Problem 1 (25p). (Uncertainty and insurance)
You are an owner of a luxurious sailing boat, worth $10, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment (0, 10).

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1, C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage $x$. The insurance contract costs $\gamma x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage $x$. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate $\gamma$ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)
Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (0, 50)$ and Bob’s endowment is $\omega^B = (50, 0)$.

The utility function of both Andy and Bob is the same and given by $U(x_1, x_2) = 3 \ln x_1 + 3 \ln x_2$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ... ).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

f) Using $MRS$ condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of $500 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).
b) Sam is a hockey player who earns $100 when young and $0 when old. Sam’s intertemporal utility is given by \( U(C_1, C_2) = \ln (c_1) + \frac{1}{1+r} \ln (c_1) \). Assuming \( \delta = r = 0 \) and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers \( C_1, C_2, S \)). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by \( y = 2K^{3/4}L^{1/4} \). Find analytically a short-run demand for labor (assume \( K = 1 \)). Find analytically equilibrium real wage rate if labor supply is given by \( L^s = 16 \). Depict it in a graph.

d) You start your first job at the age of 21 and you work till 60, and then you retire. You live till 80. Your annual earnings between 21 – 60 are $100,000 and interest rate is \( r = 5\% \). You want to maintain a constant level of consumption. Write down an equation that allows to determine \( C \) (write down the equation but you do not need to solve for \( C \)).

**Problem 4 (20p).** (Producers)

Consider a producer that has the following technology
\[ y = K^{\frac{3}{4}}L^{\frac{1}{4}}. \]

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with \( \lambda \) argument).

b) Find analytically a (variable) cost function given \( w_K = w_L = 2 \). Plot it in the graph.

c) Find \( y^{MES} \) and \( ATC^{MES} \) if a fixed cost is \( F = 2 \).

d) Find analytically a supply function of the firm and show it in the graph.

**Just for fun**

Using "secrets of happiness" show that if a firm is maximizing profit by producing \( y^* \), it necessarily minimizes the cost of production of \( y^* \) (give two conditions for profit maximization and show that they imply condition for cost minimization).
Problem 1 (25p). (Uncertainty and insurance)

You are an owner of a luxurious sailing boat, worth $4, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to \( \frac{1}{2} \)) that completely destroys it. Thus, your boat is in fact a lottery with payment (0, 4).

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by \( u(c) = c^2 \). Give von Neuman-Morgenstern utility function over lotteries \( U(C_1; C_2) \). (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to \( u(c) = \ln c \). Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage \( x \). The insurance contract costs \( \gamma \cdot x \) where the premium rate is equal to \( \gamma = \frac{1}{2} \). Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage \( x \). Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate \( \gamma \) for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)

Consider an economy with apples and oranges. Andy is initially endowed with \( \omega^A = (20, 0) \) and Bob’s endowment is \( \omega^B = (0, 20) \).

The utility function of both Andy and Bob is the same and given by

\[
U(x_1, x_2) = 5 \ln x_1 + 5 \ln x_2
\]

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ... ).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies \( MRS_A = MRS_B \). Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

f) Using \( MRS \) condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)

a) Your sister has just promised to send you pocket money of $100 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).
b) Sam is a hockey player who earns $200 when young and $0 when old. Sam’s intertemporal utility is given by $U(C_1, C_2) = \ln (c_1) + \frac{1}{1+r} \ln (c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers $C_1, C_2, S$). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2K^{3}L^{\frac{1}{2}}$. Find analytically a short-run demand for labor (assume $K = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a graph.

d) You start your first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between 21 – 60 are $50,000 and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine $C$ (write down the equation but you do not need to solve for $C$).

**Problem 4 (20p).** (Producers)

Consider a producer that has the following technology

$$y = K^{\frac{3}{4}}L^{\frac{1}{4}}.$$ 

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with $\lambda$ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) find $y^{MES}$ and $ATC^{MES}$ if a fixed cost is $F = 2$.

d) Find analytically a supply function of the firm and show it in the graph.

**Just for fun**

Using "secrets of happiness" show that if a firm is maximizing profit by producing $y^*$, it necessarily minimizes the cost of production of $y^*$ (give two conditions for profit maximization and show that they imply condition for cost minimization).
Problem 1 (25p). (Uncertainty and insurance)
You are an owner of a luxurious sailing boat, worth $6, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment $(6, 0)$.

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1, C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage $x$. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage $x$. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate $\gamma$ for which you will only partially insure your boat. (one number)

Problem 2 (30p). (Edgeworth box, and equilibrium)
Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (40, 0)$ and Bob’s endowment is $\omega^B = (0, 40)$.
The utility function of both Andy and Bob is the same and given by $U(x_1, x_2) = 2 \ln x_1 + 2 \ln x_2$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ... ).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$.

Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

f) Using $MRS$ condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

Problem 3 (25p). (Short questions)
a) Your sister has just promised to send you pocket money of $50 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).
b) Sam is a hockey player who earns $1000 when young and $0 when old. Sam’s intertemporal utility is given by \( U(C_1, C_2) = \ln(c_1) + \frac{1}{1+r} \ln(c_1) \). Assuming \( \delta = r = 0 \) and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers \( C_1, C_2, S \)). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by \( y = 2K^{3}L^{\frac{1}{2}} \). Find analytically a short-run demand for labor (assume \( K = 1 \)). Find analytically equilibrium real wage rate if labor supply is given by \( L^s = 16 \). Depict it in a graph.

d) You start your first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between 21 – 60 are $40,000 and interest rate is \( r = 5\% \). You want to maintain a constant level of consumption. Write down an equation that allows to determine \( C \) (write down the equation but you do not need to solve for \( C \)).

**Problem 4 (20p). (Producers)**

Consider a producer that has the following technology

\[ y = K^{\frac{3}{4}}L^{\frac{1}{4}}. \]

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with \( \lambda \) argument).

b) Find analytically a (variable) cost function given \( w_K = w_L = 2 \). Plot it in the graph.

c) Find \( y^{MES} \) and \( ATC^{MES} \) if a fixed cost is \( F = 2 \).

d) Find analytically a supply function of the firm and show it in the graph.

**Just for fun**

Using "secrets of happiness" show that if a firm is maximizing profit by producing \( y^* \), it necessarily minimizes the cost of production of \( y^* \) (give two conditions for profit maximization and show that they imply condition for cost minimization).
Econ 301
Intermediate Microeconomics
Prof. Marek Weretka

Midterm 2 (Group D)

You have 70 minutes to complete the exam. The midterm consists of 4 questions (25,30,25 and 20 points).

**Problem 1 (25p). (Uncertainty and insurance)**
You are an owner of a luxurious sailing boat, worth $2, that you use for recreation on Mendota lake. Unfortunately, there is a good (50%) chance of a tornado in Madison (probability is equal to $\frac{1}{2}$) that completely destroys it. Thus, your boat is in fact a lottery with payment (2, 0).

a) What is the expected value of the "boat" lottery? (give one number)

b) Suppose your Bernoulli utility function is given by $u(c) = c^2$. Give von Neuman-Morgenstern utility function over lotteries $U(C_1; C_2)$. (formula) Are you risk averse, neutral or risk loving? (two words). Find the certainty equivalent (CE) of the "boat lottery" (one number). Which is bigger, CE or the expected value of a lottery from a)? Why? (one sentence)

c) Your Bernoulli utility function changes to $u(c) = \ln c$. Give von Neuman-Morgenstern utility function. (give a formula). Are you risk averse now?

d) You can insure your boat by buying insurance policy in which you specify coverage $x$. The insurance contract costs $\gamma \cdot x$ where the premium rate is equal to $\gamma = \frac{1}{2}$. Find analytically and depict in the graph your budget constraint. Mark the point that corresponds to no insurance.

e) Find optimal level of coverage $x$. Are you going to fully insure your boat? (one number and yes-no answer). Depict optimal consumption plan on the graph.

f) Propose a premium rate $\gamma$ for which you will only partially insure your boat. (one number)

**Problem 2 (30p). (Edgeworth box, and equilibrium)**
Consider an economy with apples and oranges. Andy is initially endowed with $\omega^A = (10, 0)$ and Bob’s endowment is $\omega^B = (0, 10)$.

The utility function of both Andy and Bob is the same and given by

$$U(x_1, x_2) = 8 \ln x_1 + 8 \ln x_2$$

a) Plot the Edgeworth box and mark the allocation representing the initial endowment.

b) Provide general definition of Pareto efficiency (one sentence starting with: Allocation is Pareto efficient if ... ).

c) Prove, that an allocation is Pareto efficient if and only in such allocation satisfies $MRS^A = MRS^B$. Start with necessity by showing that if the MRS condition does not hold then allocation is not Pareto efficient. Then proceed to sufficiency by showing that if the condition MRS is satisfied then indeed allocation is efficient (use a graph and write two sentences for each of the two conditions).

d) Find analytically a collection of all Pareto efficient allocations (contract curve) and depict it in the graph.

e) Find the competitive equilibrium (give six numbers).

g) Give some other prices that are consistent with competitive equilibrium (give two numbers).

f) Using $MRS$ condition verify that equilibrium allocation is Pareto efficient and hence an invisible hand of a free (and competitive) market guides selfish Andy and Bob to a socially optimal outcome.

**Problem 3 (25p). (Short questions)**
a) Your sister has just promised to send you pocket money of $200 each month starting next month and she will keep doing it forever. What is the present value of "having such sister" if monthly interest rate is equal to 5% (one number).
b) Sam is a hockey player who earns $1000 when young and $0 when old. Sam’s intertemporal utility is given by $U(C_1, C_2) = \ln (c_1) + \frac{1}{1+r} \ln (c_1)$. Assuming $\delta = r = 0$ and using magic formulas find optimal consumption plan and optimal saving strategy (give three numbers $C_1, C_2, S$). Does Sam smooth his consumption? (yes/ no + one sentence) Is Sam tilting his consumption? (yes/ no + one sentence)

c) A production function is given by $y = 2K L^{\frac{3}{4}}$. Find analytically a short-run demand for labor (assume $K = 1$). Find analytically equilibrium real wage rate if labor supply is given by $L^s = 16$. Depict it in a graph.

d) You start you first job at the age of 21 and you work till 60, and then your retire. You live till 80. Your annual earnings between 21 – 60 are $60,000 and interest rate is $r = 5\%$. You want to maintain a constant level of consumption. Write down an equation that allows to determine $C$ (write down the equation but you do not need to solve for $C$).

Problem 4 (20p). (Producers)
Consider a producer that has the following technology

$$y = K^{\frac{1}{4}} L^{\frac{3}{4}}.$$ 

a) What returns to scale are represented by this production function? (choose between CRS, IRS or DRS; prove your statement with $\lambda$ argument).

b) Find analytically a (variable) cost function given $w_K = w_L = 2$. Plot it in the graph.

c) Find $y^{MES}$ and $ATC^{MES}$ if a fixed cost is $F = 2$.

d) Find analytically a supply function of the firm and show it in the graph.

Just for fun
Using "secrets of happiness" show that if a firm is maximizing profit by producing $y^*$, it necessarily minimizes the cost of production of $y^*$ (give two conditions for profit maximization and show that they imply condition for cost minimization).
Econ 703  
Intermediate Microeconomics  
Prof. Marek Weretka

Answer Keys to midterm 2 (Group A)

“X and Y (2pt):” means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.]
   a) $0.5 \cdot 10 + 0.5 \cdot 0 = $5(2pt).
   b) With the Bernoulli utility function $u(c) = c^2$, the v.N.M. expected utility function is $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Since $u(c) = c^2$ is a convex function, I am risk loving (2pt). The certainty equivalent $CE$ is the amount of sure money s.t. $U(CE, CE) = CE^2 = U(0, 10) = 50$, i.e. $CE = 5\sqrt{2}$ (2pt). $CE$ is larger than EV, because I am risk loving (2pt).
   c) With the Bernoulli utility function $u(c) = c^2$, the v.N.M. expected utility function is $U(C_T, C_N) = 0.5 \ln C_T + 0.5 \ln C_N$ (1pt). Yes, I’m risk averse (2pt), since $u(c) = \ln c$ is a concave function.
   d) As $C_T = (1 - \gamma)x$ and $C_N = 4 - \gamma x$ with $\gamma = .5$, we obtain the budget constraint $C_T + C_N = 10$ (2pt). Its graph has the $C_T$ intercept on $(C_T, C_N) = (10, 0)$, the $C_N$ intercept on $(C_T, C_N) = (0, 10)$, and the slope -1 on the $C_T$-$C_N$ plane (2pt). The endowment point should be plotted on $(C_T, C_N) = (0, 10)$ (1pt).
   e) Now I should maximize the utility $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ on the constraint $C_T + C_N = 10$. The magic formula yields $C_T = (1/2) \cdot (10/1) = 5$ (1pt) and $C_N = (1/2) \cdot (10/1) = 5$ (1pt). Plugging this into $C_N = 4 - \gamma x$, we obtain $x = 10$ (2pt). The optimal point should be plotted on $(5, 5)$ (1pt). Yes, I am fully insured (1pt) since $C_T = C_N$.
   f) e.g. $\gamma = 1$ (2pt). Actually I would be partially insured, i.e. $C_T < C_N$ under any premium rate larger than 0.5.

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.]
   a) The Edgeworth box should have length of 50 on each axis (1pt). The endowment is $(50, 0)$ looked from A’s origin, i.e. $(0, 50)$ from B’s origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.]
   b) ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [MRSA = MRSB: no point since it is just a mathematical equivalent property and not the definition.]  
   c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve’s name, namely “an indifference curve”, should be clarified.]

Necessity (4pt): If $MRSA \neq MRSB$ at an allocation $x$, both people’s indifference curves should cross each other at $x$ and thus we can find a point between them. Because this point is above each indifference curve looked from the people’s origin, this allocation is better than $x$ for both and thus the allocation $x$ is not Pareto efficient. [The proof should start with $MRSA \neq MRSB$ and end with Pareto inefficiency of $x$. Graph is needed. On the graph, you need to specify another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRSA = MRSB$ at an allocation $x$, both people’s indifference curves should be tangent to each other at $x$ and thus no point is below A’s indifferent curve looked from A’s origin, i.e. worse for A than x, or below B’s indifferent curve looked from B’s origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than $x$ for both people and $x$ is Pareto efficient. [The proof should start with $MRSA = MRSB$ at $x$ and end with Pareto efficiency of $x$. Graph is needed. On the graph, you need to clarify who is worse off than $x$ in each region defined by the two indifference curves.]

   d) As we proved above, the Pareto efficiency is equivalent to $MRSA = MRSB$, given the feasibility of the allocation $x_1^A + x_1^B = 50, x_2^A + x_2^B = 50$. So we solve

$$MRSA(x_1^A, x_2^A) = \frac{3/x_1^A}{3/x_2^A} = \frac{3/(50 - x_1^A)}{3/(50 - x_2^A)} = MRSB(50 - x_1^A, 50 - x_2^A).$$

1A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.
Then we obtain \( x_1^A = x_2^A \) or \( x_1^B = x_2^B \) (3pt). This is the equation for the contract curve. [You need to clarify whose consumption it is.] Graphically it is the line starting from the origin of \( A \) with slope 1, i.e. the diagonal line connecting the two origins of the Edgeworth box (1pt).

e) Let the equilibrium price be \((p_1, p_2)\). Then, Andy should maximize his utility \( U^A(x_1^A, x_2^A) = 3 \ln x_1^A + 3 \ln x_2^A \) on the budget constraint \( p_1 x_1^A + p_2 x_2^A = 50 p_1 \). The magic formula yields his optimal consumption bundle

\[
    x_1^A = \frac{150 p_1}{2} p_1 = 25, \quad x_2^A = \frac{150 p_1}{2} p_2 = 25 \frac{p_1}{p_2}.
\]

Bob should maximize his utility \( U^B(x_1^B, x_2^B) = 3 \ln x_1^B + 3 \ln x_2^B \) on the budget constraint \( p_1 x_1^B + p_2 x_2^B = 50 p_2 \). The magic formula yields his optimal consumption bundle

\[
    x_1^B = \frac{150 p_2}{2} p_1 = 25 \frac{p_2}{p_1}, \quad x_2^B = \frac{150 p_2}{2} p_2 = 25.
\]

The feasibility (a.k.a. market clearing) of the allocation requires

\[
    x_1^A + x_1^B = 25 + 25 \frac{p_2}{p_1} = 50, \quad . \quad p_2 = p_1 \neq 0.
\]

Plugging this into the above optimal bundles, we obtain \( x_1^A = 25 \) (2pt), \( x_2^A = 25 \) (2pt), \( x_1^B = 25 \) (2pt) and \( x_2^B = 25 \) (2pt). The equilibrium price \((p_1, p_2)\) can be any pair of two positive numbers as long as \( p_1 = p_2 \); for example, \( p_1 = 1, p_2 = 1 \) (2pt). [No partial credit for only \( p_1 \) or \( p_2 \).]

f) As we argued, \( p_1, p_2 \) can be any pair of two positive numbers as long as \( p_1 = p_2 \) and different from the answer in e): for example, \( p_1 = 2, p_2 = 2 \) (2pt).

g) At the equilibrium allocation \( ((x_1^A, x_2^A), (x_1^B, x_2^B)) = ((25, 25), (25, 25)) \), the two’s MRSs are

\[
\]

So we have \( MRS^A = -1 = MRS^B \) and thus this equilibrium allocation is Pareto efficient (2pt). [MRS must be calculated.]

Problem 3.  

a) \( PV = 100/(1.05) + 100/(1.05)^2 + \ldots = 10000 \) (dollars, 4pt).

b) Sam should maximize his utility \( U = \ln C_1 + \ln C_2 \) on the budget constraint \( C_1 + C_2 = 200 \) (as \( C_1 + S = 200, C_2 = S \)). The magic formula yields his optimal consumption bundle \( C_1 = (1/2) \cdot (200/1) = 50 \) (2pt), \( C_2 = (1/2) \cdot (200/1) = 50 \) (2pt). Plugging this into \( C_2 = S \), we have \( S = 50 \) (2pt). Yes, he’s smoothing (1pt) as \( C_1 = C_2 \). No, he’s not tilting (1pt) as \( C_1 = C_2 \). [If you answered only either one question and did not clarify which question you answered, you get no point.]

c) The production function \( y = 2K^3L^{1/2} \) implies the marginal productivity of labor \( MP_L = (1/2) \cdot 2K^3 L^{-1/2} = K^3 L^{-1/2} \). In particular, \( MP_L = L^{-1/2} \) at \( K = K^* = 1 \). Solving the secret of happiness \( MP_L = L^{-1/2} = w/p \), we find the short-run labor demand \( L^D = (w/p)^{-2} \) where \( p \) is the product’s price and \( w \) is wage (4pt). [Thus \( w/p \) is the real wage rate. It is not enough to state only the secret of happiness; the demand \( L^D \) should be explicitly determined.] Solving the demand-supply equality \( L^D = (w/p)^{-2} = 16 = L^S \), we obtain the equilibrium real wage \( w/p = 1/4 \) (2pt). The equilibrium point \((L, w/p) = (16, 1/4)\) must be plotted on a graph (1pt).

d) (6pt.) The annual consumption \( C \) (thousand dollars) is determined from

\[
    100 \frac{100}{1.05} + \cdots + 100 \frac{100}{1.05^{40}} = C \frac{1.05}{1.05} + \cdots + C \frac{1.05}{1.05^{60}} \quad \therefore \quad \left( 1 - \frac{1}{1.05^{40}} \right) \frac{100}{1.05} = \left( 1 - \frac{1}{1.05^{60}} \right) \frac{C}{1.05}.
\]

[Further simplification gets full points.]

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[2] We do not have to consider the market clearing of the other good 2: Walras’s theorem. Notice that if \( p_1 = 0 \) then \( p_2/p_1 = \infty \) and the equation does not hold; so we need \( p_1 \neq 0 \) too.

[4] Also I saw so many answers \( “L^D = L^{-1/2}; this does not make sense at all, as it is read as the short-run labor demand \( L^D \) is the inverse of the square root of \( L \) and we must ask what is \( L \). \( L = L^D \) is the solution of \( MP_L = L^{-1/2} = w/p \), but not a number on either side of this equation.

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2
Problem 4. a) DRS (1pt). This is because \( F(\lambda K, \lambda L) = (\lambda^{1/4} K^{1/4})(\lambda^{1/4} L^{1/4}) = \lambda^{1/2} K^{1/4} L^{1/4} = \lambda^{1/2} F(K, L) \) if \( \lambda > 1 \) (4pt). [Here \( F(k, l) \) is the output from \( K = k \) and \( L = l \)].

b) The secret of happiness is

\[
\frac{MP_K}{MP_L} = \frac{0.25K^{-3/4}L^{1/4}}{0.25K^{1/4}L^{-3/4}} = \frac{2}{2} = \frac{w_K}{w_L} \quad \therefore K = L
\]

To achieve the production of \( y = F(K, L) \), we need

\[
y = F(K, K) = K^{1/2}, \quad \therefore K = L = y^2
\]

So the cost function is \( C = 2K + 2L = 2y^2 + 2y^2 = 4y^2 \) (4pt).\(^4\) Graph should be drawn on the \( y-C \) plane (1pt).

c) Solving \( MC(y) = 8y = (4y^2 + 2)/y = ATC(y) \), we obtain \( y^{MES} = 1/\sqrt{2} \) (2pt) and \( ATC^{MES} = ATC(y^{MES}) = MC(y^{MES}) = 4\sqrt{2} \) (2pt).\(^5\)

d) (6pt for giving both the function and the graph.) The optimal supply should satisfy \( p = 8y^* = MC(y^*) \), i.e. \( y^* = p/8 \). But when \( p < ATC^{MES} = 4\sqrt{2} \), the firm cannot get positive profit even from the optimal supply and thus should quit the production.

The supply function \( S(p) \) is therefore

\[
S(p) = \begin{cases} 
p/8 & \text{if } p \geq 4\sqrt{2} \\
0 & \text{if } p \leq 4\sqrt{2}.
\end{cases}
\]

On the \( y-p \) plane, the graph is \( y = p/8 \) (i.e. \( p = 8y \)) for \( p \geq 4\sqrt{2} \) and \( y = 0 \) (a part of the vertical axis) for \( p \leq 4\sqrt{2} \).

Just for fun The secret of happiness for profit maximization is

\[
MP_K = pw_K, \quad MP_L = pw_L.
\]

Here \( p \) is the product price, \( MP_i \) is the marginal productivity of factor \( i \), and \( w_i \) is the price of factor \( i \). These two equations imply

\[
\frac{MP_K}{MP_L} = \frac{w_K}{w_L};
\]

i.e. the secret of happiness for cost minimization.\(^6\)

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\(^4\)Or, you can think of maximization of \( Y = F(K, L) = K^{1/4} L^{1/4} \) on the constraint \( 2K + 2L = c \), thinking \( Y \) as a variable and \( c \) as a constant. Then the magic formula of Cobb-Douglas (utility) maximization implies \( K = (1/2)(c/2) = c/4 \) and \( L = (1/2)(c/2) = c/4 \). Then we obtain at the maximum \( Y = (c/4)^{1/4}(c/4)^{1/4} = (c/4)^{1/2} \), i.e. \( c = 4Y^2 \). That is, when \( Y = y \) is given, the budget/cost \( C = 4y^2 \) is needed to achieve this \( y \) at the optimum.

\(^5\)Maybe \( ATC^{MES} \) is easier to calculate from \( MC(y^{MES}) \) than from \( ATC(y^{MES}) \), though they should yield the same number.

\(^6\)So there’s a close link between maximization and minimization. This link is called duality and was a driving force of mathematical economic theory during 1970s-80s: see Varian’s textbook for graduate and advanced undergraduate, *Microeconomic Analysis*. And, you will use it in undergraduate linear programming, like Computer Science 525: see Ferris, Mangasarian, and Wright, *Linear Programming with MATLAB*, SIAM-MPS, 2007.
Solutions to midterm 2 (Group B)

“X and Y (2pt)” means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.]

a) $2 (2pt).$ b) $U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Risk loving (2pt). $CE = 2\sqrt{2}$ (2pt). Larger than EV, because I am risk loving (2pt). c) $U(C_T, C_N) = 0.5\ln C_T + 0.5\ln C_N$ (1pt). Yes, I’m risk averse (2pt). d) $C_T + C_N = 4$ (2pt). Graph is needed on the $C_T$-$C_N$ plane and its position must be clarified with slope and intercepts (2pt). Plot a point on $(C_T, C_N) = (0, 4)$ for endowment (1pt). e) $C_T = 2$ (1pt). $C_N = 2$ (1pt). $x = 4$ (2pt). Plot a point on $(2, 2)$ (1pt). Yes, fully insured (1pt). f) e.g. $\gamma = 1$ (2pt). [Any number larger than 0.5 because we need $C_T > C_N$.]

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.] a) The Edgeworth box should have length of 20 on each axis (1pt). The endowment is $(20, 0)$ looked from A’s origin, i.e. $(0, 20)$ from B’s origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.] b) ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). $[MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve’s name, namely “an indifference curve”, should be clarified.] Necessity (4pt): If $MRS^A \neq MRS^B$ at an allocation $x$, both people’s indifference curves should cross each other at $x$ and thus we can find a point between them. Because this point is above each indifference curve looked from the people’s origin, this allocation is better than $x$ for both and thus the allocation $x$ is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of $x$. Graph is needed. On the graph, you need to specify another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation $x$, both people’s indifference curves should be tangent to each other at $x$ and thus no point is below A’s indifferent curve looked from A’s origin, i.e. worse for A than $x$, or below B’s indifferent curve looked from B’s origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than $x$ for both people and $x$ is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at $x$ and end with Pareto efficiency of $x$. Graph is needed. On the graph, you need to clarify who is worse off than $x$ in each region defined by the two indifference curves.]

d) $x_1^A = x_2^A$ [or $x_1^B = x_2^B$] (3pt). [You need to clarify whose consumption it is.] The diagonal line connecting the two origins of the Edgeworth box (1pt). e) $x_1^A = 10$ (2pt). $x_2^A = 10$ (2pt). $x_1^B = 10$ (2pt). $x_2^B = 10$ (2pt). $p_1 = 1$, $p_2 = 1$ (2pt). [$p_1$, $p_2$ can be any pair of two positive numbers as long as $p_1 = p_2$. No partial credit for only $p_1$ or $p_2$.] f) $p_1 = 2$, $p_2 = 2$ (2pt). [$p_1$, $p_2$ can be any pair of two positive numbers as long as $p_1 = p_2$ and different from your answer in e.).] g) $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). $[MRS$ must be calculated.]

Problem 3. a) $2000$ (4pt). b) $C_1 = 100$ (2pt). $C_2 = 100$ (2pt). $S = 100$ (2pt). Yes, he’s smoothing (1pt). No, he’s not tilting (1pt). [You answered only either one question and did not clarify which question you answered, you get no point.] c) Demand: $L^D = (w/p)^{-2}$ where $p$ is the product’s price and $w$ is wage (4pt). [Thus $w/p$ is the real wage rate.] Equilibrium real wage: $w/p = 1/4$ (2pt). The point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt). d) (6pt.) The annual consumption $C$ (thousand dollars) is determined from $\{1 - (1.05)^{-40}\} : 50/1.05 = \{1 - (1.05)^{-60}\} C/1.05$. [Further simplification gets full points.]

Problem 4. a) DRS (1pt). This is because $F(t(K, tL) = t^{1/2}K^{1/4}L^{1/4} = t^{1/2}F(K, L) < tF(K, L)$ [if $t > 1$] (4pt). [Here $F(k, l)$ is the output from $K = k$ and $L = l$.] b) $C = 4y^2$ (4pt). Graph is needed on the $y-C$ plane (1pt). c) $y^{MRS} = 1/\sqrt{2}$ (2pt). $ATC^{MRS} = 4\sqrt{2}$ (2pt). d) (6pt for giving both the function and the graph.) The supply function $S(p)$ is $p/8$ for $p \geq 4\sqrt{2}$, and 0 for $p \leq 4\sqrt{2}$. On the $y-p$ plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$.

1A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.
Solutions to midterm 2 (Group C)

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.]

a) $\$3 \ (2pt) . \ b) \ U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2 \ (1pt) . \ Risk \ loving \ (2pt) . \ CE = 3\sqrt{2} \ (2pt) . \ Larger \ than \ EV, \ because \ I \ am \ risk \ loving \ (2pt) . \ c) \ U(C_T, C_N) = 0.5\ln C_T + 0.5\ln C_N \ (1pt) . \ Yes, \ I'm \ risk \ averse \ (2pt) . \ d) \ C_T + C_N = 6 \ (2pt) . \ Graph \ is \ needed \ on \ the \ C_T-C_N \ plane \ and \ its \ position \ must \ be \ clarified \ with \ slope \ and \ intercepts \ (2pt) . \ Plot \ a \ point \ on \ (C_T, C_N) = (0, 0) \ for \ endowment \ (1pt) . \ e) \ C_T = 3 \ (1pt) . \ C_N = 3 \ (1pt) . \ x = 6 \ (2pt) . \ Plot \ a \ point \ on \ (3, 3) \ (1pt) . \ Yes, \ fully \ insured \ (1pt) . \ f) \ e.g. \ \gamma = 1 \ (2pt) . \ [Any \ number \ larger \ than \ 0.5 \ because \ we \ need \ C_N > C_T]$

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.] a) The Edgeworth box should have length of 40 on each axis (1pt). The endowment is (40, 0) looked from A’s origin, i.e. (0, 40) from B’s origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.] 

b) ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). [MRS^A = MRS^B: no point since it is just a mathematical equivalent property and not the definition.]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve’s name, namely “an indifference curve”, should be clarified.] Necessity (4pt): If MRS^A ≠ MRS^B at an allocation x, both people’s indifference curves should cross each other at x and thus we can find a point between them. Because this point is above each indifference curve looked from the people’s origin, this allocation is better than x for both and thus the allocation x is not Pareto efficient. [The proof should start with MRS^A ≠ MRS^B and end with Pareto inefficiency of x. Graph is needed. On the graph, you need to specify another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If MRS^A = MRS^B at an allocation x, both people’s indifference curves should be tangent to each other at x and thus no point is below A’s indifferent curve looked from A’s origin, i.e. worse for A than x, or below B’s indifferent curve looked from B’s origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than x for both people and x is Pareto efficient. [The proof should start with MRS^A = MRS^B at x and end with Pareto efficiency of x. Graph is needed. On the graph, you need to clarify who is worse off than x in each region defined by the two indifference curves.]

d) $x_1^A = x_2^A \ [or \ x_1^B = x_2^B] \ (3pt) . \ [You \ need \ to \ clarify \ whose \ consumption \ it \ is.] \ The \ diagonal \ line \ connecting \ the \ two \ origins \ of \ the \ Edgeworth \ box \ (1pt) . \ e) \ x_1^A = 20 \ (2pt) . \ x_2^A = 20 \ (2pt) . \ x_1^B = 20 \ (2pt) . \ x_2^B = 20 \ (2pt) . \ p_1 = 1, p_2 = 1 \ (2pt) . \ [p_1, p_2 \ can \ be \ any \ pair \ of \ two \ positive \ numbers \ as \ long \ as \ p_1 = p_2. \ No \ partial \ credit \ for \ only \ p_1 \ or \ p_2.] \ f) \ p_1 = 2, p_2 = 2 \ (2pt) . \ [p_1, p_2 \ can \ be \ any \ pair \ of \ two \ positive \ numbers \ as \ long \ as \ p_1 = p_2 \ and \ different \ from \ your \ answer \ in \ e.] \ g) \ MRS^A = -1 = MRS^B \ and \ thus \ this \ equilibrium \ allocation \ is \ Pareto \ efficient \ (2pt) . \ [MRS \ must \ be \ calculated].$

Problem 3. a) $\$1000 \ (4pt) . \ b) \ C_1 = 500 \ (2pt) . \ C_2 = 500 \ (2pt) . \ S = 500 \ (2pt) . \ Yes, \ he’s \ smoothing \ (1pt) . \ No, \ he’s \ not \ tilting \ (1pt) . \ [If \ you \ answered \ only \ either \ one \ question \ and \ did \ not \ clarify \ which \ question \ you \ answered, \ you \ get \ no \ point.] \ c) \ Demand: \ L^D = (w/p)^{-2} \ where \ p \ is \ the \ product’s \ price \ and \ w \ is \ wage \ (4pt) . \ [Thus \ w/p \ is \ the \ real \ wage \ rate.] \ Equilibrium \ real \ wage: \ w/p = 1/4 \ (2pt) . \ The \ point \ (L, w/p) = (16, 1/4) \ must \ be \ plotted \ on \ a \ graph \ (1pt) . \ d) \ (6pt) . \ The \ annual \ consumption \ C \ (thousand \ dollars) \ is \ determined \ from \ \{1 - (1.05)^{-40}\} \cdot 40/1.05 = \{1 - (1.05)^{-60}\} \cdot C/1.05. \ [Further \ simplification \ gets \ full \ points].$

Problem 4. a) DRS \ (1pt) . \ This \ is \ because \ F(tK, tL) = t^{1/2}K^{1/4}L^{1/4} = t^{1/2}F(K, L) < tF(K, L) \ [if \ t > 1] \ (4pt) . \ [Here \ F(k, l) \ is \ the \ output \ from \ K = k \ and \ L = l.] \ b) \ C = 4y^2 \ (4pt) . \ Graph \ is \ needed \ on \ the \ y-C \ plane \ (1pt) . \ c) \ y^{MES} = 1/\sqrt{2} \ (2pt) . \ ATC^{MES} = 4\sqrt{2} \ (2pt) . \ d) \ (6pt \ for \ giving \ both \ the \ function \ and \ the \ graph) \ Supply \ function \ S(p) \ is \ p/8 \ for \ p \geq 4\sqrt{2} \ and \ 0 \ for \ p \leq 4\sqrt{2} \ \text{on} \ \text{the} \ y-p \ \text{plane, \ the \ graph \ is} \ y = p/8 \ \text{(i.e.} \ p = 8y) \ \text{for} \ p \geq 4\sqrt{2} \ \text{and} \ y = 0 \ (a \ \text{part \ of \ the \ vertical \ axis) \ for} \ p \leq 4\sqrt{2}.$

1A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.
“X and Y (2pt).” means that you get 2 pts if you answered both X and Y, and no pts if you missed either (or both).

Problem 1. [Here I denote by T the state with tornado and by N the state without tornado, instead of 1 and 2.] 

a) $1 (2pt). b) U(C_T, C_N) = 0.5C_T^2 + 0.5C_N^2$ (1pt). Risk loving (2pt). $CE = \sqrt{2}$ (2pt). Larger than EV, because I am risk loving (2pt). c) $U(C_T, C_N) = 0.5\ln C_T + 0.5\ln C_N$ (1pt). Yes, I’m risk averse (2pt). d) $C_T + C_N = 2$ (2pt). Graph is needed on the $C_T$-$C_N$ plane and its position must be clarified with slope and intercepts (2pt). Plot a point on $(C_T, C_N) = (0, 2)$ for endowment (1pt). e) $C_T = 1$ (1pt). $C_N = 1$ (1pt). $x = 2$ (2pt). Plot a point on $(1, 1)$ (1pt). Yes, fully insured (1pt). f) e.g. $\gamma = 1$ (2pt). [Any number larger than 0.5 because we need $C_N > C_T$.]

Problem 2. [Here I denote apple by 1, orange by 2, Andy by A and Bob by B. You could use another notation, as long as you clarified it.] a) The Edgeworth box should have length of 10 on each axis (1pt). The endowment is $(10, 0)$ looked from A’s origin, i.e. $(0, 10)$ from B’s origin (1pt). [This is a single point in an Edgeworth box. And, it cannot be an origin.] b) ... if none could not be better off (by another feasible allocation) unless anyone is worth off (2pt). $[MRS^A = MRS^B$: no point since it is just a mathematical equivalent property and not the definition.]

c) [Need to prove the claim directly from the definition of Pareto efficiency. Arguing only slopes or tangency is wrong. Besides, logical sequence (especially starting assumptions and ending conclusions) must be clarified. The curve’s name, namely “an indifference curve”, should be clarified.] Necessity (4pt): If $MRS^A \neq MRS^B$ at an allocation $x$, both people’s indifference curves should cross each other at $x$ and thus we can find a point between them. Because this point is above each indifference curve looked from the people’s origin, this allocation is better than $x$ for both and thus the allocation $x$ is not Pareto efficient. [The proof should start with $MRS^A \neq MRS^B$ and end with Pareto inefficiency of $x$. Graph is needed. On the graph, you need to specify another allocation that improves their utilities. If you wrote two separate points and two separate curves for a single allocation, you misunderstand an Edgeworth box.]

Sufficiency (4pt): If $MRS^A = MRS^B$ at an allocation $x$, both people’s indifference curves should be tangent to each other at $x$ and thus no point is below A’s indifferent curve looked from A’s origin, i.e. worse for A than $x$, or below B’s indifferent curve looked from B’s origin, i.e. worse for B, or below both. So any point (allocation) cannot be better than $x$ for both people and $x$ is Pareto efficient. [The proof should start with $MRS^A = MRS^B$ at $x$ and end with Pareto efficiency of $x$. Graph is needed. On the graph, you need to clarify who is worse off than $x$ in each region defined by the two indifference curves.]

d) $x_A^1 = x_A^2$ [or $x_B^1 = x_B^2$] (3pt). [You need to clarify whose consumption it is.] The diagonal line connecting the two origins of the Edgeworth box (1pt). e) $x_A^1 = 5$ (2pt). $x_A^2 = 5$ (2pt). $x_B^1 = 5$ (2pt). $x_B^2 = 5$ (2pt). $p_1 = 1, p_2 = 1$ (2pt). $[p_1, p_2]$ can be any pair of two positive numbers as long as $p_1 = p_2$. No partial credit for $p_1 = 2, p_2 = 2$ (2pt). $[p_1, p_2]$ can be any pair of two positive numbers as long as $p_1 = p_2$ and different from your answer in e.) f) $p_1 = 2, p_2 = 2$ (2pt). $[p_1, p_2]$ can be any pair of two positive numbers as long as $p_1 = p_2$ and different from your answer in e.) g) $MRS^A = -1 = MRS^B$ and thus this equilibrium allocation is Pareto efficient (2pt). $[MRS$ must be calculated.]

Problem 3. a) $4000 (4pt). b) C_1 = 500 (2pt). C_2 = 500 (2pt). S = 500 (2pt). Yes, he’s smoothing (1pt). No, he’s not tilting (1pt). [If you answered which question you answered, you get no point.] e) Demand: $L Đi (w/p)^{−2}$ where $p$ is the product’s price and $w$ is wage (4pt). [Thus $w/p$ is the real wage rate.] Equilibrium real wage: $w/p = 1/4$ (2pt). The point $(L, w/p) = (16, 1/4)$ must be plotted on a graph (1pt). d) (6pt.) The annual consumption $C$ (thousand dollars) is determined from $\{1 - (0.15)^{−40}\} \cdot 60/1.05 = \{1 - (0.15)^{−60}\}$ C/1.05. [Further simplification gets full points.]

Problem 4. a) DRS (1pt). This is because $F(tK, tL) = t^{1/2}K^{1/4}L^{1/4} - t^{1/2}F(K, L) < tF(K, L)$ if $t > 1$ (4pt). [Here $F(k, l)$ is the output from $K = k$ and $L = l$.] b) $C = 4y^2$ (4pt). Graph is needed on the $y-C$ plane (1pt). c) $y^{MES} = 1/\sqrt{2}$ (2pt). $ATC^{MES} = 4\sqrt{2}$ (2pt). d) (6pt for both the function and the graph.) The supply function $S(p)$ is $p/8$ for $p \geq 4\sqrt{2}$, and 0 for $p \leq 4\sqrt{2}$. On the $y-p$ plane, the graph is $y = p/8$ (i.e. $p = 8y$) for $p \geq 4\sqrt{2}$ and $y = 0$ (a part of the vertical axis) for $p \leq 4\sqrt{2}$. 

1A Pareto efficient allocation may not hold this equation, unless the preference is convex, monotone, and smooth.