assignment in a general equilibrium dynastic framework. In contrast to the earlier literature, the model captures the persistent over-representation of students from high-income households in the top-ranked colleges. The model features several sources of inefficiency: rank externalities, peer effects, and incomplete insurance markets. Rank externalities capture the fact that parents worry about losing a spot at the top-ranked colleges because other families make larger investments and thus push them down in the college tournament. Peer effect is not internalized by households, so families would like to get into better colleges without realizing that their human capital affects the effective quality of these colleges. Market incompleteness features the fact that parents are unable to purchase insurance on the ability of their grandchildren. The model allows me to quantify the efficiency losses and to explore the contributions of different sources of inefficiency by shutting them down one-by-one.

I calibrate the model to match moments in the National Longitudinal Survey of Youth of 1997 (NLSY97). The model matches the patterns of human capital investment, college choice, student body composition and tuition schedules at colleges of different qualities. I then characterize the efficient allocation and quantify the output losses generated by incompleteness and the various inefficiencies in the decentralized equilibrium. I find that the race to the top induces parental overinvestment in pre-college human capital which lowers output by 3.6%. In counterfactual experiments, I shut down the rank externalities, so parents can no longer use their investment to improve the chances of their children to get into the top-ranked colleges. I find that sorting students on ability alone decreases investment by 19.5% because investment no longer plays a role in the access to colleges, and output declines by 2.1%. Next, I shut down the peer effects and I estimate that investment drops by 6.8% below the efficient level, and output falls by 5.5%. Lastly, I use the model to conduct a policy experiment where I impose sorting of students by their pre-college human capital regardless of income. I find that this policy promotes human capital investment, raises output due to improved effective

college quality, and enhances mobility for low-income families.

Related Literature

This paper builds on several strands of the literature. The first models and quantifies the transmission of human capital, educational choice, and inequality in an intergenerational framework. For instance, Blandin and Herrington (2021), Erosa and Koreshkova (2007), Fernández (2003), Lee and Seshadri (2019), and Restuccia and Urrutia (2004) focus on the role of parental investment in children in the transmission of inequality. In contrast to this paper, they model a representative college, and households making binary decisions about college enrollment. In my model, households choose between colleges of different quality, which allows me to study the sorting of heterogeneous students across heterogeneous colleges. Compared to the most closely related papers that focus on higher education and allow for the heterogeneity in college quality (Cai and Heathcote (2021) and Capelle (2020)), a key novelty of my framework is that I endogenize student's human capital, which allows me to investigate the interaction between college sorting and human capital investment decisions and quantify the size of the rank externality.

Another branch of the literature explicitly models the admission and tuition setting decisions of colleges accounting for rich heterogeneity in colleges and student types (Bodoh-Creed and Hickman (2018), Epple et al. (2006), Fu (2014), Rothschild and White (1995), among others). I capture some of these features, such as heterogeneity in student and college types, and peer effects. There is also the literature that explore human capital formation in the college admission tournament (Buchmann et al. (2010), Dang (2007), Gurun and Millimet (2008), Krishna et al. (2018), Liu (2020), among others). This paper differs from the literature because it embeds sorting into an intergenerational framework and focuses on the equilibrium aggregate output losses rather than sorting itself.

Finally, this paper complements the empirical literature on the role of parental background for achievements and access to top colleges (e.g. Bailey and Dynarski (2011), Chetty et al. (2011), Hoxby and Turner (2019)). It takes a quantitative approach to show a large role of parental income, and thus human capital investment, in promoting access to selective colleges. It also

builds on the literature on the effects of college choice on the labor market outcomes. A large body of literature has estimated the return to college quality. On the one hand, the evidence on the effects of college selectivity on earnings is mixed. Several papers, including Hoxby (2001), find significant gains to attending more selective colleges. Others, such as Dale and Krueger (2002), Dale and Krueger (2011), and Mountjoy and Hickman (2020), have concluded that selectivity is a poor predictor of value added. However, Dale and Krueger (2002) and Dale and Krueger (2011) report that there are significant returns to college quality for minorities and for children of less-educated parents. On the other hand, non-peer college inputs more strongly predict labor market outcomes. Using multiple proxies for college quality, Black and Smith (2004) and Black and Smith (2006) show that the returns to college quality are significant. Dillon and Smith (2019) find evidence for the complementarity between college quality and student's ability for long-run earnings outcomes.

The structure of the paper is as follows: Section 2 lays out the model. Section 3 characterizes the planner's problem. Section 4 focuses on the decentralized equilibrium. Section 5 describes the data used for the calibration. Section 6 explains the calibration strategy. The quantitative results and counterfactual experiments are presented in Section 7. Section 8 concludes.

2 Model

This model combines a two-period OLG model with endogenous human capital investment, à la Becker and Tomes (1986) and a student-to-college assignment model with differential rents as in Sattinger (1979), Terviö (2008), and Alder (2016) and peer effects as in Rothschild and White (1995). The model features sorting on students' willingness to pay and human capital. It generates the strong over-representation of high-income families at the top of the income distribution and captures patterns in the composition of high-income and low-income students at each college and the size of human capital investment.

2.1 Agents and Preferences

The model is populated by two types of agents: households and colleges. There is a mass $N > 1$ of households. Individuals live for two periods: one as a child and one as an adult. Each adult has one child. A household is characterized by parental income and parental ability. For the purpose of tractability, I assume two levels of income: high and low. The endogenous share of high-income parents is denoted by μ_t. In the first period, a child with ability a lives with her parent with income y_s, gets education, and does not consume. Without loss of generality, I assume income to be either high y_H or low y_L. Her parent imperfectly transmits ability to her child, chooses consumption c, size of pre-college human capital investment m, and college quality. In the second period, a child separates from her parent and becomes a parent herself. She consumes, invests in her child, and pays for the child's education. Parents are dynastically altruistic and cannot borrow against the future income of their children. Households have preferences over their own consumption c and the expected welfare of their children:

$$V(a, y_s) = u(c) + \lambda\beta EV(a', y')$$

$$y'_s = \begin{cases} y_H & \text{with endogenous probability } p, \\ y_L & \text{with endogenous probability } 1-p. \end{cases}$$

A child is characterized by her inherited ability. Her expected income is a function of parental human capital investment and college choice. The heterogeneity across parents generates heterogeneity in investment and, thus, human capital and access to colleges of higher quality for their children. This, in turn, generates cross-sectional heterogeneity in incomes and intergenerational persistence.

There is a measure $M = 1$ of colleges with quality q drawn from an exogenous distribution F_q and a fixed number of seats. For simplicity, I assume that each college can admit the equivalent of one student, possibly a convex combination of students with high and low-income parents. Colleges produce after-college human capital and collect tuition fees from the parents.

Colleges are assumed to be owned by risk-neutral entrepreneurs. A college with effective

quality $\tilde{q}$ maximizes tuition revenue collected from high-income students $\tau[l_H, \tilde{q}|y_H]$, denoted as $\tau_H[l_H, \tilde{q}]$, and tuition collected from low-income students $\tau[l_L, \tilde{q}|y_L]$, denoted as $\tau_L[l_L, \tilde{q}]$, by choosing the composition of high-income and low-income students. College can differentiate between low and high-income students.

$$\int \mathbb{1}_{\tilde{q}=\Omega_H(l_H)} \tau_H[l_H, \tilde{q}] dG(l_H)\mu N + \int \mathbb{1}_{\tilde{q}=\Omega_L(l_L)} \tau_L[l_L, \tilde{q}] dG(l_L)(1-\mu)N,$$

where $G(l_s)$ is an endogenous distribution of human capital given income y_s, and $\tilde{q} = \Omega_s(l)$ is an income-specific matching function between students and colleges. One can think of it as a Dean or faculty maximizing revenue collected from undergraduate students and investing it in research.

I assume that colleges perfectly observe students' human capital and their parents' income level. In the real world, colleges cannot perfectly observe either of these. The environment that allows for asymmetric information is an interesting extension.

The government provides public pre-college education to all students. Its cost is financed with a labor income tax.

2.2 Technology

2.2.1 Pre-college technology

Pre-college human capital depends on parental investment m, public per student expenditure on pre-college education $\bar{m}$, and inherited ability a which follows an intergenerational AR(1) process:

$$l = l(a, m) = a(\bar{m} + m)^{\alpha_1}$$

$$\log(a') = (1-\rho_a)(\mu_a - \sigma_a^2/2) + \rho_a \log(a) + \epsilon_a,$$

$$\text{where } \epsilon_a \sim N(0, (1-\rho_a^2)\sigma_a^2).$$

2.2.2 College technology

Colleges combine students' pre-college human capital l and effective college quality $\tilde{q}$ to produce post-college human capital h:

$$h = h(l, \tilde{q}) = l\tilde{q}^{\alpha_2}.$$

Effective college quality is a function of the exogenous underlying college quality q and the average human capital of all students that attend the college:

$$\tilde{q} = q \cdot \Big(1 + (s_H l_H + (1 - s_H) l_L)\Big)^{\theta}$$

$$\log(q) \sim F_q,$$

where s_H is a share of high-income students. Colleges internalize this externality through tuition.

After-college human capital affects the probability of being a high-income individual upon becoming an adult. This probability is increasing in after-college human capital:

$$p = p(h) \in [0, 1]$$

3 Planner's Problem

I define the efficient allocation as an allocation in which it is not possible to have more consumption at some date without having less consumption at some other date. The planner determines the college choice $q_t^*(a)$, investment $m_t^*(a)$, and consumption $c_t^*(a)$ for each child with ability a. Any allocation that maximizes the following is efficient.

$$\max_{c_t(a), m_t(a) \geq 0, q_t(a)} \sum_{t \geq 0} \beta^t \int c_t(a) dF_t(a)$$

subject to

$$\int c_t(a)dF_t(a)N + \int m_t(a)dF_t(a)N = y_H \mu N + y_L(1-\mu)N$$

$$\log(a') = (1-\rho_a)(\mu_a - \sigma_a^2/2) + \rho_a \log(a) + \epsilon_a$$

$$\epsilon_a \sim N(0, (1-\rho_a^2)\sigma_a^2)$$

$$l = a(\bar{m} + m)^{\alpha_1}$$

$$h = l\tilde{q}^{\alpha_2}$$

$$p = 1 - \exp(-\alpha_3 h)$$

$$\tilde{q} = q \cdot \Big(1 + (s_H l_H + (1-s_H)l_L)\Big)^{\theta}$$

$$\log(q) \sim F_q$$

$$\mu_{t+1} = \int p(m_t(a), a)dF_t(a)$$

Without loss of generality, consider the problem:

$$\max_{c_t(a), m_t(a) \geq 0, q_t(a)} c_t + \beta c_{t+1}$$

The planner does not take into account parental labor market productivity and allocates students to colleges according to their ability a. The resulting allocation features positive assortative matching (PAM) in ability a and college quality q. Given sorting, the planner faces the problem of investment allocation:

$$\max_{m_t(a)} \mu y_H + (1-\mu)y_L - \int m_t(a)dF_t(a) + y_L + (y_H - y_L)\int p(a, m_t)dF_t(a) - \int m_{t+1}(a)dF_{t+1}(a)$$

The first order condition (FOC) captures the fact that the planner internalizes the peer effect and invests in a child until the point where the marginal return equals the extra dollar of investment. The first order condition is:

$$1 = \beta(y_H - y_L) \cdot \frac{\partial p}{\partial h} \cdot \Big(\frac{\partial h}{\partial l} + \frac{\partial h}{\partial \tilde{q}} \cdot \underbrace{\frac{\partial \tilde{q}}{\partial l}}_{\text{peer effect}}\Big) \cdot \frac{\partial l}{\partial m}$$

4 Decentralization

4.1 Household Problem - HP

The parent of a child with ability a and with income y_s chooses human capital investment m, consumption c, and college quality $\tilde{q}$ to maximize:

$$V(a, y_s) = \max_{c,m\geq 0,\tilde{q}} u(c) + \lambda\beta EV(a', y')$$

subject to

$$c + m + \tau_s[l, \tilde{q}] = y_s^d$$

$$\log(a') = (1-\rho_a)(\mu_a - \sigma_a^2/2) + \rho_a \log(a) + \epsilon_a$$

$$\epsilon_a \sim N(0, (1-\rho_a^2)\sigma_a^2)$$

$$l = a(\bar{m} + m)^{\alpha_1}$$

$$h = l\tilde{q}^{\alpha_2}$$

$$p = 1 - \exp(-\alpha_3 h)$$

$$y' = \begin{cases} y_H & \text{with endogenous probability } p, \\ y_L & \text{with endogenous probability } 1-p \end{cases}$$

The household pays income-specific tuition τ_s to the college attended. The college choice of a household with income y_s and pre-college human capital l is characterized by the matching function $\Omega_s(l)$. The assignment mechanism is described in Section 4.3.

A household's investment choice affects effective college quality. However, households do not internalize this peer effect when they make their decisions about investment. They would like to attend colleges of higher quality without realizing that if student's human capital is low, it decreases effective college quality. The household takes the effective college quality

and tuition schedule as given. Then, her first order condition is the following:

$$u'(c) \cdot \Big(1 + \frac{\partial \tau_s}{\partial \Omega_s} \cdot \underbrace{\frac{\partial \Omega_s}{\partial l}}_{\text{slope of matching function}} \cdot \frac{\partial l}{\partial m}\Big) =$$

$$= \lambda\beta\Big(EV(a', y_H) - EV(a', y_L)\Big) \cdot \frac{\partial p}{\partial h} \cdot \Big(\frac{\partial h}{\partial l} + \frac{\partial h}{\partial \Omega_s} \cdot \underbrace{\frac{\partial \Omega_s}{\partial l}}_{\text{slope of matching function}}\Big) \cdot \frac{\partial l}{\partial m},$$

where $\tilde{q} = \Omega_s(l)$ is a matching function. By changing human capital, the household could move along the college quality distribution, ignoring the fact that her decision changes the distribution of college qualities.

4.2 College Problem - CP

Since colleges are owned by risk-neutral entrepreneurs, they maximize tuition revenue collected from students. Each college admits the equivalent of one student, possibly the convex combination of high and low-income students. Colleges compete with other colleges for students with the highest human capital and the highest willingness to pay. Colleges charge income- and human capital-specific tuition fees $\tau_s(l, q)$.

Colleges maximize total tuition subject to a fixed number of seats by selecting the composition of high-income and low-income students who are paying tuition fees $\tau_H(l_H, q)$ and $\tau_L(l_L, q)$.

$$\max_{l_H, l_L} \underbrace{\int \mathbb{1}_{\tilde{q}=\Omega_H(l_H)} \tau_H[l_H, \tilde{q}] dG(l_H) \mu N}_{\text{tution from high-income students}} + \underbrace{\int \mathbb{1}_{\tilde{q}=\Omega_L(l_L)} \tau_L[l_L, \tilde{q}] dG(l_L)(1-\mu) N}_{\text{tution from low-income students}}$$

subject to:

$$\int \mathbb{1}_{\tilde{q}=\Omega_H(l_H)} dG(l_H) \mu N + \int \mathbb{1}_{\tilde{q}=\Omega_L(l_L)} dG(l_L)(1-\mu) N = \frac{\partial F_{\tilde{q}}}{\partial \tilde{q}} M,$$

where $G(l_s)$ is an endogenous distribution of human capital given income y_s and $F_{\tilde{q}}$ is an endogenous distribution of effective college quality. The assignment mechanism behind the matching function $\Omega_s(l)$ is described in Section 4.3.

4.3 Assignment

The mechanism assigns one college to the equivalent of one student, possibly a convex combination of students with high and low-income parents. I sort student to colleges from two different income groups. Thus, there is an income-specific matching function between students and colleges $\tilde{q} = \Omega_s(l)$. Given that a college selects a high-income student endowed with a particular l_H and a low-income student endowed with a particular l_L rather than marginal units, this problem does not have standard first-order necessary conditions. Stability requires that the matching function and the payoffs satisfy resource, sorting, and participation constraints.

Given income and pre-college human capital, inherited ability is known. Therefore, I label each high student a_H and each low student a_L. $\tilde{q}$ identifies the effective college quality associated with a particular college with quality rank j. Then, $\tau_H[a, j]$ is a tuition fee associated with a particular pair (a_H, j), and $\tau_L[a, j]$ is a tuition fee associated with a particular pair (a_L, j).

The assignment problem's sorting and participation conditions are the following:

Conditional on income level s, household $\hat{a}$ prefers college j to any other college $j' \neq j$:

$$u(y_s - m - \tau_s[\hat{a}_s, j]) + \lambda\beta p(l(\hat{a}_s), \tilde{q}[j])EV(a', y_H) + \lambda\beta(1 - p(l(\hat{a}_s), \tilde{q}[j]))EV(a', y_L) \geq$$

$$\geq u(y_s - m - \tau_s[\hat{a}_s, j']) + \lambda\beta p(l(\hat{a}_s), \tilde{q}[j'])EV(a', y_H) + \lambda\beta(1 - p(l(\hat{a}_s)\tilde{q}[j']))EV(a', y_L) \quad \text{(SC1)}$$

$$u(y_s - m - \tau_s[\hat{a}_s, j]) + \lambda\beta p(l(\hat{a}_s), \tilde{q}[j])EV(a', y'_H) + \lambda\beta(1 - p(l(\hat{a}_s), \tilde{q}[j]))EV(a', y_L) \geq$$

$$\geq u(y_s - m) + \lambda\beta p(l(\hat{a}_s), q_0)EV(a', y_H) + \lambda\beta(1 - p(l(\hat{a}_s), q_0))EV(a', y_L) \quad \text{(PC1)}$$

Conditional on income level s, college j prefers student $\hat{a}$ to any other student $a \neq \hat{a}$:

$$\tau_s[\hat{a}_s, j] \geq \tau_s[a_s, j] \quad \text{(SC2)}$$

$$\tau_s[\hat{a}_s, j] \geq 0 \quad \text{(PC2)}$$

4.4 Government

The government spends $\bar{m}$ on the public pre-college education of each child. These expenditures are financed by labor income taxes. The tax schedule is characterized by the average tax rate t_a and progressivity parameter t_m. A household's disposable income is:

$$y^d = (1 - t_a) y_s^{1-t_m} T,$$

where T is an endogenous parameter such that t_a is an average income tax rate. The government budget is balanced each period.

4.5 Equilibrium

4.5.1 Definition

A stationary equilibrium is characterized by tuition schedules for high and low-income students $\tau_H[\hat{a}_H, j]$ and $\tau_L[\hat{a}_L, j]$, rank cutoffs for college attendance, parental investment m, and consumption c such that:

1. Households solves HP;
2. Colleges solves CP;
3. The allocation is in the core of the assignment game;
4. College market clears:

$$(1 - F_H(\hat{a}))\mu N + (1 - F_L(\hat{a}))(1 - \mu)N = (1 - j)M;$$

5. Government has balanced budget;
6. The share of high-income households μ is stationary.

4.5.2 Characterization of the Equilibrium

Households solve HP. The investment decision depends on the college assignment and tuition schedule that are taken as given.

Within income group. Conditional on income level, the equilibrium involves perfect sorting by human capital. Two types of conditions must hold in a competitive equilibrium. Student's sorting and participation constraints (SC1 and PC1) and college's sorting and participation constraints (SC2 and PC2) must be satisfied.

The marginal student $\underline{a}$ is the one who is indifferent between attending and not attending college.

$$u(y_s - m - \tau_s[\underline{a}, \underline{j}]) = u(y_s - m) + \lambda\beta(EV(a', y'_H) - EV(a', y'_L))(p(l(\underline{a}), q_0) - p(l(\underline{a}), \underline{j}))$$

Re-grouping sorting constraints yield the slope of the tuition schedule:

$$\tau'[\hat{a}_s, j] = \frac{\lambda\beta(EV(a', y_H) - EV(a', y_L))}{u(c)'} \frac{\partial p(l[\hat{a}], \tilde{q})}{\partial \tilde{q}} \frac{\partial \tilde{q}}{\partial q} \frac{\partial q}{\partial j}$$

The marginal student is indifferent between going to college and not. Therefore, from the participation constraint for students (PC1), tuition for a marginal college $\underline{j}$ is given by:

$$v_s = \tau_s[\underline{a_s}, \underline{j}].$$

The derivation of tuition for a marginal college is presented in the Appendix.

Relative to the standard assignment model, the slope of the tuition profile depends not only on the marginal contribution of college to surplus and the slope of college quality distribution but also on the household's marginal utility.

The tuition profile is characterized by:

$$\tau[\hat{a}_s, j] = v_s + \int_{\underline{j}}^{j} \tau'[\hat{a}_s, k] dk.$$

The derivation of tuition schedule can be found in the Appendix.

The conditions for the stable match to be positive assortative and a one-to-one correspondence between of students and colleges are in the Appendix. Let us assume the conditions for the assortative matching are satisfied.

Across income groups. Given human capital l, high-income students have higher willingness to pay than low-income students, therefore, colleges can extract higher tuition fee from high-income students. If there are no peer effects, colleges sort everyone by willingness to pay.

Now, let us consider the case when the quality of students affects college quality, for example, college quality depends on the average human capital of all students:

$$\tilde{q} = q \cdot \Big(1 + (s_H l_H + (1 - s_H) l_L)\Big)^{\theta},$$

where s_H is a mass of high-income students.

In this case, colleges face a trade-off between attracting students with higher willingness to pay and lower human capital or students with lower willingness to pay and higher human capital. In the former case, colleges extract higher tuition from the group with higher willingness to pay. Attracting students with lower human capital reduces effective college quality, and thus, tuition collected from both groups will be lower due to negative peer effects. In the latter case, colleges collect lower tuition from the group with lower willingness to pay. This strategy increases effective college quality, and thus, college could extract higher tuition from both groups due to positive peer effects.

Assume that there is a mapping: $\tilde{q} = \Omega_s(l)$. Colleges maximize total tuition subject to the fixed number of seats by selecting the composition of high-income and low-income students who are paying tuition fees $\tau(l, q|y_s)$.

$$\max_{l_H, l_L} \int \mathbb{1}_{\tilde{q}=\Omega_H(l_H)} \tau_H[l_H, \tilde{q}] dG(l_H) \mu N + \int \mathbb{1}_{\tilde{q}=\Omega_L(l_L)} \tau_L[l_L, \tilde{q}] dG(l_L)(1-\mu) N$$

subject to the capacity constraint:

$$\int \mathbb{1}_{\tilde{q}=\Omega_H(l_H)} dG(l_H) \mu N + \int \mathbb{1}_{\tilde{q}=\Omega_L(l_L)} dG(l_L)(1-\mu) N = \frac{\partial F_{\tilde{q}}}{\partial \tilde{q}} M$$

The FOC for the split is presented in the Appendix.

Marginal students $\underline{l_H}$ and $\underline{l_L}$ are pinned down by the capacity constraint of all colleges:

$$(1 - F_{\tilde{q}}(\underline{\tilde{q}}))M = (1 - G_H(\underline{l_H}))\mu N + (1 - G_L(\underline{l_L}))(1 - \mu)N$$

5 Data

The core dataset is the restricted-use version of NLSY97. It contains data on parental income, college attended, and household's income. The sample consists of students who have reported information on parental income, college attendance, and children's income. I complement this data with data on college characteristics from the Integrated Postsecondary Education Data System (IPEDS), data on the income distribution from Census and American Community Surveys (ACS) (Ruggles et al. (2021)), and data on expenditures from the Consumer Expenditure Survey (CEX). All prices are calculated in 1990 dollars.

Colleges. The restricted-use version of NLSY97 data provides information on the college identifiers that are linked to the IPEDS. The IPEDS annual surveys provide college-level information on tuition, enrollment, and the distribution of test scores within each college. I aggregate information from 2000-2009 surveys, when the NLSY cohort was attending colleges. The sample consists of 1,774 public and private not-for-profit 4-year colleges. Using Barron's selectivity criteria, highly selective colleges account for 13% of the sample.

I construct a latent college quality index that is used for ranking colleges. I measure it using the first principal component of two indicators reported in IPEDS: the average salary of all instructional faculty and the student-faculty ratio. First, I estimate the first principal component using these quality measures based on the colleges that report both indicators. Second, I use factor loadings to construct a weighted average of these quality measures. Finally, I calculate percentiles of this index, weighted by the number of undergraduate students.

Next, the IPEDS data provides information on average tuition and fees. Figure 1 shows the distribution of average tuition and fees, after deducting discounts and allowances, in log-log space. The plot shows that the empirical distribution of colleges is fitted best by a log-

normal distribution. The maximum likelihood estimates for the shape and scale parameters are $\mu = 8.565$ and $\sigma = 0.671$, respectively.

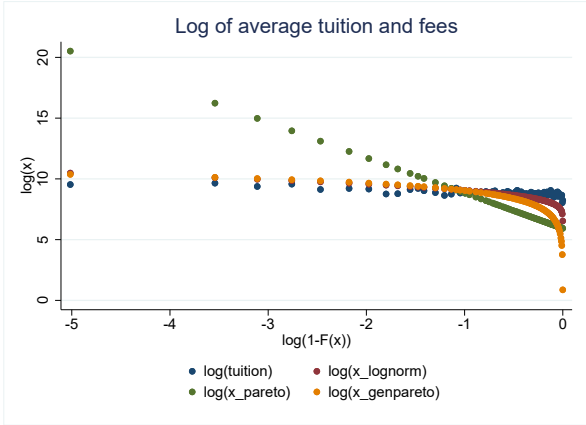

Figure 1: Average tuition and fees distribution

Income. I use 1990 and 2000 Census and 2010 ACS data to explore the distribution of household income for families with children of age 0-18. Figure 2 shows the distribution of household income for families with children of age 0-18 (in 1990 dollars). The distribution is illustrated up to 0.995. The 75th percentile corresponds to an income of $57,614 in 1990 dollars. Individuals with income below $57,614 are considered low-income, then those with the income above it are classified as high-income. Low income corresponds to the median income of the bottom 75%, which is $28,055 in 1990 dollars.

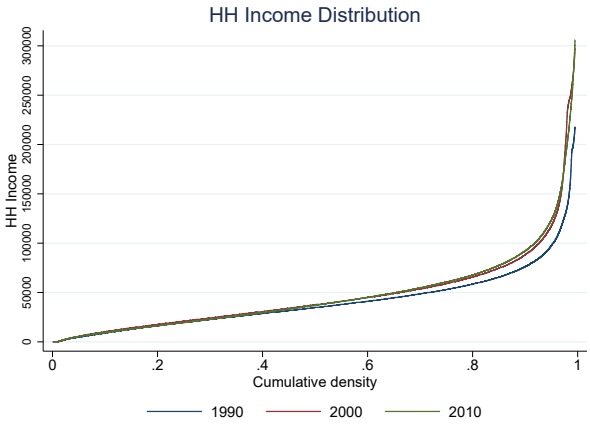

Figure 2: Income distribution

Investment. Because the NLSY97 does not contain data on child pre-college expenditures, I

complement NLSY data with the data on private expenditures from the CEX. I examine CEX surveys from 1992 to 2003, when individuals from the NLSY97 were attending school. To capture pre-college investments, I focus on households with children of age 0-18 and calculate average annual expenditures per child. Similar to Blandin and Herrington (2021), I include a range of expenditures on children that may plausibly contribute to human capital formation, including books, toys, games, computers, musical instruments, childcare, primary/secondary school tuition, and tutoring. Money investments in children are extremely noisy. In addition, the measure captures some consumption goods rather than investment, so I assume that only half of it is human capital investment (Lee and Seshadri (2019)).

The National Center for Education Statistics (NCES) reports that annual public expenditures on K-12 is $10,800 in 2008 dollars. This corresponds to $6,571 in 1990 dollars. Since these expenditures do not cover children from 0 to 6, the average annual expenditures per one year of a child's life are $4,746 per child annually.

6 Calibration

The aim of the calibration is to parametrize the economy to match aggregate and micro moments that characterize pre-college parental investment, college sorting, student body composition, and tuition schedules for the NLSY97 cohort to their model counterparts. I first discuss parameters set independently, followed by 10 parameters that are computed from a method of moments by numerically simulating the model. Parameters are summarized in Table 1.

6.1 Parameters set independently

Utility has constant relative risk aversion over consumption.

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

Following recent papers in the college attainment literature, I set $\gamma = 2$ (e.g., Lee and Seshadri (2019), and Blandin and Herrington (2021)). An annual discount rate β is set to the standard

of 0.98.

Next, the ratio of income levels y_h/y_l is informed by the ratio of median income of the top 25% to median income of the bottom 75% which is estimated to be approximately 2.776 in 1990, 2000, and 2010. Low income y_l is a numéraire and corresponds to \$28,055 in 1990 dollars.

A student-to-college ratio is set to match the 3-year moving average of the percent of recent high school completers enrolled in college reported by the National Center for Education Statistics (NCES) in 2002. The number of colleges is a numéraire, then the number of students is 1.57.

The slope of the income tax schedule $t_m = 0.23$ is estimated by Heathcote et al. (2017). Public pre-college expenditures are estimated to be 11.7% of total income. Hence, I set $b = 0.117$. Since public expenditures on education are financed by labor income tax, the average income tax rate t_a is equal to the share of per-student public expenditure in total income.

6.2 Method of Moments

The remaining 10 parameters are set jointly by targeting empirical moments. In particular, I choose them to minimize the distance between moments simulated by the model and their empirical counterparts from the NLSY97, IPEDS, and CEX data. In this section, I explain which moment is important for each parameter. Table 2 presents targeted moments and model fit.

Ability of a child follows an AR(1) process:

$$\log(a) = (1 - \rho_a)(\mu_a - \sigma_a^2/2) + \rho_a \log(a_p) + \epsilon_a$$

$$\text{where } \epsilon_a \sim N(0, (1 - \rho_a^2)\sigma_a^2)$$

The mean of abilities, μ_a, governs the probability of being high-income if an individual did not attend college. The estimate is calculated from NLSY97. The variance of abilities, σ_a^2, determines the average ratio of 75th to 25th percentiles in SAT scores. I calculate the ratios for each college using IPEDS and take their average. The exogenous persistence of abilities, ρ_a,

Parameter	Description	Value	Source
γ	Risk aversion	2	Lee and Seshadri (2019)
β	Discount factor (annual)	0.98	standard
y_h/y_l	High income-to-low income ratio	2.776	Census
N/M	Student-to-college ratio	1.57	NCES
t_m	Tax function param.	0.23	Heathcote et al. (2017)
b	Public spend./ Tot. income	0.117	NCES
μ_a	Mean of learning abilities	0.85	NLSY97
σ_a	Std of learning abilities	0.3	IPEDS
ρ_a	Persistence of learning abilities	0.28	NLSY97
μ_q	Mean college quality	1.6	NLSY97
σ_q	Std of college quality	0.45	NLSY97
α_1	Elasticity in pre-college HC fn	0.77	CEX
α_2	Elasticity in college HC fn	0.62	NLSY97
α_3	Probability fn parameter	0.25	Baseline
λ	Altruism	0.46	CEX
θ	Peer effect	0.1	NLSY97

Table 1: Parameters

Parameter	Description	Moment	Data	Model
μ_a	Mean of learning abilities	Prob of high income after no college	0.077	0.064
σ_a	Std of learning abilities	Aver. SAT p75/p25	1.25	1.25
ρ_a	Persistence of learning abilities	IG persistence of low income	0.173	0.193
μ_q	Mean college quality	Prob of high income after attending top 20% coll	0.42	0.45
σ_q	Std of college quality	Ratio of high-income adults from high-income rel. to low-income background	2.5	2.5
α_1	Elasticity in pre-college HC fn	Public invest/Total invest	0.829	0.843
α_2	Elasticity in college HC fn	IG persistence of high income	0.429	0.429
α_3	Probability fn parameter	Share of high-income families	0.25	0.26
λ	Altruism	Aver. tot. invest/Aver. income	0.141	0.139
θ	Peer effect	Share of low-income at top 20% coll	0.50	0.44

Table 2: Targeted Moments

governs the probability of becoming a low-income adult if one's parents were low-income. I measure it from the NLSY97 and verify the number by comparing it with Chetty et al. (2017). The parameters are close to their analogs in Lee and Seshadri (2019).

College quality is drawn from a log-normal distribution:

$$\log(q) \sim N(\mu_q, \sigma_q^2)$$

I calibrate the mean of underlying college quality, μ_q, to match the probability of becoming a high-income adult after attending a college in the top 20% of colleges. The moment calculated

from the NLSY97 is similar to the one reported by Chetty et al. (2017). I use the share of high-income households from high-income background over the share of high-income students from low-income background in order to calibrate the variance, σ_q.

The effective college quality has a form of generalized mean function:

$$\tilde{q} = q \cdot \Big(1 + (s_H l_H + (1 - s_H) l_L)\Big)^{\theta}$$

The parameter θ indicates the strength of the peer effect. It governs the share of students from low-income families among all students who attend the top 20% of colleges, which is measured using NLSY97 data. The parameter is difficult to compare with other papers such as Capelle (2020) and Cai and Heathcote (2021) because their functional forms differ from mine and reflect the amount of educational services and student ability instead of human capital.

The altruism parameter λ is related to the ratio of average total investment to the average income. The value of the parameter is between the values from Lee and Seshadri (2019) and Daruich (2020).

The pre-college human capital production function is assumed to be a standard Cobb-Douglas function:

$$l = a(\bar{m} + m)^{\alpha_1}$$

The parameter α_1 determines the ratio of public investment relative to total investment, estimated from NCES data.

I assume that the post-college human capital production function is the following:

$$h = l\tilde{q}^{\alpha_2}$$

I calibrate the parameter α_2 to match intergenerational persistence in high income.

The probability of being a high-income individual has the following form:

$$p = 1 - \exp(-\alpha_3 h)$$

The parameter α_3 governs the share of high-income households in the economy.

6.3 Untargeted Moments

In order to validate the model, I explore its fit for the untargeted moments. Table 3 presents the model fit for the untargeted moments. The model delivers a good fit for the tuition schedule by capturing its mean and variance. Although money investment data is extremely noisy, the model fit is also reasonable for the untargeted moments related to private investment, such as private investment dispersion and income gap in private investment.

Moment	Data	Model
Mean tuition	8.565	8.491
Std of tuition	0.671	0.723
Private investment	885	979
Std of private investment	1,674	1,823
Income gap in private investment	3.6	3.0

Table 3: Untargeted Moments

7 Quantitative Results

The decentralized equilibrium of the model features positive assortative matching on human capital once conditioned on income. However, there are several inefficiencies that generate a strong persistence of students from high-income families at the top of college distribution. Rank externalities, when parents compete with each other for a fixed number of seats at high-quality colleges by making larger investments, and the inability of parents to insure against the ability of their grandchildren leads to the overinvestment, while uninternalized peer effect partially reduces the level of investment. In this section, I quantify the size of the efficiency losses relative to the first best and explore the role of the rank externality and the peer effects separately.

7.1 Decentralized Equilibrium vs. Planner

I examine the investment decisions and associated output relative to the first best. Decentralization leads to overinvestment in response to the rank externality and incomplete insurance markets. The right panel of Figure 3 shows patterns in the investment by ability and income level. Relative to the efficient allocation, there is an overinvestment in most students. Families with children whose abilities fall into the ends of the ability distribution have the corner solution because of the flat profile of public investment. The public investment combined with the private is too high. The overinvestment is especially large for high-income families. It leads to crowding out of low-income students. The left panel of Figure 3 illustrates sorting between college rank and student ability. In the decentralized equilibrium, college access for students from low-income families is limited. They must have exceptionally high ability in order to get into colleges of high rank.

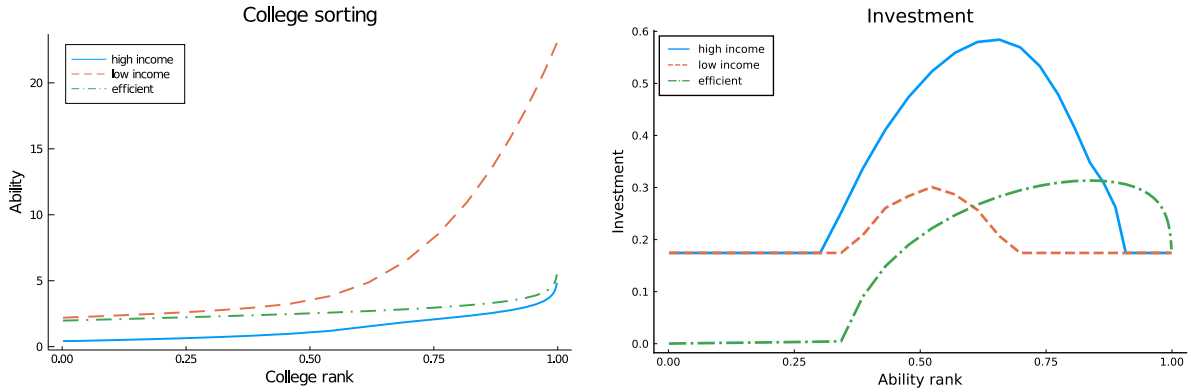

Figure 3: College Sorting and Pre-college Investment

Quantitatively, 17.5% of current investment is overinvested relative to the planner's solution. This leads to higher human capital and lower consumption. Although human capital is larger, the total output is lower by 3.6% in the decentralized equilibrium relative to the efficient allocation because talented low-income students have limited access to top colleges, which has a negative effect on the average human capital and thus effective college quality at the selective colleges. Then, this lower college quality reduces total output. Consumption is lower because, with higher human capital, tuition charged by colleges is higher. Table 4 compares planner's solution and decentralized equilibrium. The gap is calculated as the ratio

of the value in the decentralized equilibrium relative to the planner's solution

	Baseline		
	Planner	CE	Gap
Total investment	0.254	0.308	+17.5%
Total private investment	-	0.057	-
Total human capital	0.884	0.903	+2.1%
Aver. effective quality	5.665	5.469	-3.6%
Total consumption	2.077	1.864	-11.4%
Total output	3.575	3.451	-3.6%

Table 4: Planner vs Decentralized Equilibrium

7.2 Counterfactuals

Next, I use counterfactual experiments to explore the roles of the rank externality and the peer effect separately. I acknowledge that there are might be some complementarities between the key components of the model which require further investigation.

7.2.1 Sorting on Ability

First, I shut down the effects of endogenous sorting and thus rank externalities. I do this by sorting students on ability alone instead of human capital. Since parental pre-college investment no longer affects college sorting, parents do not have incentives to invest in order to compete for the limited number of seats at the high rank colleges. Table 5 compares the baseline economy with the economy with sorting on ability alone. This decreases pre-college human capital investment by 19.5%. In fact, it falls below the planner's solution, which highlights the fact that peer effect is not internalized by households. Total human capital in the economy decreases and consumption rises. As a result, average effective quality drops, and output declines by 2.1%.

7.2.2 No Peer Effect, $\theta = 0$

Next, I shut down the peer effect by setting $\theta = 0$. Students are sorted based on their willingness to pay only which amplifies the role of income. The experiment reveals that investment

	Baseline		Sorting on ability		Difference
	Planner	CE	Planner	CE	
Total investment	0.254	0.308	0.254	0.248	-19.5%
Total private investment	-	0.057	-	0.001	-98.2%
Total human capital	0.884	0.903	0.884	0.843	-6.6%
Aver. effective quality	5.665	5.469	5.665	5.307	-3.0%
Total consumption	2.077	1.864	2.077	1.832	-1.7%
Total output	3.575	3.451	3.575	3.378	-2.1%
Probability (High | High parent)	-	0.429	-	0.248	-18.1pp
Probability (High | Low parent)	-	0.193	-	0.248	+5.5pp

Table 5: Sorting on Ability

and output drop in the planner's solution and in the decentralized equilibrium. This change is larger in the decentralized equilibrium where investment drops by 6.8%, resulting in lower human capital and effective college quality. Output declines by 5.5%. The results are presented in Table 6.

	Baseline		No peer effect		Difference
	Planner	CE	Planner	CE	
Total investment	0.254	0.308	0.241	0.287	-6.8%
Total private investment	-	0.057	-	0.047	-17.5%
Total human capital	0.884	0.903	0.872	0.889	-1.6%
Aver. effective quality	5.665	5.469	5.279	5.137	-6.1%
Total consumption	2.077	1.864	2.053	1.713	-8.1%
Total output	3.575	3.451	3.533	3.260	-5.5%
Probability (High | High parent)	-	0.429	-	0.369	-6.0pp
Probability (High | Low parent)	-	0.193	-	0.147	-4.6pp

Table 6: No Peer Effect

7.3 Policy Experiment

Finally, I conduct a policy experiment where I impose a sorting on human capital alone, regardless of income level. Colleges take this assignment as given and charge students as high tuition as possible such that students do not deviate. This assignment might be not optimal for colleges, but one could think that there is a punishment for income discrimination that prevents colleges from deviating from this allocation. Then, the matches are more productive than before, which increases effective college quality. It leads to higher marginal return to investment, resulting in substantially higher investment and output gains of 3.2%. The policy

is also effective at promoting mobility of students from low-income families.

In the second policy experiment, colleges receive a subsidy for admitting low-income students. The subsidy is financed by lump-sum tax. This policy also promotes output and mobility for low-income families. The results are presented in Table 7.

	Planner	CE	Gap	Income-blind	College subsidy
Total investment	0.254	0.308	+17.5%	+31.2%	+12.3%
Total private investment	-	0.057	-	+28.1%	+11.1%
Total human capital	0.884	0.903	+2.1%	+12.8%	+7.1%
Aver. effective quality	5.665	5.469	-3.6%	+4.8%	+2.9%
Total consumption	2.077	1.864	-11.4%	+3.1%	+0.7%
Total output	3.575	3.451	-3.6%	+3.2%	+2.1%
Probability (High | High parent)	-	0.429	-	-9.0pp	-8.1pp
Probability (High | Low parent)	-	0.193	-	+13.4pp	+10.6pp

Table 7: Policy Experiment

8 Conclusion

In this paper, I study how pre-college parental investment in human capital of children affects sorting of students into colleges. I then estimate the effectiveness of the decentralized allocation and explore the implications of pre-college investment for intergenerational mobility. I find that the race to the top induces overinvestment in pre-college human capital and associated output losses relative to the first best. In counterfactual experiments, I estimate that sorting students on ability substantially decreases investment. Next, I shut down the peer effect and estimate that investment drops by 6.8% and output falls by 5.5%. Lastly, I use the model to conduct a policy experiment where I impose sorting of students by their pre-college human capital regardless of income. The policy promotes investment and output due to improved effective college quality and is an effective tool for improving income mobility for low-income families. Another policy that helps to improve upon the outcome is subsidizing colleges for admitting low-income students.

In my analysis, I made several simplifying assumptions. I assume that colleges care only about their profits. Quality of students matters for them because it increases tuition that could be collected. There is no consensus in the literature about the objective of colleges.

For instance, some papers suggest that colleges care about reputation or have some social objective, such as student diversity. Exploring different objective college functions might be fruitful. Another assumption that I made is that there is no social benefit from going to college. The qualitative literature supports the importance of distance to college for students and parents. Omitting it could affect social gains from sorting. Finally, I assume that information is perfect. The environment that allows for asymmetric information is an interesting extension.

Appendix

Tuition profile

The tuition schedule is derived from the sorting constraint for students (SC1):

$$u(y_s - m - \tau_t[a_s, j]) + \lambda\beta p(l[a_s], \tilde{q}[j])EV(a', y'_H) + \lambda\beta(1 - p(l[a_s], \tilde{q}[j]))EV(a', y'_L) \geq$$

$$\geq u(y_s - m - \tau_s[a_s, j']) + \lambda\beta p(l[a_s], \tilde{q}[j'])EV(a', y'_H) + \lambda\beta(1 - p(l[a_s], \tilde{q}[j']))EV(a', y'_L) \quad \text{(SC1)}$$

Re-arranging the inequality:

$$[u(y_s - m - \tau_s[a_s, j]) + \lambda\beta(EV(a', y'_H) - EV(a', y'_L))p(l[a_s], \tilde{q}[j])] - [u(y_s - m - \tau_s[a_s, j - \epsilon]) +$$

$$+\lambda\beta(EV(a', y'_H) - EV(a', y'_L))p(l[a_s], \tilde{q}[j - \epsilon])] \geq 0$$

$$u(y_s - m - \tau_s[a_s, j]) - u(y_s - m - \tau_s[i_s, j - \epsilon]) + \lambda\beta(EV(a', y'_H) - EV(a', y'_L))(p(l[a_s], \tilde{q}[j]) - p(l[a_s], \tilde{q}[j - \epsilon]) \geq 0$$

$$\frac{u(y_s - m - \tau_s[a_s, j]) - u(y_s - m - \tau_s[a_s, j - \epsilon])}{\epsilon} + \lambda\beta(EV(a', y'_H) - EV(a', y'_L))\frac{p(l[a_s], \tilde{q}[j]) - p(l[a_s], \tilde{q}[j - \epsilon])}{\epsilon} \geq$$

As $\epsilon \to 0$,

$$u'(c)\tau'[a_s, j] = \lambda\beta(EV(a', y'_H) - EV(a', y'_L))\frac{\partial p(l[a_s], \tilde{q}[j])}{\partial \tilde{q}}\frac{\partial \tilde{q}[j]}{\partial q}\frac{\partial q[j]}{\partial j}$$

$$\tau'[a_s, j] = \lambda\beta(EV(a', y'_H) - EV(a', y'_L))\frac{\frac{\partial p(l[a_s], \tilde{q}[j])}{\partial \tilde{q}}\frac{\partial \tilde{q}[j]}{\partial q}\frac{\partial q[j]}{\partial j}}{u'(c)}$$

Then, the tuition profile is characterized by:

$$\tau[a_s, j] = v_t + \int_{\underline{j}}^{j} \tau'[a_s, k]dk$$

The tuition schedule for the marginal student $\underline{a_s}$, who is indifferent between going to college and not, is derived from the participation constraint for students (PC1):

$$u(y_s - m - \tau_s[a_s, j]) + \lambda\beta p(l[a_s], \tilde{q}[j])EV(a', y'_H) + \lambda\beta(1 - p(l[j_s], \tilde{q}[j]))EV(a', y'_L) \geq$$

$$\geq u(y_s - m) + \lambda\beta p(l[a_s], q_0)EV(a', y'_H) + \lambda\beta(1 - p(l[a_s], q_0))EV(a', y'_L) \qquad \text{(PC1)}$$

$$u(y_s - m - \tau_s[\underline{a_s}, \underline{j}]) + \lambda\beta p(l[\underline{a_s}], \tilde{q}[\underline{j}])EV(a', y'_H) + \lambda\beta(1 - p(l[\underline{a_s}], \tilde{q}[\underline{j}]))EV(a', y'_L) =$$

$$= u(y_s - m) + \lambda\beta p(l[\underline{a_s}], q_0)EV(a', y'_H) + \lambda\beta(1 - p(l[\underline{a_s}], q_0))EV(a', y'_L)$$

$$u(y_s - m) - u(y_s - m - \tau_s[\underline{a_s}, \underline{j}]) = \lambda\beta(EV(a', y'_H) - EV(a', y'_L))(p(l[\underline{a_s}], \tilde{q}[\underline{q}]) - p(l[\underline{a_s}], q_0))$$

Then tuition for the marginal college $\underline{j}$ is found from:

$$u(y_s - m - \tau_s[\underline{a_s}, \underline{j}]) = u(y_s - m) - \lambda\beta(EV(a', y'_H) - EV(a', y'_L))(p(l[\underline{a_s}], \tilde{q}[\underline{j}]) - p(l[\underline{a_s}], q_0))$$

College choice across income groups

A college maximizes total tuition revenue by substituting low-income students with high-income students subject to the college capacity constraint:

$$\max_{\epsilon_H, \epsilon_L} \tau_H(l[i_H - \epsilon_H], \tilde{q}[i_H - \epsilon_H, i_L + \epsilon_L])(di_H - \epsilon_H)\mu N + \tau_L(l[i_L + \epsilon_L], \tilde{q}[i_H - \epsilon_H, i_L + \epsilon_L])(di_L + \epsilon_L)(1 - \mu)N$$

subject to:

$$\epsilon_H \mu N = \epsilon_L(1 - \mu)N$$

$$(di_H - \epsilon_H)\mu N + (di_L + \epsilon_L)(1 - \mu)N = djM$$

After substitution:

$$\max_{\epsilon_H} \tau_H(l[i_H - \epsilon_H], \tilde{q}[i_H - \epsilon_H, i_L + \frac{\mu}{1-\mu}\epsilon_H])(di_H - \epsilon_H)\mu N +$$

$$+\tau_L(l[i_L + \frac{\mu}{1-\mu}\epsilon_H], \tilde{q}[i_H - \epsilon_H, i_L + \frac{\mu}{1-\mu}\epsilon_H])(di_L + \frac{\mu}{1-\mu}\epsilon_H)(1-\mu)N$$

FOC:

$$\Big\{\Big(\frac{\partial \tau_{H_2}(\hat{\imath}_H, \tilde{q}[\hat{\imath}_H, \hat{\imath}_L])}{\partial \tilde{q}} - \frac{\partial \tau_{L_2}(\hat{\imath}_L, \tilde{q}[\hat{\imath}_H, \hat{\imath}_L])}{\partial \tilde{q}}\Big)\Big[\frac{\partial \tilde{q}[\hat{\imath}_H, \hat{\imath}_L]}{\partial \hat{\imath}_H} - \frac{\partial \tilde{q}[\hat{\imath}_H, \hat{\imath}_L]}{\partial \hat{\imath}_L}\frac{\mu}{1-\mu}\Big] +$$

$$+\frac{\partial \tau_{H_1}(\hat{\imath}_H, \tilde{q}[\hat{\imath}_H, \hat{\imath}_L])}{\partial \hat{\imath}_H} + \frac{\partial \tau_{L_1}(\hat{\imath}_L, \tilde{q}[\hat{\imath}_H, \hat{\imath}_L])}{\partial \hat{\imath}_L}\frac{\mu}{1-\mu}\Big\}(di_H - \epsilon_H) +$$

$$+\tau_H(\hat{\imath}_H - \epsilon_H, \tilde{q}[\hat{\imath}_H - \epsilon_H, \hat{\imath}_L + \frac{\mu}{1-\mu}\epsilon_H]) - \tau_L(\hat{\imath}_L + \frac{\mu}{1-\mu}\epsilon_H, \tilde{q}[\hat{\imath}_H - \epsilon_H, \hat{\imath}_L + \frac{\mu}{1-\mu}\epsilon_H]) =$$

$$= \Big\{\frac{\partial \tau_{L_1}(\hat{\imath}_L, \tilde{q}[\hat{\imath}_H, \hat{\imath}_L])}{\partial \hat{\imath}_L}\frac{\mu}{1-\mu} - \frac{\partial \tau_{L_2}(\hat{\imath}_L, \tilde{q}[\hat{\imath}_H, \hat{\imath}_L])}{\partial \tilde{q}}\Big[\frac{\partial \tilde{q}[\hat{\imath}_H, \hat{\imath}_L]}{\partial \hat{\imath}_H} - \frac{\partial \tilde{q}[\hat{\imath}_H, \hat{\imath}_L]}{\partial \hat{\imath}_L}\frac{\mu}{1-\mu}\Big]\Big\}\frac{djM}{\mu N}$$

This FOC characterizes the optimal combination of high-income students and low-income students at one college.

Conditions for PAM(NAM)

In this subsection, I present the conditions for PAM(NAM) between students and colleges. Let $\phi(q, l, w)$ be the maximum utility that college q generates when matched with a student l, if the student l receives utility w.

$$\phi(q, l, w) = \max_{\tau} \tau$$

$$u(y - m(l) - \tau) + \lambda\beta\{p(q, l)EV(a', y'_H) + (1 - p(q, l))EV(a', y'_L) \geq w\}$$

Let $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

δ is the Lagrange multiplier.

$$\tau = y - m(l) - \delta^{-\frac{1}{\gamma}}$$

$$u(y - m(l) - y + m(l) + \delta^{-\frac{1}{\gamma}}) + \lambda\beta\{p(q, l)EV(a', y'_H) + (1 - p(q, l))EV(a', y'_L)\} = w$$

$$\frac{\delta^{1-\frac{1}{\gamma}}}{1-\gamma} = w - \lambda\beta\{p(q,l)EV(a',y'_H) + (1-p(q,l))EV(a',y'_L)\}$$

$$\delta = (1-\gamma)^{\frac{\gamma}{\gamma-1}}(w - \lambda\beta\{p(q,l)EV(a',y'_H) + (1-p(q,l))EV(a',y'_L)\})^{\frac{\gamma}{\gamma-1}}$$

$$\phi(q,l,w) = \tau = y - m(l) - (1-\gamma)^{\frac{1}{1-\gamma}}(w - \lambda\beta\{p(q,l)EV(a',y'_H) + (1-p(q,l))EV(a',y'_L)\})^{\frac{1}{1-\gamma}}$$

If a high type q is willing to pay more of w for an increment in its partner's type l, the assignment is PAM:

$$\phi_{ql}(q,l,w) \geq \frac{\phi_l(q,l,w)}{\phi_w(q,l,w)}\phi_{qw}(q,l,w)$$

$$(1-\gamma)^{\frac{1}{1-\gamma}}(w - \lambda\{p(q,l)EV(a',y'_H) + (1-p(q,l))EV(a',y'_L)\})^{\frac{1}{1-\gamma}} \cdot \frac{\partial^2 p}{\partial l \partial q} \geq \gamma \cdot \frac{\partial p}{\partial q} \cdot \frac{\partial m}{\partial l}$$

$$(y - m(l) - \tau) \cdot \frac{\partial^2 p}{\partial l \partial q} \geq \gamma \cdot \frac{\partial p}{\partial q} \cdot \frac{\partial m}{\partial l}$$

NAM if:

$$\phi_{ql}(q,l,w) \leq \frac{\phi_l(q,l,w)}{\phi_w(q,l,w)}\phi_{qw}(q,l,w)$$

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