Households

- The preferences of a representative household defined over a composite consumption good $C_t$, real money balances $M_t/P_t$, and leisure $1 - N_t$, where $N_t$ is the time devoted to market employment.

- Households maximize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ C_{t+i}^{1-\sigma} \frac{1}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right].$$  \hspace{1cm} (1)

- The composite consumption good consists of differentiate products produced by monopolistically competitive final goods producers (firms). There are a continuum of such firms of measure 1, and firm $j$ produces good $c_j$. 

The composite consumption good that enters the household’s utility function is defined as

\[ C_t = \left[ \int_0^1 c_{jt}^{\theta-1} \, dj \right]^{\theta} \quad \theta > 1. \]  

(2)

The parameter \( \theta \) governs the price elasticity of demand for the individual goods.
- The household’s decision problem can be dealt with in two stages.

1. Regardless of the level of $C_t$, it will always be optimal to purchase the combination of the individual goods that minimize the cost of achieving this level of the composite good.

2. Given the cost of achieving any given level of $C_t$, the household chooses $C_t$, $N_t$, and $M_t$ optimally.
Dealing first with the problem of minimizing the cost of buying $C_t$, the household’s decision problem is to

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj$$

subject to

$$\left[ \int_0^1 \left( c_{jt} \right)^{\frac{1}{\theta - 1}} dj \right]^{\frac{\theta}{\theta - 1}} \geq C_t,$$  \hspace{1cm} (3)

where $p_{jt}$ is the price of good $j$. Letting $\psi_t$ be the Lagrange multiplier on the constraint, the first order condition for good $j$ is

$$p_{jt} - \psi_t \left[ \int_0^1 \left( c_{jt} \right)^{\frac{1}{\theta - 1}} dj \right]^{\frac{1}{\theta - 1}} c_{jt}^{\frac{1}{\theta - 1}} = 0.$$
Rearranging, \( c_{jt} = \left( \frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \). From the definition of the composite level of consumption (2), this implies

\[
C_t = \left[ \int_0^1 \left( \frac{p_{jt}}{\psi_t} \right)^{-\theta} C_t \right]^{\frac{\theta-1}{\theta}} \frac{\theta}{\theta-1} = \left( \frac{1}{\psi_t} \right)^{-\theta} \left[ \int_0^1 p_{jt}^{1-\theta} \, dj \right]^{\frac{\theta}{\theta-1}} C_t.
\]

Solving for \( \psi_t \),

\[
\psi_t = \left[ \int_0^1 p_{jt}^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}} \equiv P_t.
\]
The Lagrange multiplier is the appropriately aggregated price index for consumption.

The demand for good \( j \) can then be written as

\[
c_{j} = \left( \frac{p_{jt}}{P_{t}} \right)^{-\theta} C_{t}. \tag{5}
\]

The price elasticity of demand for good \( j \) is equal to \( \theta \). As \( \theta \to \infty \), the individual goods become closer and closer substitutes, and, as a consequence, individual firms will have less market power.
Given the definition of the aggregate price index in (4), the budget constraint of the household is, in real terms,

\[ C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left( \frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + R_{t-1} \left( \frac{B_{t-1}}{P_t} \right) + \Pi_t, \quad (6) \]

where \( M_t \) (\( B_t \)) is the household’s nominal holdings of money (one period bonds). Bonds pay a gross nominal rate of interest \( R_t \). Real profits received from firms are equal to \( \Pi_t \).

In the second stage of the household’s decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (1) subject to (6).
Households

The following conditions must also hold in equilibrium

1. the Euler condition for the optimal intertemporal allocation of consumption

\[ C_t^{-\sigma} = \beta R_t E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{-\sigma}; \]  

(7)

2. the condition for optimal money holdings:

\[ \gamma \left( \frac{M_t}{P_t} \right)^{-b} \frac{C_t^{-\sigma}}{C_t^{-\sigma}} = \frac{R_t - 1}{R_t}; \]  

(8)

3. the condition for optimal labor supply:

\[ \frac{\chi N_t^\eta}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \]  

(9)
Firms

Firms maximize profits, subject to three constraints:

1. The first is the production function summarizing the technology available for production. For simplicity, we have ignored capital, so output is a function solely of labor input $N_{jt}$ and an aggregate productivity disturbance $Z_t$:

   $$c_{jt} = Z_t N_{jt}, \quad \mathbb{E}(Z_t) = 1.$$

2. The second constraint on the firm is the demand curve each faces. This is given by equation (5).

3. The third constraint is that each period some firms are not able to adjust their price. The specific model of price stickiness we will use is due to Calvo (1983).
Each period, the firms that adjust their price are randomly selected: a fraction $1 - \omega$ of all firms adjust while the remaining $\omega$ fraction do not adjust.

- The parameter $\omega$ is a measure of the degree of nominal rigidity; a larger $\omega$ implies fewer firms adjust each period and the expected time between price changes is longer.

For those firms who do adjust their price at time $t$, they do so to maximize the expected discounted value of current and future profits.

- Profits at some future date $t + s$ are affected by the choice of price at time $t$ only if the firm has not received another opportunity to adjust between $t$ and $t + s$. The probability of this is $\omega^s$. 
First consider the firm’s cost minimization problem, which involves minimizing $W_t N_{jt}$ subject to producing $c_{jt} = Z_t N_{jt}$. This problem can be written as

$$\min_{N_{jt}} W_t N_{jt} + \varphi^n_t (c_{jt} - Z_t N_{jt}).$$

where $\varphi^n_t$ is equal to the firm’s nominal marginal cost. The first order condition implies

$$W_t = \varphi^n_t Z_t,$$

or $\varphi^n_t = W_t / Z_t$. Dividing by $P_t$ yields real marginal cost as $\varphi_t = W_t / (P_t Z_t)$. 

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The firm’s pricing decision problem then involves picking $p_{jt}$ to maximize

$$
E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \Pi \left( \frac{p_{jt}}{P_{t+i}}, \varphi_{t+i}, c_{t+i} \right) =
E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ \left( \frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left( \frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i},
$$

where the discount factor $\Delta_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\sigma}$
and profits are

$$
\Pi(p_{jt}) = \left[ \left( \frac{p_{jt}}{P_{t+i}} \right) c_{jt+i} - \varphi_{t+i} c_{jt+i} \right]
$$
Price adjustment

- All firms adjusting in period $t$ face the same problem, so all adjusting firms will set the same price.

- Let $p^*_t$ be the optimal price chosen by all firms adjusting at time $t$. The first order condition for the optimal choice of $p^*_t$ is

$$
E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[ (1 - \theta) \left( \frac{1}{p_{jt}} \right) \left( \frac{p^*_t}{P_{t+i}} \right)^{1-\theta} + \theta \varphi_{t+i} \left( \frac{1}{p^*_t} \right) \left( \frac{p^*_t}{P_{t+i}} \right)^{-1} \right].
$$

- Using the definition of $\Delta_{i,t+i}$,

$$
\left( \frac{p^*_t}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}. \quad (10)
$$
The case of flexible prices

If all firms are able to adjust their prices every period ($\omega = 0$):

\[
\left( \frac{p^*_t}{P_t} \right) = \left( \frac{\theta}{\theta - 1} \right) \varphi_t = \mu \varphi_t.
\]  \hspace{1cm} (11)

Each firm sets its price $p^*_t$ equal to a markup $\mu > 1$ over nominal marginal cost $P_t \varphi_t$.

When prices are flexible, all firms charge the same price, and $\varphi_t = \mu^{-1}$. 

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The case of sticky prices

- When prices are sticky ($\omega > 0$), the firm must take into account expected future marginal cost as well as current marginal cost when setting $p_t^*$.
- The aggregate price index is an average of the price charged by the fraction $1 - \omega$ of firms setting their price in period $t$ and the average of the remaining fraction $\omega$ of all firms who set prices in earlier periods.
- Because the adjusting firms were selected randomly from among all firms, the average price of the non-adjusters is just the average price of all firms that was prevailing in period $t - 1$.
- Thus, the average price in period $t$ satisfies

$$P_t^{1-\theta} = (1 - \omega)(p_t^*)^{1-\theta} + \omega P_t^{1-\theta}. \quad (12)$$
Using the first order condition for $p_t^*$ and approximating around a zero average inflation, flexible-price equilibrium,

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} \hat{\varphi}_t$$  \hspace{1cm} (13)

where $\hat{\varphi}_t$ is a measure of the real marginal cost $W_t/(P_t Z_t)$ and:

$$\tilde{\kappa} = \frac{(1 - \omega)(1 - \beta \omega)}{\omega}$$

Equation (13) is often referred to as the New Keynesian Phillips curve.
The New Keynesian Phillips curve is forward-looking; when a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods.

Solving forward,

\[ \pi_t = \tilde{\kappa} \sum_{i=0}^{\infty} \beta^i E_t \hat{\phi}_{t+i}, \]

Inflation is a function of the present discounted value of current and future real marginal cost.

Inflation depends on real marginal cost and not directly on a measure of the gap between actual output and some measure of potential output or on a measure of unemployment relative to the natural rate, as is typical in traditional Phillips curves.
Define the output gap as the deviation of output from what it would be with flexible prices.

First equation relates output gap to real interest rate:

$$x_t = -\phi(R_t - E_t \pi_{t+1}) + E_t x_{t+1} + g_t$$

Linearized consumption Euler equation/\textit{IS} curve.

Second equation is the New Keynesian Phillips curve relating inflation and real activity:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$$

Linearized pricing decisions of firms with sticky prices

Consistent with

- optimizing behavior by households and firms
- budget constraints
- market equilibrium

Two equations but three unknowns: $x_t$, $\pi_t$, and $i_t$ – need to specify monetary policy
The basic new Keynesian inflation adjustment equation without cost shocks is:

\[ \pi_t = \kappa x_t + \beta E_t \pi_{t+1} \]

We can solve this equation forward to get an expression for inflation:

\[ \pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} \]

Inflation is a function of the present discounted value of current and future output gaps.

The absence of a stochastic disturbance implies there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero.

Just set \( x_{t+i} = 0 \) for all \( i \); keeps inflation equal to zero.
Thus, the key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.

No policy conflicts.

When prices are sticky but wages are flexible, the nominal wage can adjust to ensure labor market equilibrium is maintained in the face of productivity shocks. Optimal policy should then aim to keep the price level stable.
Models that combine optimizing agents and sticky prices have very strong policy implications.

When the price level fluctuates, and not all firms are able to adjust, price dispersion results. This causes the relative prices of the different goods to vary. If the price level rises, for example, two things happen.

1. The relative price of firms who have not set their prices for a while falls. They experience an increase in demand and raise output, while firms who have just reset their prices reduce output. This production dispersion is inefficient.

2. Consumers increase their consumption of the goods whose relative price has fallen and reduce consumption of those goods whose relative price has risen. This dispersion in consumption reduces welfare.
The solution is to prevent price dispersion by stabilizing the price level.

What is critical for this result is that nominal wages are assumed to be completely flexible.

But the same argument would apply if wages are sticky and prices flexible. With sticky wages and flexible prices, monetary policy should stabilize the nominal wage.
Woodford versus Friedman

- The basic new Keynesian model suggests price stability (i.e., zero inflation) is optimal.
  - Zero inflation eliminates inefficient price dispersion.

- Friedman rule: zero nominal rate of interest is optimal.
  - Zero nominal rate eliminates inefficiency in money holdings.
  - Optimal inflation is negative (deflation) at rate equal to real rate of interest.

- Khan, King, and Wolman (2000) analyze model with both distortions. They conclude optimal inflation is closer to zero than to the Friedman rule.
Now assume

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t$$

where $u_t$ represents an inflation or cost shock, which is serially correlated:

$$u_t = \rho_u u_{t-1} + \epsilon_t^u$$

Then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i E_t u_{t+i}$$

Cannot keep both $x$ and $\pi$ equal to zero. Trade-offs must be made.
Objective of Policy

- Policy objective in general is to maximize welfare of agent:

\[ E_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right], \]

where

\[ \Omega = \frac{1}{2} \bar{Y} U_c \left[ \frac{\omega}{(1 - \omega)(1 - \omega \beta)} \right] \left( \theta^{-1} + \eta \right) \theta^2 \]

and

\[ \lambda = \left[ \frac{(1 - \omega)(1 - \omega \beta)}{\omega} \right] \frac{(\sigma + \eta)}{(1 + \eta \theta) \theta}. \]

- Greater nominal rigidity (larger \( \omega \)) reduces \( \lambda \).
- Calvo specification implies \( \lambda \) is small – Taylor specification leads to larger weight on output gap.
- If there are distortions in the economy (such as monopoly power), optimal level of output gap is positive.
Policy Problem

- Suppose central bank targets positive output gap $\bar{x} > 0$. Chooses interest rate policy each period to minimize loss, taking as given private expectations.
- Easiest here to suppose central bank directly controls inflation and output gap, then use IS to back out optimal interest rate choice.
- Represent the central bank’s problem as a Lagrangian:

$$\mathcal{L} = \frac{1}{2} \left( \lambda (x_t - \bar{x})^2 + \pi_t^2 \right) + \mu (\kappa x_t + \beta E_t \pi_{t+1} + u_t - \pi_t)$$

- The first order conditions are:

$$\lambda (x_t - \bar{x}) + \mu \kappa = 0 \text{ and } \pi_t = \mu$$

or

$$x_t = -\frac{\kappa}{\lambda} \pi_t + \bar{x}$$
\[ x_t = -\frac{\kappa}{\lambda} \pi_t + \bar{x} \]

- Substitute back into Phillips:

\[ \pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \]

\[ (1 + \kappa^2 / \lambda) \pi_t = \kappa \bar{x} + \beta E_t \pi_{t+1} + u_t \]

- Guess \( \pi_t = k_0 + k_1 u_t \). Then

\[ E_t \pi_{t+1} = k_0 + k_1 E_t u_{t+1} = k_0 + k_1 \rho_u u_{t-1}. \]

Substitute:

\[ \pi_t = \frac{\lambda}{\kappa^2 + \omega(1 - \beta \rho)} u_t + \frac{\lambda}{\kappa} \bar{x} \]

- Then from optimality get:

\[ x_t = -\frac{\kappa}{\lambda} \pi_t + \bar{x} = \frac{-\kappa}{\kappa^2 + \lambda(1 - \beta \rho)} u_t \]
Note $E\pi_t = \frac{1}{\kappa} \bar{x}$ but $E x_t = 0$. Target gap $\bar{x}$ only affects mean inflation rate.

- Government tries to push output above potential, in equilibrium only leads to higher inflation.
- This is just as in the earlier analysis, but more direct/explicit.
- Policymakers have an incentive to announce they will be tough on inflation to affect people’s expectations, then actually to pursue loose policy.
- In equilibrium, people will come to expect this. With rational expectations (as we’ve used), this only leads to higher inflation.
Optimal Discretionary Policy

- With $\bar{x} = 0$:
  \[
  \pi_t = \frac{\omega}{\kappa^2 + \omega(1 - \beta \rho)} u_t, \quad x_t = \frac{-\kappa}{\kappa^2 + \omega(1 - \beta \rho)} u_t
  \]

- Can then get optimal interest rate response from IS:
  \[
  x_t = -\phi(R_t - E_t \pi_{t+1}) + E_t x_{t+1} + g_t \\
  R_t = E_t \pi_{t+1} + (1/\phi)(E_t x_{t+1} - x_t + g_t) \\
  = \gamma E_t \pi_{t+1} + (1/\phi)g_t, \text{ where } \gamma > 1.
  \]

- (i) Cost push shocks $u_t$ imply inflation/output tradeoff.
- (ii) If expected inflation rises, nominal interest rates should rise by more ($\gamma > 1$) so real rates increase.
- (iii) Policy should offset demand shocks $g_t$, accommodate movements in potential output (say productivity shocks).
• When forward-looking expectations play a role, discretion leads to a stabilization bias even though there is no average inflation bias.

• Under optimal commitment, central bank at time $t$ chooses both current and expected future values of inflation and the output gap.

• Minimize

$$-\Omega E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (x_{t+i} - x^*)^2 \right]$$

subject to

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t.$$ 

• Set $x^* = 0$: Fiscal subsidy to offset distortion from monopolistic competition.
The central bank’s problem is to pick $\pi_{t+i}$ and $x_{t+i}$ to minimize

$$E_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda x_{t+i}^2 + \psi_{t+i} (\pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}) \right].$$

The first order conditions can be written as

$$\pi_t + \psi_t = 0 \quad (14)$$

$$E_t (\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}) = 0 \quad i \geq 1 \quad (15)$$

$$E_t (\lambda x_{t+i} - \kappa \psi_{t+i}) = 0 \quad i \geq 0. \quad (16)$$

Dynamic inconsistency – at time $t$, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = -(E_t \psi_{t+1} - \psi_t)$. When $t + 1$ arrives, a central bank that reoptimizes will again obtains $\pi_{t+1} = -\psi_{t+1} -$ the first order condition (14) updated to $t + 1$ will reappear.
An alternative definition of an optimal precommitment policy requires the central bank to implement conditions (15) and (16) for all periods, including the current period so that

\[ \pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} = 0 \quad i \geq 0 \]

\[ \lambda x_{t+i} - \kappa \psi_{t+i} = 0 \quad i \geq 0. \]

Woodford (1999) has labeled this the “timeless perspective” approach to precommitment.
Under the timeless perspective optimal commitment policy, inflation and the output gap satisfy

$$\pi_{t+i} = -\left(\frac{\lambda}{\kappa}\right)(x_{t+i} - x_{t+i-1})$$  

(17)

for all $i \geq 0$.

Woodford (1999) has stressed that, even if $\rho = 0$, so that there is no natural source of persistence in the model itself, $a > 0$ and the precommitment policy introduces inertia into the output gap and inflation processes.

This commitment to inertia implies that the central bank’s actions at date $t$ allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap.
Responses of to a 1% inflation shock under the optimal commitment (solid) and discretion (dashed) policies.
Improved trade-off under commitment

- The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion.
- Consider a positive inflation shock, \( e > 0 \).
- A given change in current inflation can be achieved with a smaller fall in \( x \) if expected future inflation can be reduced:

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t
\]

- Requires a commitment to future deflation.
- By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in \( E_t \pi_{t+1} \) at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.
Gains from Commitment

**FIGURE 2**

Output and Inflation Tradeoffs

- **standard deviation of output gap**
- **standard deviation of inflation**