Credit Lines and Bank Risk*

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November 5, 2022
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Abstract

This paper studies how much providing credit lines to firms contributes to bank risk and its welfare implications. I develop a quantitative model in which banks lend to heterogeneous firms both through term loans and credit lines. Credit lines give firms liquidity insurance against crisis times and help overcome financing frictions. At the same time, credit lines also introduce a new channel for banks to be exposed to excessive risk. I estimate the model to match both aggregate and heterogeneous contract data. I find that 20% of bank losses in crisis times can be attributed to credit lines, and that credit lines help stabilize banks during crises. My model suggests banks are over-lending in both contracts compared to a planner, but the relative shares of contracts are broadly correct. I find bank capital ratios should be 3% higher and show how to implement optimal policy. Additionally, I show how a model with only term loans would underpredict optimal capital ratios. *JEL Classification Numbers: E44, G21, G28.*

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*I am deeply indebted to Dean Corbae, Rishabh Kirpalani, and Erwan Quintin for their invaluable guidance and support. I also thank Hengjie Ai, Manuel Amador, Anmol Bhandari, Job Boerma, Carter Braxton, Francesco Celentano, Briana Chang, Philip Coyle, Pablo D’Erasmo, Alessandro Dovis, Natalie Duncombe, Tim Kehoe, Annie Soyean Lee, Paolo Martellini, April Mechl, Dmitry Orlov, Mark Rempel, Roberto Robatto, Anson Zhou, and seminar and conference participants at the University of Wisconsin-Madison, Minnesota-Wisconsin Macro/International workshop for helpful comments and suggestions on this paper. I gratefully acknowledge the financial support from the Ko and Ying Shih Dissertation Fellowship.

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1 Introduction

How much does providing credit lines to firms contribute to bank risk and what are its welfare implications? Excessive risk in the banking sector is a core interest to regulators and has created a rich literature in banking. The conventional approach is to model bank lending as term loans contracts — per period loans where every time there is new loan issuance, the firm and bank sign a new contract. This means that in some states of the world banks may charge higher interest rates on new loans or even deny them outright. This approach disregards an important data fact: in the US, more than half of bank loans to firms — commercial and industrial (C&I) loans — originate through credit line contracts (Greenwald et al. (2021)), not term loans. More importantly for bank risk, undrawn loan commitments are incredibly large — they are 140% of already issued loans. Credit lines pose a unique risk to banks. With term loans, the risk is mainly in whether the loan may default. However with credit lines, banks are also giving firms insurance from liquidity shocks. The crucial feature in these contracts is that firms lock in relatively low interest rates from which they can access funds during the duration of the contracts. In return, firms pay banks an annual fee, even when the credit line isn’t used, called a commitment fee — very much like an insurance premium. Several papers have empirically documented that during aggregate downturns firms draw on these credit lines en masse showing up as large increases in bank loans; during the global financial crisis, Cornett et al. (2011), and again during the COVID-19 outbreak Li et al. (2020). Furthermore, evidence suggests banks are affected by credit line risks. Acharya et al. (2021) show that during the COVID-19 outbreak, stock prices of banks with large ex-ante exposure to undrawn credit lines, and large ex-post drawdowns declined significantly more. U.S. firms with pre-arranged credit lines from banks drew down their credit lines with a far greater intensity than in past recessions. Acharya and Steffen (2021) suggest “[...] bank credit lines the new source of financial fragility”, and call for higher bank capital buffers.

While credit lines are quantitatively significant, a normative assessment of whether banks should or should not have this much risk must recognize that these contracts exist in the world for a reason and

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1 Sufi (2009) finds that the median maturity on credit line contracts is approximately 3 years.
2 While spreads can in principle be renegotiated, Greenwald et al. (2021) show that spreads on more than 90% of credit lines remain completely unchanged from 2012-2019.
3 As an example, Ford Motor Company drew down USD 15.4 billion from its credit line in March 2020. Ford usually carries USD 20bn in cash, so the credit line draw was significant. Ford’s credit line contract only charges 125bps on the drawn credit. For this facility, Ford was paying 15bps in commitment fees on the whole 15.4 billion undrawn credit line amounting to USD 23.1 million annually. See Acharya et al. (2021) for more details.
that they provide benefits to firms. Firms benefited from having such access to pre-arranged credit lines during the pandemic when capital market funding froze (e.g., Chodorow-Reich et al. (2021); Greenwald et al. (2021)). Firms that did not have credit lines that they could draw on were more likely to sharply reduce investment, significantly reduce R&D, reduce dividend payouts and total debt following these crises. It isn’t clear a priori that less credit lines would reduce bank risk, and discouraging credit lines could in fact be welfare decreasing.

In this paper, I consider an environment in which contracts between firms and banks are endogenous. The key friction in the model I consider is a two-sided moral hazard of firms and banks. Credit line contracts arise endogenously and help overcome firm moral hazard to achieve higher efficiency. However, bank moral hazard means some of this risk borne by banks may be socially inefficient. Building on Holmström and Tirole (1998), firms have productive projects that receive uncertain liquidity shocks.

In a completely frictionless world, we can achieve the first best — all firms who receive small liquidity shocks and still have positive surplus projects are the only ones who will receive additional funding. All firm failures are efficient, and even bank failures are efficient. In other words, firms and banks are both holding exactly the efficient amount of risk. Adding firm moral hazard introduces an inefficiency; there will be some firms with positive surplus that will be unable to get additional funding when hit with liquidity shocks. Firms and banks are able to partially overcome this inefficiency through a committed long-term contract, which I show is implemented through credit lines. Therefore, banks ability to commit gives rise endogenously to credit line contracts, which increases efficiency. However, when we also add bank moral hazard, in the form of deposit insurance and limited liability, banks fail to internalize the social cost of their default and some of the risk from credit lines become inefficient — introducing the need for regulation. Whether banks have too much risk through credit lines, and how much should be regulated becomes a quantitative question.

More specifically, I model a dynamic general equilibrium environment in which banks intermediate lending between households and firms. In contrast to existing literature, banks may lend to firms both through credit lines or term loan contracts. In my model, credit lines and term loans arise endogenously through implementations of optimal contracts. The optimal contracting microfoundation crucially guides substitution across loan contract types in the aggregate. Banks cannot extract all the surplus from firms due to a moral hazard effort choice by the firms. This moral hazard wedge implies long term contracts, in the form of credit lines, are necessary to commit financing in face of liquidity shocks.
Consistent with empirical evidence, I show that the optimal contracts to small firms are implemented through only term loans, whereas large firms also receive credit lines that insure against crisis times. Furthermore, loan sizes are larger, and interest rates lower for larger, less financially constrained firms. While credit lines are necessary to implement the optimal contract and achieve higher efficiency, banks also face a moral hazard problem, from deposit insurance as in Karaken and Wallace (1978), introducing the potential need to regulate term loans and credit lines. Because of deposit insurance, banks with limited liability do not internalize the social cost of their default. Thus, if left unregulated, banks become excessively risky from the perspective of households. This is relevant to households because consumption is affected by bank activities through the net return on deposit savings, bank equity income and taxes related to the cost of deposit insurance. Ultimately, regulatory policies trade off a reduced cost of deposit insurance and resource loss with reduced bank profits and firm output.

To quantify excessive risk and specify optimal regulatory policy, I estimate the model via simulated method of moments (SMM). At the micro level, to ensure the distribution of firms by their characteristics is accurate, I use heterogeneous loan contract moments from bank supervisory data FR Y-14Q. The optimal contract framework serves as a mapping of firm characteristics to loan characteristics and I exploit this to back out the firm distribution banks are facing. At the macro level, I discipline the aggregate banking industry by using Call Report data.

I find that 20% of bank losses during crisis times are from credit line lending and the remaining 80% loss are from term loan lending. The losses coming from credit lines are quantitatively large especially given that banks are selecting to extend credit lines to large firms with low liquidity cost. While naively it may seem like eliminating credit lines will reduce bank default risk proportionally, banks react to changes in their environment. I find that a counterfactual equilibrium with no credit lines actually doubles bank default rate from 1% to 2%. This is because while there is a direct effect of eliminating the risk from credit lines, there is also a substitution effect in which the bank gives term loans to firms who formerly received credit lines. These firms were cheap to insure against liquidity shocks, but are now forced to be over-liquidated resulting in higher firm default which leads to higher bank default. I find that the term loan part of bank portfolios gives negative gross returns — in other words, defaults — at moderate shock sizes, but the credit lines portion only fails at very large shocks. Therefore, with moderate shocks, the credit lines actually help offset losses from term loans that might otherwise lead to bank default, and help increase financial stability. However, I also find that as bank size grows from
a sequence of low shocks, its default probability increases from near 0% to the steady state value of 1%. When the growing bank increases both term loans and credit lines, the size of the shock required to default the term loan portions remains constant, but the credit lines’ decreases. In this way, while in level terms the term loans are the bigger reason for bank default, the marginal increase in bank default probability is driven by the increasing risk in the credit lines.

In normative exercises, I study a planner who is subject to the same frictions as private agents, but internalizes the social cost of bank default. The planner finds that the decentralized bank is overlending on both loan types, but that the share of each contracts is broadly correct. The planner economy has a bank capital ratio of 11.2%, while the Basel III rule is currently 8%. I show that we can implement the planner’s allocation using modified macroprudential policies. We can fully implement the efficient allocation by using three instruments: a conventional capital requirement, leverage requirement, and a loan commitment constraint. I also show that when the regulator is constrained to using only one policy tool, a risk-weighted capital requirement can achieve 95% of the optimal welfare. Finally, I show that if use a model with only term loans, as in conventional models, we would underpredict optimal capital ratios and conclude that that the current Basel III 8% is sufficient, leaving us with lower welfare and higher bank default than is desirable. This is because the decrease in bank profits from increasing capital requirements is larger with term loans than it is with credit lines. Therefore model misspecification leads the planner to overestimate the costs of increasing capital ratios.

Related literature

This paper relates to a growing literature of analyzing banking regulation in quantitative general equilibrium models. Seminal papers include Van den Heuvel (2008), Corbae and D’Erasmo (2021), Begenau (2019); Begenau and Landvoigt (2018). Recent contributions include Pancost and Robatto (2022) which studies how capital requirements can increase risk in non-financial firms, and Dempsey (2020) which shows how aggregate risk in the economy can shift to non-banks. In these papers, banks lend through term loan contracts — banks do not have committed long-term loan obligation to borrowers. I argue that these papers miss an important feature of bank lending — that half of bank lending to firms occur through credit line contracts and that these loan commitment exposures are large and important to bank risk. In this paper, I explicitly model banks choice to lend through either

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\(^4\)See also for quantitative banking models, Bianchi and Bigio (2022), Pandolfo (2021), Nguyen (2018), Davydiuk (2019), and others.
term loan and/or credit line contracts, thereby allowing banks to trade-off higher returns and higher risk and allowing risk shifting across contract types in response to regulation. In contrast to much of the quantitative banking literature, I apply an optimal contracting framework to allow contracts to change in response to regulation, and explicitly model the benefits of credit lines as well as the costs. The micro-foundation of the contracts is an extension of seminal work by Holmström and Tirole (1998), which shows that firm moral hazard necessitates loan contracts with commitments to avoid inefficient liquidations.

There is a large corporate finance literature studying the prevalence and use of credit lines by firms\(^5\). Complementing this literature, this paper focuses on the supply side of credit lines, endogenous choice in contracts, and the implications of drawdown risks for banks and its welfare consequences. Several empirical work examine bank exposure to credit line risk during market turmoils. Ivashina and Scharfstein (2009) and Cornett et al. (2011) show that following the failure of Lehman Brothers, banks with more credit exposure decreased their new lending more. As firms draw on existing credit lines, this acts similarly to a liquidity shock and constrains both new term loan lending and new credit line extensions. More recently, Li et al. (2020) document the same phenomenon during the COVID market panic of March 2020. In both these episodes, bank deposits inflow increased as banks were simultaneously perceived to be safe institutions, a phenomenon Gatev and Strahan (2006); Gatev and Schuermann (2009) first recorded\(^6\). However, this inflow is not sufficient and the papers conclude that government intervention helped significantly in managing bank stress.

Recent papers such as Greenwald et al. (2021); Chodorow-Reich et al. (2021) use the Federal Reserve’s Y-14 data to outline firms’ credit line usage during crises and show firm heterogeneity in credit lines has important implications for how we think about policy. They show that only large firms had access to credit lines during COVID, and that virtually all the increase in corporate cash occurred specifically through these existing credit line contracts. I extensively rely on the data moments reported in these two papers in the estimation of my model. In a closely related work, Payne (2020) shows bank-firm search frictions amplifies shocks to banks to the real economy. While this paper also uses an optimal contract framework to highlight the role of long term loan contracts, like credit lines, in contrast, my paper features banks with rich balance sheet problems and default on-the-equilibrium path, making it more suitable to study banking regulation. In recent work, Benetton et al. (2022) also

\(^5\)See Bolton et al. (2011), Nikolov et al. (2019), Sufi (2009), and others.  
\(^6\)Kashyap et al. (2009) show that synergy with deposit taking make credit lines a uniquely bank product.
model multi-product banks which extend both term loans and credit lines. They use an IO approach to estimate the benefits of economies of scope across products versus the cost of market power exploitation by these banks. In this paper, I focus on the risk implications of credit lines to banks and incorporate endogenous contracts.

Roadmap

The remainder of the paper is organized as follows: Section 2 outlines a simple contracting environment meant to capture what occurs within a period in the full model, Section 3 describes the full environment, Section 4 presents the equilibrium, Section 5 describes how we map the model to the data, Section 6 conducts a quantitative analysis of the model, and Section 7 concludes.

2 The optimal bank-firm contract

First, I outline a simple environment where a single firm with moral hazard contracts with a frictionless, deep pocketed bank. There are two subperiods and both the firm and bank are risk neutral. The environment builds on Holmström and Tirole (1998), where depending on fundamentals, the optimal contract can be implemented through credit lines, or only with term loans. This environment will provide the microfoundation for what occurs within a period in the full model — why and when are credit lines necessary. In Section 3, I will embed this contracting framework into a dynamic general equilibrium environment, introducing a heterogeneous distribution of firms and a dynamic bank with a moral hazard problem of its own.

2.1 Environment

2.1.1 Firm

A firm lives for subperiods $t = 1, 2$. At $t = 1$, the firm start with capital $k$ and chooses an investment size $I \in \mathbb{R}^+$ to a risky project with linear technology: the project returns $R \cdot I$ at the end of $t = 2$ with probability $p \in \{p_L, p_H\}$ and 0 with $1 - p$, where $\Delta \equiv p_H - p_L > 0$. This probability $p$ is a hidden effort choice from the firm. At the beginning of $t = 2$, the firm receives a liquidity shock of $\ell \cdot I$ with probability $P$, or 0 with probability $1 - P$. The project continues if the liquidity shock is 0. If the liquidity shock is $\ell \cdot I$, then the liquidity cost must be paid for to continue. If it is not paid for, the
project is destroyed, there is a salvage value of $\chi I$, and the model ends. If the project continues, then the firm chooses its project effort $p \in \{p_L, p_H\}$. This effort is subject to moral hazard; choosing low effort $p_L$ gives the firm a private benefit $B \cdot I$. At the end of the period, project returns are realized. The firm has limited liability and only has to pay the bank if the project succeeds.

Figure 1: Timing: Firms

2.1.2 Bank

The bank is deep-pocketed, frictionless, offers take-it-or-leave-it contracts to the firm, and maximizes its expected profits. Through monitoring technology, there is no private information - all parameters in the environment are known to the bank, except for the single hidden effort choice $p$ of the firm.

2.2 The optimal contract

The firm and bank are free to choose from a general contract space.

Lemma 1. In the optimal contract,

1. The split of the project returns $RI$ between firm and bank is not state-contingent

2. Any salvage value $\chi$ goes to the bank

Appendix A contains the proofs. Just for intuition, for the non-state contingency of the split of project returns, the bank will find it better to always minimize the firms share of returns for all states and rather induce the firm to participate through increasing the project size $I$, rather than through increasing the return share. For the salvage value, the intuition is that since any salvage value is earned before the project effort choice by the firm, it cannot be used to discipline the firm’s moral hazard.
**Definition 1.** A contract $C$ is a 3-tuple $(I, R^F, \iota)$ where

1. $I$: Project investment size
2. $R^F$: Per unit returns to the firm
3. $\iota$: How much liquidity funds will be paid by the bank in the event of a liquidity shock

We can show that contracts can be summarized by a 3-tuple of project investment size $I$, per unit return to the firm $R^F$, and how much liquidity insurance $\iota$ the contract gives where these three objects map to loan quantity, loan price and contract type, respectively, when taken to the data. We solve for the optimal contract over this 3-tuple space.

**Lemma 2.** The optimal contract must take one of two forms, either $C_{TL} = (I_{TL}, R_{TL}^F, 0)$ or $C_{CL} = (I_{CL}, R_{CL}^F, \ell)$, referred to as a term loan contract or a credit line contract, respectively.

Lemma 2 shows that we only need to solve for two types of contracts: ones where there are no insurance against the liquidity shock, which we refer to as term loan contracts, and ones with full insurance against the liquidity shock, which we refer to as credit line contracts. The intuition here is that since the support of the liquidity shock is two values $\{0, \ell\}$, any partial insurance will be useless — the liquidity cost must be paid in full to continue. In credit line contracts with full insurance, for simplicity we model banks as giving a transfer to firms and getting negative cash flow. However, in the real world firms are eventually paying back these additional loans drawn from credit lines. I show in Appendix B that this environment is isomorphic to one in which the bank has an outside investment opportunity, but is unable to invest in it since it provides the funds to the firm as promised. The bank would suffer an opportunity cost loss in the state of the world in which the firm receives a liquidity shock, as in the original model. The real world situation this models would be one in which the bank provides loans through the credit line contract at the contracted fixed interest rate, and is capacity constrained to provide funds at the currently higher market rate somewhere else.\(^7\) Furthermore, while I label $C_{CL}$ as the credit line contract, the implementation of it can be through a term loan to fund the project plus a credit line, or only with a credit line where there is an initial draw to fund the project. The crucial point is that it must include a credit line, as explained further in Lemma 1.

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\(^7\)Acharya et al. (2021) show that there may be several reasons why banks would be constrained in lending during credit line drawdowns. Funding liquidity to source new loans can become a binding constraint if the bank does not have enough deposits or liquid assets. Also, the credit line drawdowns can lock up scarce bank capital through regulatory constraints such as capital requirements.
Assumption 1. The project is NPV positive to invest in, but the entrepreneur is constrained in her borrowing by moral hazard: $p_H \left( R - \frac{B}{\Delta P} \right) < \min \left\{ 1 + P\ell, \frac{1 - P\chi}{1 - P} \right\} < p_H R$.

Taking Lemma 1, we compare the optimal term loan contract solution and the optimal credit line contract solution to determine the optimal contract. The optimal term loan contract $C_{TL} = (I_{TL}, R_{TL}^F, 0)$ solves,

$$\pi_{TL} = \max_{I_{TL}, R_{TL}^F} (1 - P) p_H \left( R - R_{TL}^F \right) I_{TL} + P\chi I_{TL} - \left( I_{TL} - k \right)$$

where firm incentive compatibility constraint (IC) is

$$p_H R_{TL}^F I \geq p_L R_{TL}^F I + BI$$

and firm participation constraint (PC) is

$$(1 - P) p_H R_{TL}^F I_{TL} \geq k$$

The objective function shows, the return to the bank in the event of no liquidity shock, the salvage value it receives if there is a shock, and the initial loan quantity. The incentive compatibility constraint shows that the firm’s earnings upon its exerting high effort must be greater than if it had exerted low effort. Finally, the participation constraint shows that the firm must be willing to put in its initial capital $k$ instead of eating it. Similarly, the optimal credit line contract $C_{CL} = (I_{CL}, R_{CL}^F, 0)$ solves,

$$\pi_{CL} = \max_{I_{CL}, R_{CL}^F} p_H \left( R - R_{CL}^F \right) I_{CL} - \left( I_{CL} - k \right) - P\ell I_{CL}$$

where firm incentive compatibility constraint (IC) is

$$R_{CL}^F \geq \frac{B}{\Delta P}$$

and firm participation constraint (PC) is

$$p_H R_{CL}^F I_{CL} \geq k$$
The objective function shows that the project return to the bank is always the same since the project is always continued. However, there is a potential cost for the bank for insuring the firm against the liquidity cost, $\ell I_{CL}$. These two problem clearly shows the trade-off the bank faces when deciding which contract to give. In the term loan contract, there is a potential liquidation of the project and therefore lower return; in the credit line contract, there is a guarantee of the project surviving, but at a potential additional cost to the bank.

**Lemma 3.** The optimal term loan contract is $C_{TL} = (I_{TL}, R_{TL}^F, 0) = \left( \frac{k}{(1-P)p_H \frac{R}{\Delta p}}, \frac{B}{\Delta p}, 0 \right)$, while the optimal credit line contract is $C_{CL} = (I_{CL}, R_{CL}^F, 0) = \left( \frac{k}{p_H \frac{R}{\Delta p}}, \frac{B}{\Delta p}, \ell \right)$.

The bank chooses the contract that maximizes its profits, $C^* = \arg\max \{\pi_{TL}, \pi_{CL}\}$.

**Lemma 4.** The optimal contract offered by the bank is the credit line contract $C_{CL}$ iff

$$\ell < \frac{1 - \chi}{1 - \bar{P}}$$

(7)

In this simple version, there is a clear cutoff value for the firms per-unit liquidity cost $\ell$. If the cost is sufficiently low, it is better ex-ante to insure the firm against this risk and prevent liquidation. If the cost is sufficiently high, it is better to not pay the cost and allow the project to liquidate. The cutoff value depends also on how generous the salvage value is and how likely the liquidity shock will hit. When the bank chooses the credit line contract $C_{CL}$, there are some cases where the bank would receive negative profits from paying the liquidity cost and continuing the project, and therefore find it time-inconsistent. The bank commits it will pay and to continue in these states of the world. Therefore, this commitment technology is key to being able to convince the firm that its participation constraint is satisfied, and be able to extend the efficient optimal contract.

**Proposition 1.** If $p_H R > \ell > p_H \left( R - \frac{R}{\Delta p} \right) - \chi$, optimal contracts are renegotiation-proof:

1. The credit line contract cannot be implemented through two term loans in a wait-and-see policy, and can only be implemented through a committed long term contract.

2. The term loan contract cannot be renegotiated in the event a liquidity shock is realized. That is, the firm cannot get a second loan to continue the project.
The optimal contract implementations are distinct from each other because the moral hazard friction necessitates long term committed contracts. The intuitive argument here is that if moral hazard did not exist, the firm and the bank would always value the project equally. Whenever the project is net present value positive, both the firm and the bank will want to continue by paying the liquidity cost. Therefore, we don’t need prior commitments. We can achieve the first best without commitments. However, when moral hazard is sufficiently high, there is a situation where it is ex-ante efficient to proceed with the project, but if the liquidity shock arrives, the banks end share of the project returns $p_H \left( R - \frac{R}{\Delta p} \right)$ are lower than the liquidity cost $\ell$, so the bank will not want to continue. That is, the liquidity cost is even higher than what the bank would receive from the successful project. Then, the firm will want to give the bank some of its share of the project $R^F = \frac{R}{\Delta p}$ to help convince the bank to continue the project. However, since the optimal contract already has the incentive compatibility binding, the firm will not be able to credibly give part of its share of returns to the bank — the bank knows that the firm will be tempted to put low effort and to earn private benefit. Therefore, renegotiations will fail, and the bank will not want to continue. This is ex-post inefficient because the total surplus of the project is still positive since the firms share is not only positive, but larger than the bank’s negative return $p_H \left( R - \frac{R}{\Delta p} \right) - \ell$. Therefore, in the presence of moral hazard, credit line contracts must be implement through committed long term contracts, instead of through a wait-and-see policy of two separate loans. This commitment technology is what enables the bank to credibly satisfy the firm’s participation constraint under the credit line contract. The intuition is similar that a term loan contract will not lead to renegotiation with the firm receiving a second loan it was not promised initially.

Finally, this simple contract has sharp predictions on how the optimal contract terms will vary according to firm characteristics — the underlying parameters. In particular, Lemma 5 introduces three comparative static which I specifically introduce because they will play a critical role in the estimation of the model.

**Lemma 5.** The optimal contracts have the following features:

1. Firms with lower $\ell$ get credit line contracts, as shown in Lemma 4
2. Firms with higher $k$ get larger loans, i.e., loan size $(I - k)$ is increasing in firm size $k$
3. Firms with lower $B$ pay lower interest rates, i.e., the implied interest rate $r$ is increasing in moral
hazard $B$, i.e. $\frac{\partial r}{\partial r} > 0$.

In Section 5 where we take the model to the data, we will see that the optimal contract correctly predicts the contract data across the firm distribution. In fact, the optimal contract can be seen as a mapping from the set of firm characteristics to the set of contract characteristics. This mapping will be exploited later in the estimation procedure and illuminates why we see heterogeneous contracts in equilibrium.

3 The general equilibrium environment

We now embed the contract into a dynamic general equilibrium environment with bank moral hazard. In the previous section, we showed that firm moral hazard endogenously gives rise to credit line contracts. So far, everything is efficient — credit line contracts help firms and bank partially overcome a friction to achieve higher efficiency. Now, in the presence of bank moral hazard, we’ll see that not all bank risk is efficient, and that credit lines pose another avenue to expose banks to socially costly default.

Time is discrete and infinite. Each period is divided into two subperiods: an investment stage and a drawdown stage, which correspond to the subperiods in the simple contract of Section 2. There are four types of agents in the economy: firms, bank, households, and a government. Each period, the bank intermediates between firms who have profitable projects, and households who want to save. The government operates a deposit insurance fund with a balanced budget. All consumption occurs at the end of the period.

3.1 Environment

3.1.1 Firms

Each period $t$, a distribution of firms indexed by $(i,j)$ are born. The firms are heterogeneous along two orthogonal dimensions: initial asset $k_i$, private benefit $B_i$, liquidity cost $\ell_i$, drawn from a distribution $F(k,B,\ell)$, and project technology $R_j$ drawn from $G(R)$. At the beginning of the investment stage, each firm starts with initial asset $k_i$ and choose an investment size $I_{ij}$ to a risky project that returns $R_jI_{ij}$ at the end of the model period with probability $p \in \{p_L,p_H\}$ and 0 with $1-p$. In the drawdown stage, an aggregate state $P \in [0,1]$ is drawn from $H(P)$: this $P$ is common to all firms. Then, firms receive an iid liquidity shock of $\ell_iI_{ij}$ with probability $P$, or 0 with probability $1-P$. The liquidity shock
must be paid for the project to continue. If it is not paid for, there is a salvage value of $\chi I_{ij}$. If liquidity shock is paid for, firms choose their project effort $p \in \{p_L, p_H\}$. There is no private information except for the hidden action choice $p$ which is subject to moral hazard — banks know the full distributions of firm characteristics. Choosing low effort $p_L$ gives firms private benefit $B_i I_{ij}$. At the end of the period, project returns are realized. Then, the period $t$ cohort of firms dies, and a new identical cohort is born at the beginning of period $t + 1$, giving a stationary distribution of firms.

### 3.1.2 Households

An infinitely lived, risk neutral household with discount factor $\beta$ is endowed with $\omega$ units of goods each period. The objective of the household is to maximize the presented discount value of consumption $C$. At the beginning of the period, households start with endowment $\omega$ and can save through either deposits $D$ or riskless storage technology $a$. At the end of the period, households earn returns from deposit $R_D D$, storage technology $R_f a$, pay state-contingent lump sum taxes $T$, and earn dividends from the bank, own both the bank and firms, and consume $C$. The state-contingent tax $T$ is used to cover deposit insurance for failing banks. While households own banks, they pay for the resource costs of default $\xi$ through deposit insurance - banks do not internalize this resource costs.

### 3.1.3 Bank

An infinitely lived, risk neutral bank, with commitment technology and the same discount factor as households $\beta$, maximizes the presented discount value of dividends $Div$. At the beginning of the investment stage, the bank starts with capital $K$ and may further choose to raise deposits $D$ from households at interest rate $R_D$. Using capital $K$ and deposits $D$, the bank issues dividends $Div > 0$, or raises equity from households $Div < 0$, and offers optimal loan contracts to each individual firm $(i, j)$. $L_{TL}$ denotes the aggregate amount of term loan contracts given to firms, and $L_{CL}$ denotes the aggregate amount of credit line contracts. At the beginning of the drawdown stage, the bank also realizes the aggregate state $P$. Because the liquidity shock draws are iid to firms, the bank knows that exactly $P$ share of firms received a liquidity shock. As per the credit line contract, the bank pays $PL I_{CL}$ to every firm that received credit line contracts; the aggregated amount is $PL$, where $L$ is the aggregate amount of liquidity the bank has committed. At the end of the period, the bank earns the returns from its investments $L_{TL}$ and $L_{CL}$, which are $(1 - P) R_{TL} + \chi R_S$ and $R_{CL}$, respectively.
\( R_{TL}(R_{CL}) \) denotes the aggregate total returns from all term loan (credit line) contracts \( L_{TL}(L_{CL}) \), and \( R_S \) denotes the aggregate salvage value from liquidated firms. It pays depositors \( R_D D \) and updates its next period capital \( K' \).

Importantly, there is deposit insurance for bank deposits which generates moral hazard for the bank against households — households are guaranteed their deposits by the government even if the bank fails. This is an institution feature in the real world that tries to address bank runs, as in Diamond and Dybvig (1983), which is unmodelled in this paper. Because of limited liability, the bank therefore has a convexified payoff. Furthermore, when the bank defaults, a fraction \( \xi \) of their assets are deadweight loss. The bank will choose an expected default probability that does not internalize the social cost of its default. If the realization of next period capital is negative \( K' < 0 \) and the bank cannot fully repay its depositors, it receives a value of 0. Once defaulted, the government steps in and takes over the bank and liquidates its assets to pay depositors.

Following Gomes (2001) and Jermann and Quadrini (2012), external financing by raising equity through \( Div < 0 \) has a convex cost. Specifically, I follow Dempsey (2020) by using a valuation function \( \zeta(\cdot) \) that captures any direct and agency costs from issuing equity. This function is strictly increasing for all \( Div \in \mathbb{R} \), strictly concave for negative \( Div < 0 \) with \( \zeta'(Div) > 1 \), but linear (\( \zeta'(Div) = 1 \)) for positive \( Div > 0 \).

Finally, the bank is subject to a standard capital requirement which states that its equity, total assets minus total liabilities, must be some fraction \( \phi^{CR} \in [0, 1] \) of total loans.

\[
\frac{L_{TL} + L_{CL} - D}{L_{TL} + L_{CL}} \geq \phi^{CR}
\]

3.1.4 Government

The government operates a per-period balanced budget. When banks enter default, government liquidates assets at a resource cost of \( \xi \) and pays depositors - remaining amount is drawn from deposit insurance fund. Deposit insurance is funded by levying a state-contingent lump sum tax \( T \) from households.
3.2 Timing

1. At the beginning of the investment stage, the bank starts with state variable capital $K$, and raises deposits $D$ from households.

2. The bank issues dividends $Div$ to households, and offers optimal contracts $(I, R^F, i)$ to each individual firm $(i, j)$.

3. Each firm starts with assets $k_i$, borrows $I - k_i$ from bank, and invests in project size $I_{ij}$. The bank has given an aggregate term loan amount of $L_{TL}$, aggregate credit line amount of $L_{CL}$ with $L$ amount of total liquidity commitments.

4. At the beginning of the drawdown stage, aggregate state $P \in [0, 1]$ is realized by all agents.

5. Firms receive liquidity shock $\ell_i I_{ij}$ with probability $P$ and 0 with $(1 - P)$ which is iid across firms.

6. In aggregate, exactly $P$ share of firms received a liquidity shock $\ell_i I_{ij}$.

7. Of those who received a liquidity shock, firms who received a credit line contract get $\ell_i I_{ij}$ from bank and continue projects; firms who received term loans liquidate their projects and receive salvage value $\chi I_{ij}$. Banks pay $PL$ to firms in total.

8. Firms who continue exert project effort $p \in \{p_L, p_H\}$.

9. Project returns $R_j I_{ij}$ are realized with $p$. Banks receive a total of $(1 - P) R_{TL} + \chi PR_S$ and $R_{CL}$.

10. If next period capital $K'$ for bank is negative, bank defaults and is liquidated by the government. Otherwise, the bank continues to next period.

11. Households are paid deposits back at $RD$ and earn $R_f a$ from their riskless storage. Taxes $T$ are levied if necessary.

12. Old firms die and new firms are born.
Figure 2: Timing: Firms and Banks

4 Equilibrium

4.1 Decentralized equilibrium

4.1.1 Bank problem

The bank starts with capital $K$, and chooses dividends $Div$, contracts $\{C_{ij}\}_{ij}$, and deposits $D$ to maximize profits. The value of the bank is

$$V^B(K) = \max_{\{Div, (C_{ij}), D\}} \zeta(Div) + \beta \mathbb{E}V^B(K')$$  \hspace{1cm} (9)$$

s.t.

$$Div + L_{TL} + L_{CL} \leq D + K$$  \hspace{1cm} (10)$$

$$K' = \max \{0, (1 - P) R_{TL} + \chi PR_S + R_{CL} - R_D D - PL\}$$  \hspace{1cm} (11)$$

$$\frac{L_{TL} + L_{CL} - D}{L_{TL} + L_{CL}} \geq \phi_{CR}$$  \hspace{1cm} (12)$$

$$R_i^F \geq \frac{BI_{ij}}{\Delta p} \forall i, j$$  \hspace{1cm} (13)$$
\[
\begin{align*}
\begin{cases}
(1 - \bar{P}) R^F_{ij} & \geq k_i & \text{if } C_{ij} \in C_{TL} \\
R^F_{ij} & \geq k_i & \text{if } C_{ij} \in C_{CL}
\end{cases}
\end{align*}
\]

where

\[
L_{TL} \equiv \int_{C_{TL}} (I_{ij} - k_i) \\
L_{CL} \equiv \int_{C_{CL}} (I_{ij} - k_i)
\]

\[
R_{TL} \equiv \int_{C_{TL}} (R_j - R^F_{ij}) \\
R_{CL} \equiv \int_{C_{CL}} (R_j - R^F_{ij})
\]

\[
\mathcal{L} \equiv \int_{C_{CL}} \ell_i I_{ij}
\]

The bank’s problem over a general contract space may seem abstract, but a visual representation of optimal contracts illustrates how monotonicity gives a clear characterization of regions in the firm characteristics space. In Figure 3 we see how firm characteristics determine which type of contract firms receive. The relevant characteristic for determining contract type is the liquidity cost \(\ell\), which is plotted on the x-axis. On the y-axis is the project quality \(R_j\) dimension. Firms in the top right quadrant have high project quality and are therefore worth funding, but their liquidity cost \(\ell_i\) is too high such that insuring them against liquidity shocks is not profitable. Firms on the top left are similarly of high project quality, but they also have low liquidity cost. Therefore, the optimal contract is to insure them with credit lines. The boundary of the term loan region and the unfunded region is flat because conditional on receiving a term loan, the liquidity cost \(\ell_i\) does not matter and only the project quality \(R_j\) matters. However, for credit line optimal firms, the liquidity cost is also relevant for accessing project profitability. Therefore, as the project quality decreases, if the firm has sufficiently low liquidity cost, the bank will to fund them, giving a downward slope. The bottom right region in grey shows the firms who have too high of a liquidity cost and/or too low of a project quality and are not funded at all. We formalize this below in Lemma 6.
Lemma 6. Bank chooses cut off values \((\ell^*, R^*)\) such that

1. For firms \((\ell_i \geq \ell^*)\) and \((R_j \geq R^*)\), the optimal contract is \((I_{TL}, R_{TL}^F, 0)\)

2. For firms \((\ell_i < \ell^*)\) and \(\left(R_j \geq R^* + \frac{\bar{p}}{\Delta p} (\ell_i - \ell^*)\right)\), the optimal contract is \((I_{CL}, R_{CL}^F, \ell)\)

3. For all other firms, the optimal contract is to not be funded

where \(R_{TL}^F = \frac{B}{\Delta p}\), \(I_{TL} = \frac{k}{(1-P)\Delta p} \) and \(I_{CL} = \frac{k}{pu \Delta p}, R_{CL}^F = \frac{B}{\Delta p}\)

4.1.2 Household problem

The problem of a representative household is

\[
V^H = \max_{D,a,C} C + \beta \mathbb{E} V^H
\]

s.t.

\[
D + a \leq \omega \tag{19}
\]

\[
C \leq R_f a + R_D D - T + Div + V^F \tag{20}
\]
Therefore, for households to hold bank deposits it must be that

\[ R_D \geq R_f \]

and households’ deposit supply function is

\[
D_S = \begin{cases} 
0 & \text{if } R_D < R_f \\
[0, \omega) & \text{if } R_D = R_f \\
\omega & \text{if } R_D > R_f
\end{cases}
\]  

(21)

4.1.3 Government problem

Government operates a balanced budget. When banks enter default, government liquidates assets at a resource cost of \( \xi \) and pays depositors - remaining amount is drawn from deposit insurance fund. Deposit insurance is funded by taking lump sum tax \( T \) from household.

\[
T = \max \{0, R_D D - \xi [(1 - P) R_{TL} + P\chi R_S + R_{CL} - P\mathcal{C}]\}
\]  

(22)

4.2 Definition of equilibrium

A recursive general equilibrium is a set bank functions \( \{D_{iv}, \{C_{ij}\}_{ij}, D^D\} \), household functions \( \{C, a, D^S\} \), and deposit rate \( R_D \) in aggregate state \( K \) such that

1. Given deposit rate, \( \{D_{iv}, \{C_{ij}\}_{ij}, D^D\} \) solves the maximization problem of the bank

2. Firm incentive compatibility (IC) and participation constraints (PC) are satisfied

3. Given deposit rate, \( \{C, a, D^S\} \) solves the maximization problem of households

4. Given deposit rate, the government budget constraint holds

5. The deposit market clears: \( D^D = D^S \)

4.3 Constrained planner’s problem

We consider a planner who’s objective is to maximize household consumption. The planner is subject to the same frictions as private agents — we want to consider an equilibrium that is achievable through
The key difference is that the planner internalizes the cost of bank default. The planner also chooses to extend optimal contracts to firms, but bears the negative profits and any resource costs. Equation 24 represents the resource constraint from the household, equation 25 the resource constraint from financial intermediation, equations 26 and 27 show the cost of default and how capital is updated, respectively. The planner’s recursive problem is

$$V^{SP}(K) = \max_{\{C, Div, D, (C_{ij})_{x_{ij}}\}} C + \beta \mathbb{E} V^{SP}(K')$$

s.t.

$$C \leq R_f (\omega - D) + R_D D - T + \zeta (Div) + V^F$$

$$Div + L_{TL} + L_{CL} \leq D + K$$

$$T = \max \{0, R_D D - [(1 - \xi) ((1 - P) R_{TL} + P \chi R_S + R_{CL}) - P L]\}$$

$$K' = \max \{0, (1 - P) R_{TL} + P \chi R_S + R_{CL} - R_D D - P L\}$$

5 Mapping the model to the data

In order to discuss counterfactual changes to macroprudential policies, I match the quantitative model with key “micro”-moments on firm heterogeneity and “macro”-moments about the aggregate banking industry. Matching the micro moments ensures that banks are facing a distribution of firms similar to that in the data. The distribution of firm fundamentals disciplines what contracts banks can give off-the-equilibrium path, crucial for counterfactuals. I do this by ensuring the heterogeneity in the quantity, price and type of model contracts are consistent with the data. Matching macro moments ensures that the model bank has an accurate balance sheet and risk choices as observed in the data.

5.1 Data sources

In order to match the heterogeneity in the loan contracts given to firm, I use moments from FR Y-14Q H.1, which is a supervisory data set maintained by the Federal Reserve to assess capital adequacy and to support stress testing. The Y14 data consists of information on loan facilities with $1 million
in committed amount or more, held by bank holding companies (BHCs) subject to the Dodd-Frank Act Stress Tests. The advantages of the Y14 dataset are that coverage is wide and includes detailed loan level information to small firms; the supervisory data covers 60% of all corporate loans, including 50,000 SMEs. Using traditional datasets on bank loans that cover mostly large firms, such as Compustat/Capital IQ or DealScan, will incorrectly imply that most firms have credit lines and make difficult to know loan contracts to smaller firms that still account for 1/3 of C&I lending by banks. Since I do not have direct access to this confidential supervisory data set, I construct and use moments from Chodorow-Reich et al. (2021) and Greenwald et al. (2021). For bank moments, I use the FDIC’s Consolidated Reports of Condition and Income (“Call reports”) for commercial banks in the US. Since this model is specifically about investing in firms, I aggregate the banks to the bank holding company (BHC) level and only use commercial and industrial (C&I) numbers. The data sample period is 2012Q3 to 2020:Q3, unless otherwise noted. Detailed definitions of data and model moments are in Appendix B.

5.2 Estimation strategy

Model estimation occurs in two stages: an external calibration, where a subset of parameters are chosen outside the model, and an internal calibration, where parameters are chosen to match a set of moments in the data via simulated method of moments (SMM). The internal calibration is divided into two categories: micro firm parameters and macro bank parameters. Table 1 summarizes the baseline parameters of the model.

External calibration

The model period is one year. The discount factor that all agents in the economy share in common is $\beta$ which I set to 0.99 to get a risk-free interest rate $R_f$ of 1%. The household storage technology pins down the deposit rate that households are willing to accept; in equilibrium $R_D = R_f$, therefore, I set this to be the observed average bank deposit rate. I normalize the project success rates $p_H$ and $p_L$ to be 1 and 0, respectively\(^8\). Resource loss upon bank default is 20% and comes from FDIC data estimates related to the liquidation expense and cost of maintaining the FDIC deposit insurance fund. This gives

\(^8\)There is a technical restriction in these parameters in that they cannot be exactly 1 or 0 since the hidden action choice would then be verifiable. As long as they are limiting to 1 or 0, this issue does not arise.
resource loss value $\xi$ of 0.8. Lastly, since the Basel II framework is already in place in the data period, the data reflects banks who are already constrained by an 8% capital requirement. Therefore, for the estimation we set $\phi^{CR} = 0.08$.

**Micro firm parameters**

These set of parameters rely on the Y14 data with heterogeneous contract information. We exploit the fact that the optimal contract is a mapping from the set of firm characteristics $(k, B, \ell)$ to the set of contract characteristics $(I, R^F, \iota)$. We use heterogeneous contract data to essentially back out the implied firm characteristics. In the Y14 data, the firm asset size distribution is heavily skewed, as it is in the universe of firms. Chodorow-Reich et al. (2021) and Greenwald et al. (2021) report that firms with asset size less than $250$ million either do not have credit lines or have credit lines that cannot be reliable drawn upon aggregate market shocks. I define *term loan firms* to be firms with asset size less than $250$ million and consider them to not have credit lines. These firms account for 86% of the mass of firms, but account for approximately 2/3 of loans in the banking sector. On the other hand, firms with asset size greater than $250$ million are defined as *credit line firms*. These firms are 14% of the mass of firms, account for 1/3 of bank loans, and have credit lines with banks that do insure against aggregate market shocks. For the estimation, I take the capital $k$ distribution of firms to be a pareto distribution which is governed by two parameters: a scale parameter $x_m$ and a shape parameter $\sigma$. The optimal contract shows that loan quantity is increasing in firm capital $k$. Therefore, we use the two parameters that govern the firm capital distribution to target loan sizes: total loans to term loan firms are $539$ billion and total loans to credit line firms are $277$ billion. For tractability, we assume that liquidity cost $\ell$ and moral hazard $B$ are direct mappings of firm size $k$: $\ell = \ell_0 k^{\ell_1}$ and $B = B_0 k^{B_1}$. This gives a flexible functional form and asks the model to tell us the relationship between characteristics: positive/negative, any curvature, etc. I target the two parameters of the moral hazard function $B_0, B_1$ such that the average interest rate to term loan firms is 415 bp while the credit line firms get 378 bp (inclusive of any fees). For $\ell_0, \ell_1$, we use two moments. First, an estimate of the cost of the total unused commitments by banks. The Y14 data shows that banks have outstanding unused commitments of $2.77$ billion dollars in total. I take the opportunity cost of these loans to be how much spreads increase during these market downturns and set it at 250bp. Second, the liquidity cost of the firms that receive term loans on the equilibrium path cannot be observed in the data because it
is strictly off the equilibrium path. So we use the moment that only the top 14% mass of firms receive credit lines - we estimate the off equilibrium path costs to be such that the bank endogeneously chooses exactly not to give the correct mass of firms credit lines.

**Macro bank parameters**

These set of parameters rely on Call Reports. Since the aggregate liquidity shock $P$ is a probability, which in the aggregate becomes the credit line drawdown rate, it must be a distribution that is bounded between $[0, 1]$. I take a bounded pareto distribution which is heavily skewed left as are aggregate credit line drawdowns in the data. The raw average drawdown rate in the data during non-crisis times is 11%. I demean the series with this such that $P$ represents only insurance against aggregate shocks, not normal times usage. For the three pareto distribution parameters $\alpha, H, L$, I target the average of the demeaned drawdown series to be 3%, an expected bank failure rate of 1% as in the data, and normalize $H = 1$. The project technology distribution is taken to be uniform $R \sim U [0, \bar{R}]$ where $\bar{R}$ is chosen to match the banks overall leverage 0.92. I choose the functional form for the dividend valuation function as in Dempsey (2020),

$$
\zeta(Div) = \begin{cases} 
1 - \exp(-Div) & \text{if } Div < 0 \\
Div & \text{if } Div \geq 0
\end{cases} 
$$

(28)

which is concave for negative $Div < 0$. The clean feature of this particular functional form from Dempsey (2020) is that it captures convex costs of equity issuance while still imposing smoothness at the potential kink at $Div = 0$, since $\lim_{Div \to 0} \zeta(Div) = 1 = \zeta'(0)$. Another key parameter is the salvage rate $\chi$ which determines how costly term loan defaults are relative to credit line payments. It is not realistic to assume that, for example, a 30% draw on credit lines will result in a 30% loss on term loans. Therefore we introduce some salvage value that makes term loan loss less costly. Using the Y-14 data, Greenwald et al. (2021) empirically show that banks who were harder hit with credit line draws issued less new loans to firms that were borrowing from them only through term loans without credit lines. These firms who did not have credit lines, in turn, sharply decreased their capital expenditures. Using their regression estimates, I calculate that the 40% aggregate credit line drawdown during COVID 2020 was associated with a 13% drop in investment by term loan firms. I match salvage rate $\chi$ such that
term loan losses equals this when the model is hit with the exact same magnitude shock. Finally, I set
the household endowment \( \omega \) such that total interest and dividend income to household total income
matches the data moment from BEA estimates for Personal Income of interest and dividend income
being 13.2% of pre-tax disposable income. This ensures that we capture household income that is not
related to return on assets and ownership of the bank and firms, such as labor income, etc. Without
this we would overestimate the welfare implications of financial regulation.

In Figure 4 I show the estimated firm distribution and the relationship between firm characteristics.
The model correctly picks up the inverse relationship between firm size \( k \) and firm liquidity cost \( \ell \) as
observed in the data.
6 Quantitative results

6.1 How much do credit lines affect bank risk?

In crisis times, banks draw a high aggregate drawdown rate \( P \). This aggregate shock causes losses on both banks term loan and credit line portfolios. Specifically, losses on term loans come because only \((1 - P)\) share of firms survive and return \( R_{TL} \), while \( P \) share of firms liquidate and only return a salvage value of \( \chi R_S \). On credit lines, banks always get \( R_{CL} \), but have to pay out \( \mathcal{L} \). How much of its total losses comes from each contract type? We can rearrange the total returns from term loans to be:

\[
(1 - P) R_{TL} + P \chi R_S = R_{TL} - P (R_{TL} - \chi R_S) \tag{29}
\]

where the first term represents a certain return from term loans, and the second term represents the variable losses from term loans. We can now easily compare the potential losses from term loans, \( R_{TL} - \chi R_S \), with the potential losses from credit lines, \( \mathcal{L} \). We find that term loans account for 72\% of potential losses and credit lines account for 28\% of potential losses. Since these losses are both linear in aggregate \( P \), the decomposition stays the same for any realization \( P \). When banks default, total
losses to banks in dollars are $83.5 million dollars, of which $59.9 million dollars for term loans and $23.6 million dollars from credit lines.

The above decomposition is using steady state values, that is, it answers what happens when the economy is in good times and gets hit with a large shock. Now we ask the opposite question, what does bank fragility look like as the bank experiences a series of good shocks? Interestingly, the model predicts that ex-ante bank default rate actually increases in good times. That is, financial fragility increases as risk builds up in good times. Specifically, the exercise we perform is to assume that the economy is off the steady state equilibrium because of large $P$ shocks. Then, the bank receives a sequence of very low $P$ shocks simulating good times. In Figure 5, we follow the transition of a bank with low capital $K_0 = 1$ and feed it a sequence of the average value of $P$. As the bank transitions back to the steady state, we see that it slowly builds up capital — it takes about 7 years to return to the steady state. The bank mostly builds up capital through retained earnings and only issues small amounts of equity due to costly issuance. During this time, the bank is borrowing constrained by the capital requirement and therefore must increase deposits slowly as well. As capital and deposits increase, the bank is able to lend more to firms and does so by increasing the issuance of both term loans and credit lines. In the left most panels in Figure 5, we see that as bank capital increases, and deposits and loans increases, the threshold $P^*$ at which the bank would default decreases — the bank becomes increasingly susceptible to smaller $P$ shocks. As a result, ex-ante bank default probability increases. This risk build up is quantitatively significant. We see that in the first several years the size of $P$ shock needed to default the bank is much greater than 1, meaning bank default rate is 0%. As risk builds up, the threshold $P^*$ eventually reaches the steady state value of 48%. What is driving this increasing financial fragility in good times? We examine this by constructing threshold $P^*$ values for the term loan portfolio and credit line portfolio separately. That is, total bank threshold is

$$P^* = \frac{R_{TL} + R_{CL} - R_D D}{R_{TL} - \chi R_S + L}$$

and the separated thresholds are

$$P_{TL}^* = \frac{R_{TL} - R_D D_L TL + L_{CL} LCL}{R_{TL} - \chi R_S} \quad P_{CL}^* = \frac{R_{CL} - R_D D_L CL + L_{CL} LCL}{L}$$

The exercise here is to pretend the term loans and credit lines are separate banks, and we split the
liabilities according to relative portfolio size. In the right most panels of Figure 5, we make several observations. Term loans are much riskier — they default at $P$ shocks of around 26% to 30%. However, the term loan riskiness does not change much. As the bank grows, the threshold only decreases from 26% to 30%. However, credit lines are the opposite. They start out very safe at a threshold of over 600%, but decreases rapidly to the steady state value of 95%. This means that in level terms term loans are consistently the riskiest part of the portfolio — they default easily. On the other hand, credit lines are very good at weathering small shocks, but are susceptible to big shocks. This is why credit lines only become a big liability to banks in big crises, not over the regular business cycle. Much of this feature is because banks select large firms with low cost to insure. However, in the transition from receiving series of a good shocks, it is the credit line risk increasing dramatically that drives the overall bank default rate from going up from 0% to near 1%.

Why is this happening? Recall the revenue of credit lines is $R_{CL}$, while its potential costs are $L$, the total loan commitments. We see that $R_{CL}$ is increasing at a decreasing returns to scale as does $R_{TL}$. However, the total loan commitment $L$ is increasing at a constant rate. As the bank lends more to marginally lower project quality $R_j$, because of the orthogonality of the project quality dimension and the firm characteristics dimension, the marginal increasing in distribution of liquidity costs $\ell$ stays constant. At this point, it may seem the orthogonality assumption affects this result. However, we see that if we added correlation between the project quality $R_j$ and liquidity cost $\ell$, that correlation should be negative, that is, lower quality firms should have higher liquidity cost $\ell$. That would make loan commitments $L$ actually increasing returns to scale, further exacerbating the drop in revenue to cost for credit lines. Therefore, we see that there is build up of financial fragility in the model that is driven by credit lines, and that this is actually a conservative estimate given the orthogonality assumption.
6.2 Are we safer without credit lines?

By insuring firms through credit lines, banks are assuming aggregate risk for firms - firm default risk is transfered to banks. Does this then mean that eliminating credit lines will decrease bank default risk? If one eliminates credit lines, banks will adjust and make new choices. Therefore, we need to perform an equilibrium analysis of such a counterfactual. In Table 2 we see the model moments from the baseline model compared to a counterfactual economy in which credit lines are eliminated and banks can only lend through term loan contracts. The first thing we see is that the bank increases term loans by substituting the credit line lending with term loans. However, it does not substitute completely as total loans decreases by $100 million. The bank borrows less to finance this smaller amount of lending by decreasing deposits by $110 million. Because the term loan quantity increases, the interest rate on term loans decreases following the demand curves derived from the optimal contracts. What is striking is that bank default rate actually increases - it doubles from 1.1% to 2.0%. This is because, while the bank does not have credit line exposure, only lending through term loans increases firm default rate. This increase in firm default rate contributing to a higher bank default rate. Output decreases from the decreased lending, and welfare decreases even further because of lower output and higher taxes from higher bank default rates.
Table 2: Counterfactual equilibrium: Term loans only

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Term loan only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term Loans $L_{TL}$</td>
<td>320</td>
<td>651</td>
</tr>
<tr>
<td>Credit Lines $L_{CL}$</td>
<td>556</td>
<td>−</td>
</tr>
<tr>
<td>Total loans</td>
<td>876</td>
<td>651</td>
</tr>
<tr>
<td>Deposits $D$</td>
<td>813</td>
<td>505</td>
</tr>
<tr>
<td>Credit line exposure $\mathcal{L}$</td>
<td>68</td>
<td>−</td>
</tr>
<tr>
<td>TL interest rate $r_{TL}$</td>
<td>500bp</td>
<td>505bp</td>
</tr>
<tr>
<td>CL interest rate $r_{CL}$</td>
<td>481bp</td>
<td>−</td>
</tr>
<tr>
<td>Capital ratio</td>
<td>8%</td>
<td>8%</td>
</tr>
<tr>
<td>Bank default rate</td>
<td>0.95%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Firm default rate</td>
<td>3.5%</td>
<td>4.1%</td>
</tr>
<tr>
<td>Share of firms: Term loans</td>
<td>83%</td>
<td>100%</td>
</tr>
<tr>
<td>Output $Y$</td>
<td>6439</td>
<td>4714</td>
</tr>
<tr>
<td>Welfare (c.e.)</td>
<td>−</td>
<td>−13%</td>
</tr>
</tbody>
</table>

Notes: Values are in $ billions. Total loans are the sum of term loans and credit lines. Welfare is calculated in consumption equivalence.

What is causing the substitution effect to dominate and increase bank default rate? Table 2 shows the aggregate moments, but we can look at the firm distribution through Figure 6. First thing we notice is that the formerly blue region of firms with credit lines are now partially replaced with term loans. These firms were of low liquidity cost $\ell$ type and were relatively cheap to insure for the bank. Now, the bank isn’t allowed to cheaply insure these loans for the firms and must let these projects fail from liquidity shocks. Instead of scaling back lending from this increased risk, the bank instead doubles down by increasing loans. We see that the project quality cut off $R^*$ is lower in the counterfactual, meaning the bank is reaching deeper into the pool of firms and lending to less efficient firms now. Furthermore, the lower triangular region from the formerly credit line firms are now unfunded. Those firms were not very efficient, but their cost of insurance was also very low. Now that the bank can’t insure them, their inefficiency becomes relevant again.
6.3 The planner’s allocation

Given that a term loan only economy has lower welfare and even higher default rate, what then is the efficient allocation? In Table 3 we see again see the baseline economy, now compared to the planner’s allocation from the earlier planner’s problem. Again, the planner is subject to the same frictions as private agents, but chooses the banks allocations by maximizing household welfare. This means the key distinction is that the planner internalizes the fiscal cost of bank default through taxes. The planner decreases both the amount of lending through term loans, by 6%, and credit lines, by 7%. The planner thinks the decentralized bank is overlending in credit lines a bit more than through term loans, but there doesn’t seem to be large differences, suggesting that banks are largely able to understand the risk coming from each type of contract. The larger channel is the quantity channel through which the bank is lending too much overall, by 12%. However, the 7% decrease in credit lines masks the fact that the planner wants to decrease credit line exposures by 11%. Because the bank optimally selects the lower liquidity cost firms first, the marginal decrease in credit lines will come first for the firms with the largest liquidity costs. Therefore decreasing the credit line quantity does not result in a linear decrease in potential losses. The interest rate on both term loans and credit lines increase following the aggregate demand curves since lending is decreased. The planner wants less lending, but also wants the funding composition of the loans to be more capital intensive. Deposits decrease by 13% and the capital ratio increases from 9.3% to 15.2%. This increase in the capital ratio helps banks half their default rate from 1.1% to 0.5%. The decrease in the amount of credit lines being larger than the
decrease in term loans is reflected in the fact that the share of firms who receive term loans increase from 85% to 86%. This causes the firm default rate to increase slightly from 3.3% to 3.34%. Output and welfare increase by 0.2% largely because the tax costs to household from bank default is going down.

We can again look at the firm distribution to see how contracts are changing in Figure 7. We see that the vertical line that denotes the cut off value for \( \ell^* \) shifts to the left for the planner. This means that the firms at the margin of the contract types are receiving insurance through credit lines when in the fact the planner thinks that is not worth the risk-return tradeoff given their liquidity cost; they are, however, efficient in project to still fund and should receive term loans. We also see that the planner moves up the triangular bottom portion of the credit line region. Similarly to firms at the margin of contract types, the firms at the boundary of receiving credit lines and no loans shifts towards funding fewer of them. These firms have low enough liquidity costs, but the planner views their project quality to be too low. Lastly, the project quality cut off \( R^* \) also moves up as the planner thinks the low project quality firms do not return enough in positive states. While the term loan region increases along the firm characteristic dimension, it decreases along the project quality dimension and ends up decreasing in the aggregate. The credit line region decreases along both dimensions and unambiguously decreases in total.

Table 3: Planner’s allocation

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Planner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Term Loans ( L_{TL} )</td>
<td>320</td>
<td>301</td>
</tr>
<tr>
<td>Credit Lines ( L_{CL} )</td>
<td>556</td>
<td>534</td>
</tr>
<tr>
<td>Total loans</td>
<td>876</td>
<td>835</td>
</tr>
<tr>
<td>Deposits ( D )</td>
<td>813</td>
<td>745</td>
</tr>
<tr>
<td>Credit line exposure ( L )</td>
<td>68</td>
<td>66</td>
</tr>
<tr>
<td>TL interest rate ( r_{TL} )</td>
<td>500bp</td>
<td>518bp</td>
</tr>
<tr>
<td>CL interest rate ( r_{CL} )</td>
<td>481bp</td>
<td>496bp</td>
</tr>
<tr>
<td>Capital ratio</td>
<td>8%</td>
<td>11%</td>
</tr>
<tr>
<td>Bank default rate</td>
<td>0.95%</td>
<td>0.56%</td>
</tr>
<tr>
<td>Firm default rate</td>
<td>3.5%</td>
<td>3.3%</td>
</tr>
<tr>
<td>Share of firms: Term loans</td>
<td>83%</td>
<td>83%</td>
</tr>
<tr>
<td>Output ( Y )</td>
<td>6439</td>
<td>6440</td>
</tr>
<tr>
<td>Welfare (c.e.)</td>
<td>–</td>
<td>+0.02%</td>
</tr>
</tbody>
</table>

Notes: Values are in $ billions. Total loans are the sum of term loans and credit lines. Welfare is calculated in consumption equivalence.
6.4 Optimal macropudential policy

How do we get to the planner’s allocation in a market economy? We can decentralize the allocation by modifying the existing framework of macroprudential policy. We first show how to fully achieve the planner’s allocation, and second, show how to maximize welfare when the set of policy tools at our disposal is limited. The planner’s allocation can be achieved through three instruments: a capital requirement, a leverage ratio, and a loan commitment constraint. The first two instruments are policies already in place under the Basel III framework. The capital requirement dictates how much capital the bank must have as a fraction of its total balance sheet. The leverage ratio constrains how much debt a bank holds relative to its capital stock. Intuitively, together these two constraints help pin down the total lending $L_{TL} + L_{CL}$ and total borrowing $D$ of the bank to match that of the planner’s. As the third instrument we propose a loan commitment constraint: a constraint on how much off-balance sheet exposure the bank has relative to its capital stock. This last constraint helps the planner split the total lending $L_{TL} + L_{CL}$ into the appropriate shares of $L_{TL}, L_{CL}$.

\[
\frac{L_{TL} + L_{CL} - D}{L_{TL} + L_{CL}} \geq \phi^{CR} \tag{30}
\]

\[
\frac{D}{L_{TL} + L_{CL} - D} \leq \phi^{LR} \tag{31}
\]

\[
\frac{L}{L_{TL} + L_{CL} - D} \leq \phi^{CC} \tag{32}
\]
However, in the real world, the political process of passing legislation and introducing new regulatory policy, such as a loan commitment constraint, can be quite difficult and lengthy. We show that even if we limit ourselves to just a single instrument, a risk-weighted capital requirement, we are able to achieve 95% of the planner’s welfare gains if implemented correctly.

\[
\frac{L_{TL} + L_{CL} - D}{L_{TL} + L_{CL} + \theta^{LC} L} \geq \phi^{CR}
\]

(33)

Table 4: Optimal policy parameters

<table>
<thead>
<tr>
<th></th>
<th>Full implementation</th>
<th>Single instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital requirement (\phi^{CR})</td>
<td>11%</td>
<td>12%</td>
</tr>
<tr>
<td>Leverage requirement (\phi^{LC})</td>
<td>8%</td>
<td>–</td>
</tr>
<tr>
<td>Commitment constraint (\phi^{CC})</td>
<td>66</td>
<td>–</td>
</tr>
<tr>
<td>Risk-weight (\theta^{LC})</td>
<td>–</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Notes: Optimal parameters are found through grid searching to maximize steady state welfare.

When constrained to only the risk-weighted capital requirement, the capital requirement is set higher than when using all three instruments (11% vs. 12%). This is because the capital requirement is used, inefficiently, to curb risk-shifting that occurs through leverage and credit lines. To be more concrete, when only the capital requirement is set at 11%, as in the full implementation, without setting leverage requirements and commitment constraints, then the bank tries to undo the decreasing default probability through other margins. Specifically, it increases leverage by increasing deposits and increasing the relative share of credit lines. Therefore, to compensate for this, the optimal policy increases capital requirements by a full percent. The regulator then uses the risk weight on loan commitments \(\theta^{LC}\) to correct the share of credit lines vs. term loans. However, we see that this risk-weight is not very large — a weight of \(\theta^{LC} = 0.1\) on \(L\), which is the opportunity cost on undrawn credit lines, translated into a weight of 0.0025 in the Basel III framework. This shows that the distortionary effects of risk-weights actually outweighs any inefficient substitution into credit lines. This is in line with the difference in the baseline equilibrium and the planner equilibrium in which the relative share of contracts is not very different — as we already saw, the decentralized bank is already setting the efficient *share* of contracts in equilibrium, but *over* lending in both.
6.5 Optimal policy from term loan only models

How can policy be inaccurate if we do not consider models with credit lines? To create the most apples-to-apples comparison, we conduct the following exercise. First, we take our model and shut down credit lines: banks are only allowed to lend through term loan contracts. This model has the same mechanisms as conventional banking models: banks take excessive risk through overleveraging term loans, and the main risk comes from non-performing loans (term loan defaults). We take this model and re-estimate to match the same data moments as our benchmark model (estimation results in table B.1 of Appendix B). Then, using the newly estimated parameters, we solve the same planner problem, but one in which the planner also thinks the economy features only term loans. When we compare the optimal capital ratio that comes from this term loan only model, we find that it significantly underpredicts what the optimal capital ratio should be from 11.2% to 8.2%, and is remarkably close to the current capital requirement of 8%. This might lead policymakers to think that the current capital ratio is sufficient.

Table 5: Optimal policy across models

<table>
<thead>
<tr>
<th></th>
<th>Optimal capital ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basel III</td>
<td>8%</td>
</tr>
<tr>
<td>Model with both contracts</td>
<td>11.2%</td>
</tr>
<tr>
<td>Model with only term loans</td>
<td>8.2%</td>
</tr>
</tbody>
</table>

Notes: Optimal parameters are found through grid searching to maximize steady state welfare.

Why is the model underpredicting the optimal capital ratio? In both models, bank default is socially costly and the planner would like to avoid it. Both models are calibrated to the same data moments, so both models feature 1% default probability. The planner can curb this costly 1% default by increasing capital buffers. What is the cost of capital buffers though? With higher capital buffers, this increases the cost to finance loans — a well known phenomenon in the literature. Therefore the planner must trade off lower profits against lower defaults. It is here where the models diverge. Because banks are able to earn higher profits through credit lines, scaling back loans is less costly with credit lines than term loans. Also, since credit lines have linearly increasing liquidity costs but decreasing returns to scale total returns, the marginal credit line loans were less profitable than marginal term loans. For these reasons it is less costly to scale back loans in the baseline model — expected profits won’t decrease as much. Therefore in this model the planner increases capital ratios to 11% to curb default risk. In the term loan only model, while the planner would like higher capital ratios to lower
default risk, the negative impact it would have on bank profits is calculated to be much higher. That is, through model mispecification, the planner overestimates the costs of increasing capital ratios.

7 Conclusion

This paper studied how much providing credit lines to firms contributes to bank risk and its welfare implications. I explicitly modelled endogenous contract choice such that credit line contracts have a purpose in the economy, but may contribute to socially excessive risk taking by banks. I show that term loans account for 80% of bank losses in crisis times and credit lines account for 20%. While term loans are always risky in level terms, increasing financial fragility in good times is driven by increasing credit line risk. In normative exercises, I show how regulators can implement the constrained efficient allocations, and how model mispecification can lead to inaccurate policy.
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A Proofs and derivations

Lemma 1

Proof. Suppose the splitting rule $R^F$ is state-contingent, i.e., $R^F$ differs on the realization of the liquidity shock. Denote $R^F_0$ return to firm when liquidity shock is 0 and $R^F_\ell$ return to firm when liquidity shock is $\ell$. Then, the banks problem is

$$
\max_{I,R^F_0,R^F_\ell} V_{CL} = P_{pH} (R - R^F_\ell) I + (1 - P) p_{pH} (R - R^F_0) I - (I - k) - P\ell I
$$

Firm incentive compatibility constraint (IC) are

$$
R^F_0 \geq \frac{B}{\Delta p}
$$

$$
R^F_\ell \geq \frac{B}{\Delta p}
$$

Firm participation constraint (PC) is

$$
P_{pH} R^F_\ell I + (1 - P) p_{pH} R^F_0 I \geq k
$$

We want to investigate the contract in which the bank receives more in the shock state so that it is naturally encouraged to continue the project and less if it doesn't have to pay the liquidity shock. Therefore, $R^F_\ell < R^F_0$. Then, it follows that the first IC constraint is slack and the second IC constraint is binding. Since the PC binds, this gives

$$
P_{pH} R^F_\ell I + (1 - P) p_{pH} R^F_0 I = k
$$

$$
\iff P_{pH} \frac{B}{\Delta p} I + (1 - P) p_{pH} R^F_0 I = k
$$

$$
\iff P_{pH} \frac{B}{\Delta p} I + (1 - P) p_{pH} R^F_0 I = k
$$

$$
\iff P \frac{B}{\Delta p} + (1 - P) R^F_0 = \frac{k}{p_{pH} I}
$$

we see that to satisfy the PC constraint with equality, the bank has to choose $R^F_0$ such that lower $R^F_0$ leads to higher project size $I$. Since the banks objective function is increasing in project size $I$ and
decreasing in $R_0^F$, it must be that the bank lowers $R_0^F$ as much as possible and both IC binds. If both IC binds, it must be that the return to firm is not state-contingent.

Lemma 2

Proof. Since the liquidity shock takes values of either $\ell$ or 0, any transfer $\hat{\ell} < \ell$ is not sufficient to continue the project. Therefore contracts that offer such underinsurance will only be costly to the bank without changing any outcomes. Similarly for $\hat{\ell} > \ell$, the additional transfer beyond the continuation cost does not affect firm outcomes and only increases firm returns so will not be offered by the bank. Therefore the only values $\hat{\ell}$ can take are either 0 or $\ell$ in equilibrium.

Lemma 3

Proof. The optimal term loan contract solves,

$$\max_{I_{TL}, R_{TL}^F} V_{TL} = (1 - P) p_H (R - R_{TL}^F) I_{TL} + P \chi I_{TL} - (I_{TL} - k)$$

Firm incentive compatibility constraint (IC) is

$$R_{TL}^F \geq \frac{B}{\Delta p}$$

Firm participation constraint (PC) is

$$(1 - P) p_H R_{TL}^F I_{TL} \geq k$$

It must be that both IC and PC are binding. Proof is by contradiction in considering the alternative cases.

1. **Neither binds**: Bank can decrease $R_f$ by epsilon and both constraints remain slack, but objective function increases. Contradiction.

2. **IC binds, but PC is slack**: Assume that

$$R_{TL}^F = \frac{B}{\Delta p}$$
then objective becomes

$$
\max_{I_{TL}, R_{TL}^F} V_{TL} = (1 - P) p_H (R - R_{TL}^F) I_{TL} + P\chi I_{TL} - (I_{TL} - k)
$$

$$
= (1 - P) p_H \left( R - \frac{B}{\Delta p} \right) I_{TL} + P\chi I_{TL} - (I_{TL} - k)
$$

$$
= \left[ (1 - P) p_H \left( R - \frac{B}{\Delta p} \right) - 1 + P\chi \right] I_{TL} + k
$$

From Assumption 1, we know that

$$
p_H \left( R - \frac{B}{\Delta p} \right) < \min \left\{ 1 + P\ell, \frac{1 - P\chi}{1 - P} \right\}
$$

$$
\iff p_H \left( R - \frac{B}{\Delta p} \right) < \frac{1 - P\chi}{1 - P}
$$

$$
\iff (1 - P) p_H \left( R - \frac{B}{\Delta p} \right) - 1 + P\chi < 0
$$

Therefore, the optimal $I_{TL} = 0$, but this contradicts PC being slack.

3. **PC binds, but IC is slack**: Assume that

$$
R_{TL}^F \geq \frac{B}{\Delta p}
$$

$$
(1 - P) p_H R_{TL}^F I_{TL} = k
$$

then,

$$
R_{TL}^F = \frac{k}{(1 - P) p_H I_{TL}}
$$
therefore objective becomes

\[
\max_{I_{TL}, R_{TL}^F} V_{TL} = (1 - P) p_H \left( R - R_{TL}^F \right) I_{TL} + P \chi I_{TL} - (I_{TL} - k)
\]

\[
= (1 - P) p_H \left( R - \frac{k}{(1 - P) p_H I_{TL}} \right) I_{TL} + P \chi I_{TL} - (I_{TL} - k)
\]

\[
= ((1 - P) p_H R - 1 + P \chi) I_{TL}
\]

From Assumption 1, we know that

\[
\min \left\{ 1 + P \ell, \frac{1 - P \chi}{1 - P} \right\} < p_H R
\]

\[
\iff \frac{1}{1 - P} < p_H R
\]

\[
\iff 0 < (1 - P) p_H R - 1
\]

Therefore, the optimal \( I_{TL} = \infty \), but then binding PC gives \( R_{TL}^F = 0 \), which then contradicts IC being slack. Therefore, it must be that both constraints are binding, giving

\[
R_{TL}^F = \frac{B}{\Delta p}
\]

\[
I_{TL} = \frac{k}{(1 - P) p_H \frac{E}{\Delta p}}
\]

Similarly, the optimal credit line contract solves,

\[
\max_{I_{CL}, R_{CL}^F} V_{CL} = p_H \left( R - R_{CL}^F \right) I_{CL} - (I_{CL} - k) - P \ell I_{CL}
\]

Firm incentive compatibility constraint (IC) is

\[
R_{CL}^F \geq \frac{B}{\Delta p}
\]

Firm participation constraint (PC) is

\[
p_H R_{CL}^F I_{CL} \geq k
\]
It must be that both IC and PC are binding. Proof is by contradiction in considering the alternative cases.

1. **Neither binds**: Bank can decrease $R_f$ by epsilon and both constraints remain slack, but objective function increases. Contradiction.

2. **IC binds, but PC is slack**: Assume that

$$R_{CL}^F = \frac{B}{\Delta p}$$

$$p_H R_{CL}^F I_{CL} \geq k$$

then objective becomes

$$\max_{I_{CL}, R_{CL}^F} V_{CL} = p_H \left( R - R_{CL}^F \right) I_{CL} - (I_{CL} - k) - P\ell I_{CL}$$

$$= p_H \left( R - \frac{B}{\Delta p} \right) I_{CL} - (I_{CL} - k) - P\ell I_{CL}$$

$$= \left[ p_H \left( R - \frac{B}{\Delta p} \right) - 1 - P\ell \right] I_{CL} + k$$

From Assumption 1, we know that

$$p_H \left( R - \frac{B}{\Delta p} \right) < \min \left\{ 1 + P\ell, \frac{1 - P\chi}{1 - P} \right\}$$

$$\iff p_H \left( R - \frac{B}{\Delta p} \right) < 1 + P\ell$$

$$\iff p_H \left( R - \frac{B}{\Delta p} \right) - 1 - P\ell < 0$$

Therefore, the optimal $I_{TL} = 0$, but this contradicts PC being slack.

3. **PC binds, but IC is slack**: Assume that

$$R_{CL}^F \geq \frac{B}{\Delta p}$$

$$p_H R_{CL}^F I_{CL} = k$$
then,

\[ R_{CL}^F = \frac{k}{p_H I_{CL}} \]

therefore objective becomes

\[
\max_{I_{CL}, R_{CL}^F} V_{CL} = p_H \left( R - R_{CL}^F \right) I_{CL} - (I_{CL} - k) - P\ell I_{CL} \\
= p_H \left( R - \frac{k}{p_H I_{CL}} \right) I_{CL} - (I_{CL} - k) - P\ell I_{CL} \\
= (p_H R - 1 - P\ell) I_{CL}
\]

From Assumption 1, we know that

\[
\min \left\{ 1 + P\ell, \frac{1 - P\chi}{1 - P} \right\} < p_H R \\
\iff 1 + P\ell < p_H R \\
\iff 0 < p_H R - 1 - P\ell
\]

Therefore, the optimal \( I_{CL} = \infty \), but then binding PC gives \( R_{CL}^F = 0 \), which then contradicts IC being slack.

Therefore, it must be that both constraints are binding, giving

\[
R_{CL}^F = \frac{B}{\Delta p} \\
I_{CL} = \frac{k}{p_H \frac{B}{\Delta p}}
\]
Lemma 4

Proof. Simply comparing the two contracts gives,

\[ V_{CL} (I_{CL}, R_{CL}^F) > V_{TL} (I_{TL}, R_{TL}^F) \]
\[ \iff p_H (R - R_{CL}^F) I_{CL} - (I_{CL} - k) - P \ell I_{CL} > (1 - P) p_H (R - R_{TL}^F) I_{TL} + P \chi I_{TL} - (I_{TL} - k) \]
\[ \iff \left[ p_H \left( R - \frac{B}{\Delta p} \right) - 1 - P \ell \right] I_{CL} + k > \left[ (1 - P) p_H \left( R - \frac{B}{\Delta p} \right) - 1 + P \chi \right] I_{TL} + k \]
\[ \iff \left( 1 - P \right) \ell < 1 - \chi \]
\[ \iff \ell < \frac{1 - \chi}{1 - P} \]

Lemma 5

Proof. Using the optimal contract terms, we see that,

\[ I_{TL} = \frac{k}{\left( 1 - P \right) p_H \frac{B}{\Delta p}} > \frac{k}{p_H \frac{B}{\Delta p}} = I_{CL} \]
and

\[ r_{TL} = \frac{p_H (R - R_{TL}^F) I_{TL}}{I_{TL} - k} \]

\[ = \frac{p_H R \frac{k}{(1-P)pH \Delta p} - \frac{k}{(1-P)}}{(1-P)pH \frac{p_H}{\Delta p} - k} \]

\[ = \frac{(1-P) \frac{R}{\Delta p} - \frac{1}{(1-P)}}{1 - (1-P)pH \frac{B}{\Delta p} B} \]

\[ = \frac{p_H (R - \frac{B}{\Delta p})}{1 - (1-P)pH \frac{B}{\Delta p}} < \frac{p_H (R - \frac{B}{\Delta p})}{1 - p_H \frac{B}{\Delta p}} \]

\[ = r_{CL} \]

similarly,

\[ \frac{\partial (I_{TL} - k)}{\partial k} = \frac{1}{(1-P)pH \frac{B}{\Delta p}} - 1 > 0 \]

\[ \frac{\partial (I_{CL} - k)}{\partial k} = \frac{1}{p_H \frac{B}{\Delta p}} - 1 > 0 \]

\[ \frac{\partial (I_{TL} - k)}{\partial B} = - \frac{1}{(1-P)pH \frac{B}{\Delta p}} k < 0 \]

\[ \left( \frac{1}{p_H \frac{B}{\Delta p}} \right)^2 < 0 \]

\[ \frac{\partial (I_{CL} - k)}{\partial B} = - \frac{1}{p_H \frac{B}{\Delta p}} k < 0 \]

\[ \left( \frac{1}{p_H \frac{B}{\Delta p}} \right)^2 < 0 \]
Lemma 6

Proof. We argue by monotonicity. By Lemma 4, we show for any given combination of \((k, B, \ell, R)\), only the \(\ell\) type determines this cutoff. For the extensive margin on whether to receive a loan or not, we check NPV of project to banks.

\[
V_{TL} > 0 \\
\iff (1 - P) p_H (R - R_{TL}^F) I_{TL} - (I_{TL} - k) > 0 \\
\iff (1 - P) p_H \left( R - \frac{B}{\Delta p} \right) \frac{k}{(1 - P) ph_B} - \frac{k}{(1 - P) ph_B} + k > 0 \\
\iff (1 - P) p_H \left( R - \frac{B}{\Delta p} \right) \frac{1}{(1 - P) ph_B} - \frac{1}{(1 - P) ph_B} + 1 > 0 \\
\iff (1 - P) p_H \left( R - \frac{B}{\Delta p} \right) > 1 - (1 - P) ph_B \\
\iff (1 - P) p_H R - (1 - P) ph_B > 1 - (1 - P) ph_B \\
\iff (1 - P) p_H R > 1 \\
\iff R > \frac{1}{(1 - P) p_H}
\]
NPV valuation of a term loan project does not depend on $\ell$ since it is off equilibrium path. For credit line contracts, $\ell$ does feature into the calculation and therefore gives an indifference relationship between $R$ and $\ell$.

$$V_{CL} > 0 \iff p_H \left( R - \frac{B}{\Delta p} \right) \frac{k}{p_H \frac{B}{\Delta p}} - \frac{k}{p_H \frac{B}{\Delta p}} - k - P\ell \frac{k}{p_H \frac{B}{\Delta p}} > 0$$

$$\iff p_H \left( R - \frac{B}{\Delta p} \right) \frac{k}{p_H \frac{B}{\Delta p}} - \frac{k}{p_H \frac{B}{\Delta p}} + k - P\ell \frac{k}{p_H \frac{B}{\Delta p}} > 0$$

$$\iff p_H \left( R - \frac{B}{\Delta p} \right) \frac{1}{p_H \frac{B}{\Delta p}} - \frac{1}{p_H \frac{B}{\Delta p}} + 1 - P\ell \frac{1}{p_H \frac{B}{\Delta p}} > 0$$

$$\iff p_H \left( R - \frac{B}{\Delta p} \right) \frac{1}{p_H \frac{B}{\Delta p}} - p_H \frac{B}{\Delta p} \frac{1}{p_H \frac{B}{\Delta p}} - \frac{1}{p_H \frac{B}{\Delta p}} + 1 - P\ell \frac{1}{p_H \frac{B}{\Delta p}} > 0$$

$$\iff p_H - \frac{1}{p_H \frac{B}{\Delta p}} - P\ell \frac{1}{p_H \frac{B}{\Delta p}} > 0$$

$$\iff R - \frac{1}{p_H} - P\ell > 0$$

$$\iff p_H R - 1 - P\ell > 0$$

$$\iff R > \frac{1 + P\ell}{p_H}$$

Proposition 1

Proof. Recall credit line contracts are optimal iff $(1 - P)\ell < 1$. Assume further that $p_H R > \ell > p_H \left( R - \frac{B}{\Delta p} \right)$, and that the firm and bank decided on a wait-and-see policy. If the firm draws a liquidity shock $\ell$, the project is still NPV positive since $p_H R > \ell$. The total surplus is still positive and it is efficient to pay $\ell$ and continue. However, if the bank gives a second loan of size $\ell$ to continue the project, the per-unit return from the project to the bank is $p_H \left( R - R_{CL}^E \right) = p_H \left( R - \frac{B}{\Delta p} \right)$, according to the optimal contract. Since, $\ell > p_H \left( R - \frac{B}{\Delta p} \right)$, the bank gets a negative return and thus will not want to continue. Therefore, a wait-and-see policy would not work. A natural question is why the firm and bank would not renegotiate and continue if there is still positive surplus. The banks return $p_H \left( R - R_{CL}^E \right)$ is already the maximum the bank can extract from the project without violating the
firm’s IC (recall the IC is binding in the optimal contract), therefore any surplus the firm tries to give
the bank to induce it to continue will not be credible.

Similarly, this type of renegotiation might be desired by the firm since the term loan contract costs
less. The expected return to the investor from issuing another term loan to cover the realization of
\( \ell \cdot I_{TL} \) is

\[
V_{TL2} = \max_{R_{TL2}^F} \left( p_H \cdot \left( R - R_{TL2}^F \right) I_{TL} - \ell \cdot I_{TL} \right)
\]

subject to firm IC

\[
R_{TL2}^F \geq \frac{B}{\Delta p}
\]

Since IC is binding, the objective function is linear in \( I_{TL} \), but \( \ell > p_H \left( R - \frac{B}{\Delta p} \right) \) means bank will not
agree to renegotiate. \( \square \)

**Model with outside investment opportunity**

Suppose in addition to the environment in the main body, there is an investment opportunity for the
bank in the intermediate period, but the bank is constrained in its lending in that period. Specifically,
the bank receives an endowment of \( y \) at the beginning of subperiod 2, and has an investment opportunity
of \( R_2 > 1 \). For the firm, those that receive a liquidity shock \( \ell I \) and pay it to continue will earn \( \ell I \) back
at the end of period 2 - the liquidity shock is purely a liquidity issue, not one that lowers the project
value. Then, for a term loan only contract, the bank’s problem is

\[
\max_{I, R^F} V_{TL} = (1 - P) p_H \left( RI - R^F \right) - (I - k) + R_2 I_2
\]

\[
R^F \geq \frac{B}{\Delta p}
\]

\[
(1 - P) p_H R^F I \geq k
\]

\[
I_2 \leq y
\]

where we see that since the bank has no obligations to pay the first firm’s liquidity cost, it may put \( y_2 \)
fully into \( R_2 \). Note that the solution to the problem does not change - the IC and PC are the same and
are still binding. Then, for a credit line contract, the bank’s problem is

\[
\max_{I, R_F} V_{CL} = p_H (R_I - R_F) - (I - k) + P \ell I + (1 - P) R_2 y + P R_2 (y - \ell I)
\]

\[
= p_H (R_I - R_F) - (I - k) + P \ell I + R_2 (y - P \ell I)
\]

\[
= p_H (R_I - R_F) - (I - k) + R_2 y - P \ell I (R_2 - 1)
\]

\[
R_F \geq \frac{B}{\Delta p}
\]

\[
p_H R_F I \geq k
\]

\[
1_{\text{shock}} \ell I + I_2 \leq y
\]

Now the bank has a non-trivial budget constraint in the intermediate period. Note the bank pays \(\ell I\) to the firm, but also knows it will receive this same \(\ell I\) amount back (just like a credit line loan). However, the bank is unable to put all its endowment \(y\) into \(R_2\). This is equivalent to a negative cash flow shock to the bank compared to the term loan contract.
## B Data and estimation

### Table B.1: Estimation of term loan only model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Term loan only model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm size distribution $k$</td>
<td>$x_m$</td>
<td>Total loans to bottom 84% firms</td>
<td>277</td>
<td>303</td>
</tr>
<tr>
<td>Firm size distribution $k$</td>
<td>$\sigma$</td>
<td>Total loans to top 16% firms</td>
<td>539</td>
<td>560</td>
</tr>
<tr>
<td>Moral hazard $B$</td>
<td>$B_0$</td>
<td>Average interest rate to bottom 84% firms</td>
<td>415bp</td>
<td>489bp</td>
</tr>
<tr>
<td>Moral hazard $B$</td>
<td>$B_1$</td>
<td>Average interest rate to top 16% firms</td>
<td>350bp</td>
<td>472bp</td>
</tr>
<tr>
<td>Liquidity cost $\ell$</td>
<td>$\ell_0$</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Liquidity cost $\ell$</td>
<td>$\ell_1$</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

### Panel A: Micro firm parameters

| Project quality distribution $\bar{R}$ | 1.07 | Bank leverage | 0.92 | 0.90 |
| Salvage rate $\chi$                  | 0.5  | TL loss in COVID | 13% | 14%  |
| Shock distribution $\alpha$          | 1.4  | Bank default rate | 1% | 0.94% |
| Shock distribution $L$                | 0.01 | Average excess drawdowns | 3% | 3% |
| Household endowment $\omega$         | –    | –              | –    | –    |

### Panel B: Macro bank parameters

| Discount factor $\beta$              | 0.99 | Risk-free rate | 1% | 1% |
| Household storage technology $R_f$   | 1.01 | Average deposit rate | – | – |
| Project success rates $(p_H, p_L)$   | (1.0)| Normalized | – | – |
| Resource loss $\xi$                  | 0.2  | FDIC liquidation cost | – | – |
| Capital requirement $\phi^{CB}$      | 0.08 | Basel II baseline | – | – |

Notes: Moments are in $ billions, unless otherwise noted.