Motivation

Many goods and assets are traded in decentralized, dynamic markets:
- Buyers and sellers arrive, are matched, and negotiate prices
- If trade doesn’t occur, return to try again

Within matches, trade may be inhibited by frictions:
- private information about gains from trade
- strategic behavior

But opportunity to match many times can mitigate these frictions.

Questions:
- What is the impact of frictions on prices and efficiency?
- Can a centralized mechanism achieve better outcomes?

We study these questions empirically using ebay auctions for iPads.
Motivation

Theoretical literature addresses these questions in a stylized auction environment:

- buyers are randomly matched to sellers in each period
- trade involves one unit of a homogenous good
- private independent values

Main result (Satterthwaite and Shneyerov (2007, 2008)): All steady state equilibria converge to the competitive (Walrasian) outcome as number of trading opportunities for sellers and buyers gets large (i.e., market is large over time).

- decentralized, dynamic markets with simple selling mechanisms can allocate supply almost efficiently
- gains from centralized mechanisms may be quite limited.

Question: does this result hold for ebay auctions?
Dynamic bidding

Two important aspects of eBay auction environments:

1. Bidders who lose, and sellers who fail to sell, can return to try again
   ⟹ Bids reflect the option value of losing, so bidders don’t bid their values
   ⟹ Start prices reflect the option value of selling again, so sellers set binding start prices even when their value is zero.

2. Buyers may actively choose in which auction to bid
   - eBay posts current highest losing bid in each auction
   - This leads to sorting: buyers and sellers are not randomly matched
     - High value buyers tend to bid in sooner-to-close auctions.
     - Low value buyers bid in later-to-close auctions with lower posted prices

Hendricks & Sorensen Dynamic Auction Markets
Objective

- Develop and estimate a structural model of dynamic bidding in the eBay auction environment
  - Primitives: distributions of buyers' and sellers' valuations and their entry, exit, and return rates
  - Provide conditions under which the primitives are identified.
  - Apply restrictions implied by steady state to test the model
- Use estimates to quantify the impact of trading frictions on prices and efficiency.
  - Competitive Benchmark: intermediary pools buyers and sellers over time and runs a uniform price, double auction
- Simulate counterfactuals to examine the effects of sorting and exit rates on convergence of equilibria to competitive outcomes
Gavazza (2016) estimates a structural model of search and bargaining in a decentralized, dynamic market for used aircraft.

Empirical auction models using eBay data:

<table>
<thead>
<tr>
<th>Static</th>
<th>Dynamic</th>
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</thead>
<tbody>
<tr>
<td>Lewis et al (2007)</td>
<td></td>
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<tr>
<td>+ others</td>
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</table>
Model: sellers

- Sellers arrive stochastically, roughly one per hour
- Intermediary conducts an auction on the seller’s behalf
- Each auction is open for a fixed amount of time (e.g. several days)
  - At any time $t$, a large number of auctions indexed by $j$ are open for bidding
  - Intermediary posts the current highest losing bid (or start price), $r_j(t)$
- Sellers who sell award item to highest bidder at price equal to second-highest bid
- Sellers who fail to sell exit with probability $\theta$, or with probability $(1 - \theta)$ return to relist at a future time
Buyers

- New buyers arrive in real time according to a Poisson process with parameter $\lambda$ per period (e.g., hour).

- Losing bidders:
  - Exit with probability $\alpha$.
  - Or with probability $(1 - \alpha)$ go into pool of losers who come back to bid again.
    - Time to return is exponential with parameter $\beta$.

- Winners exit (single unit demand).

Note: entry, exit, and return rates of sellers and buyers are exogenous.
Bidder arrivals

- $N$ new buyers with values $(x)$ drawn from $F_E$
- $L$ returning bidders, with values drawn from pool of losers.
  - If $K$ is the size of the pool, then $M$ is binomial with parameters $K$ and $\beta$
  - In the long run, $K$ fluctuates around
    $$\overline{K} = \frac{(\lambda - q)(1 - \alpha)}{\alpha \beta}$$
    where $q$ is probability that the item is sold.
  - In long run, values of returning buyers are drawn from a stationary distribution $F_L$.

**Assumption:** $\overline{K}$ is sufficiently large that $L$ is well approximated as a Poisson with parameter $\gamma = \beta \overline{K}$

$\implies$ Beliefs of new and returning buyers about $L$ are the same so behavior and value functions are same.
Buyers’ decisions

When a buyer arrives, she observes the state \( (\omega) \):

- Time remaining in each of the auctions currently open for bidding
- Posted price in each of those auctions

Arriving buyers choose

- Which auction to bid: \( \rho(x; \omega) \)
- How much to bid in that auction: \( \sigma(x; \omega) \)

Bid is a proxy bid: submits a maximum bid and intermediary bids on her behalf up to that level

- Buyers bid only once in an auction.
Let $v(x; \omega)$ be the equilibrium expected payoff to a bidder with value $x$ in observed state $\omega$.

Define the ex ante value function

$$V(x) \equiv \int v(x; \omega) d\mu(\omega)$$

where $\mu$ is the stationary distribution of the state.

**Key Assumption:** A buyer’s continuation value if she loses an auction and returns at some future time is $V(x)$

- ignores possible impact of her bid on transition law determining state of return
- does not use current state to forecast state of return
Optimal dynamic bidding

Let $G_{M_j|B}(m; \omega, b)$ be the distribution of the maximum rival bid in state $\omega$ in auction $j$ at bid $b$ when rivals play equilibrium strategies $M_j$ is not independent of $b$ because $b$ affects choices of subsequent buyers and path of posted prices.

Optimal bid in auction $j$ solves

$$\max_b \int_0^b (x - m) dG_{M_j|B}(m; \omega, b) + (1 - \alpha)(1 - G_{M_j|B}(b; \omega, b))V(x)$$

Dominant strategy to bid

$$\sigma(x) = x - (1 - \alpha)V(x)$$

Optimal bid does not depend on $\omega$ because continuation value does not depend on $\omega$. 
This result allows us to integrate over states and express continuation value as

\[ V(x) = \frac{\int_0^{\sigma(x)} (x - m) dG_{M|B}(m|\sigma(x))}{[1 - (1 - \alpha)(1 - G_{M|B}(\sigma(x)|\sigma(x)))]} \]

where \( G_{M|B}(m|\sigma(x)) \equiv \int G_{M|\rho(x;\omega)} B(p;\omega, b) d\mu(\omega) \).
Inverse bid function

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where \( G_{M|B}(m|\sigma(x)) \equiv \int G_{M\rho(x;\omega)|B}(p; \omega, b) d\mu(\omega) \).

The inverse bid function can be expressed as

\[ \eta(b) = b + \frac{(1 - \alpha) G_{M|B}(b|b)}{\alpha} (b - E[P|P < b; b]) \]

- To recover values from bids, get estimates of \( \alpha, G_{M|B}(b|b), \) and \( E[P|P < b; b] \) directly from the data
Optimal Start Prices

- Sellers have zero value for the item.

- Exit rates are heterogeneous, independently drawn from a distribution $F_S$.

- Assumption: sellers’ beliefs about the probability of sale and price conditional on sale is given by the stationary distributions of winning bids and prices.

Optimal start price solves

$$r(\theta) = (1 - \theta) \frac{E[P|P \geq r](1 - G(r))}{[1 - (1 - \theta)G(r)]}$$

- $E[P|P \geq r]$ and $G$ can be estimated directly from the data.

- Seller’s type can then be recovered from these functions and her startup price.
Overidentifying restrictions

In a period of length $\Delta$, the flow of types $x$ out of the loser pool is

$$\beta \Delta k(t) f_L(x)$$

where $k(t)$ is the number of bidders in the loser pool.

The flow into the loser pool of types equal to $x$ is

$$(1 - \alpha) \left[ 1 - G_{M|B}(\sigma(x)) \right] \left[ \beta \Delta k(t) f_L(x) + \lambda \Delta f_E(x) \right]$$

In steady state ($k(t) = \bar{K}$) these flows should be on average equal, so

$$f_L(x) = \frac{\lambda \alpha (1 - G_{M|B}(\sigma(x)))}{(\lambda - q) \left[ 1 - (1 - \alpha)(1 - G_{M|B}(\sigma(x))) \right]} f_E(x)$$

We use these conditions to test the model.
Endogenous exit

- Losing bidders face a cost to continue, drawn from some distribution $F_C$.

- Exit if cost exceeds expected continuation payoff

- Inverse bid function: $\alpha(b)$ instead of just $\alpha$
  - $\alpha(b)$ can still be estimated directly from data
  - relation between $\alpha(b)$ and $v(b)$ identifies distribution of participation costs

- But counterfactual simulations trickier
All eBay listings for iPads between Feb-Sep 2013

- Focus on auctions of used 16GB iPad Mini, WiFi-only

Observe auction and seller characteristics

Observe times and amounts of all bids, along with bidder IDs

⇒ We can track bidders across auctions

⇒ We see the winner’s bid (not just sale price)
Defining the relevant market

Focus on auctions for used 16GB WiFi-only model

- Not much substitution between 16GB WiFi model and other models
  - After losing an auction for a 16GB WiFi model, 83% of return bidders bid on the same model

- Not much substitution between used and new:
  - After losing an auction for a used item, 79% of return bidders bid on another used item
  - When we observe bidding on a sequence of 3 items, the modal sequence is used-used-used, and the next most common is new-new-new.
## Summary stats ($N = 7,159$)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>0.10</th>
<th>0.50</th>
<th>0.90</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Start price</strong></td>
<td>150.78</td>
<td>116.98</td>
<td>0.99</td>
<td>185.00</td>
<td>300.00</td>
</tr>
<tr>
<td><strong>Positive reserve price (0/1)</strong></td>
<td>0.10</td>
<td>0.31</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Reserve price (if positive)</strong></td>
<td>271.03</td>
<td>42.22</td>
<td>200.00</td>
<td>275.00</td>
<td>330.00</td>
</tr>
<tr>
<td><strong>Successfully sold (0/1)</strong></td>
<td>0.78</td>
<td>0.41</td>
<td>0.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Sale price (if sold)</strong></td>
<td>288.01</td>
<td>33.07</td>
<td>230.00</td>
<td>290.00</td>
<td>335.00</td>
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<tr>
<td><strong>Shipping fee</strong></td>
<td>7.17</td>
<td>5.60</td>
<td>0.00</td>
<td>6.60</td>
<td>15.00</td>
</tr>
<tr>
<td><strong>Number of bids</strong></td>
<td>17.03</td>
<td>17.76</td>
<td>0.00</td>
<td>11.00</td>
<td>51.00</td>
</tr>
<tr>
<td><strong>Number of unique bidders</strong></td>
<td>7.52</td>
<td>6.67</td>
<td>0.00</td>
<td>6.00</td>
<td>20.00</td>
</tr>
<tr>
<td><strong>Minutes since last auction</strong></td>
<td>49.91</td>
<td>86.33</td>
<td>1.25</td>
<td>23.25</td>
<td>183.90</td>
</tr>
<tr>
<td><strong>Cover included (0/1)</strong></td>
<td>0.18</td>
<td>0.39</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Seller feedback (#)</strong></td>
<td>5561.57</td>
<td>38342.19</td>
<td>4.00</td>
<td>106.00</td>
<td>8239.00</td>
</tr>
<tr>
<td><strong>Seller feedback (% positive)</strong></td>
<td>98.92</td>
<td>6.39</td>
<td>96.67</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

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**Dynamic Auction Markets**
Basic bidding patterns

- Roughly 1 auction per hour
- 3.94 new bidders arrive per hour \((\hat{\lambda})\)
- 3.78 returning bidders per hour
- 10% of bidders bid in an auction that ends within 5 minutes; 24% bid in auctions that end within 3 hours
- Conditional on losing an auction, 50.4% of bidders return to bid again \((\hat{\alpha} = 0.496)\)
  - 50% return within an hour
Sellers and reserve prices

- Many sellers choose low effective reserve prices
  - 21% below $1, 41% below $180

- Sellers who fail to sell: re-list about 44% of the time

- \( \text{Prob(sell)} \) decreasing in reserve price; \( \text{Prob(relist)} \) increasing

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**Graph**

- \( \text{Prob(sold)} \) and \( \text{Prob(relisted if not sold)} \) as a function of reserve price.
No intra-auction dynamics?

- 43% of bidders engage in incremental bidding (multiple bids submitted in same auction)
- We consider only the maximum bid of each bidder in each auction
No intra-auction dynamics?

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Exogenous exit?

- Exits appear to be approximately independent of bids
Stationary bidding strategies?

- Bidders tend to bid more aggressively when they return after losing, but effect is quantitatively insignificant (25 cent increase, on average)
- Regression of bids on auction number (in bidder’s sequence) and bidder fixed effects gives an $R^2$ of 0.80
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Current standing bids uninformative about future state?

- Median number of auctions before a bidder returns: 6
  $\Rightarrow$ Median number of state transitions $\approx 54$
- Correlation between current vector of standing bids and the vector 6 periods (auctions) later: $\approx 0$
Inverting the bids

Use normalized bids, based on regression of prices on item characteristics

To invert bids, need $G_{M|B}(b|b)$ and $E[P|P < b; b]$
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$G_{M|B}(b|b)$:

- $G_{M|B}(b|b)$ is the probability “type” $b$ wins in the set of auctions in which it bids.


- Note: not the same as the marginal distribution of the winning bid!
Inverting the bids

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$E[P|P < b; b]$:

- $E[P|P < b; b]$ is the expected price in auctions won by a bidder with bid $b$.
- Estimated by regressing prices on cubic B-splines of the winning bid.
Estimate of $G_{M|B}(b|b)$

- **Normalized bid**
- **Prob(win|bid)**
- **ECDF of winning bid**

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Dynamic Auction Markets
Estimating $f_E$ and $f_L$

1. Invert all normalized bids to get $\hat{x}$’s

2. Fit kernel densities to $\hat{x}$’s for new and returning bidders separately
   - Ignore bids below $100$
Kernel densities of inverted bids

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Dynamic Auction Markets
Comparison to efficient centralized market

Efficient mechanism: buyers and sellers pooled, market cleared with a uniform auction

Given our estimate of $f_E$:

- Market clearing price is $278.22$
- Average gross surplus is $303.00$

In the data:

- Average price is $276.35$, std. dev. is $25.55$
- Estimated average gross surplus is $291.28$
- 35% of winners have values below $278.22$

Test of model restriction on $f_L$
Open auctions: need to impose an auction selection rule

We use the data to estimate a logit model of auction choice

- Preferred auction depends on time to close, with preferences depending on $x$
  - higher types have stronger preferences for soon-to-close auctions
  - this rationalizes the “gap” between $G_{M|B}(b|b)$ and ECDF of winning bid
Auction choice probabilities

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Dynamic Auction Markets
Preliminary conclusions

Accounting for dynamic bidding incentives is important

- Otherwise, level and dispersion of buyers’ valuations would be understated
- Efficiency and revenue gains of centralized mechanism would be overstated

Impact of bidder sorting on prices and efficiency

- Closed auctions: intermediary provides no info on start prices or bids
- Equilibrium: bidders bid in next-to-close auction $\Rightarrow$ random matching
- Results: sorting lowers dispersion of prices, increases efficiency and revenues.

Impact of lower exit rates on convergence to Walrasian allocation

- Simulation outcomes for $\alpha \rightarrow 0$ converge to benchmark allocation
- TBD: simulate the model for $\theta \rightarrow 1$
- Conjecture: patient sellers select posted price mechanism, impatient sellers select auctions.