Lectures 8-9: Fiscal Policy in the Growth Model and Optimal Taxation

Economics 714, Spring 2017

1 Fiscal Policy in the Growth Model

1.1 Lump-Sum Taxes

Level of taxes and spending (even if lump sum) matter because of wealth effects.

Assume $U(C_t, N_t) = U(C_t)$, so $N_t \equiv 1$, define $F(K, 1) = f(K)$.

Equilibrium conditions:

$$\begin{aligned} u'(C_t) &= \beta u'(C_{t+1})[r_{t+1} + 1 - \delta] \\ &= \beta u'(C_{t+1})[f'(K_{t+1}) + 1 - \delta] \\ K_{t+1} &= f(K_t) + 1 - \delta K_t - C_t - G_t \end{aligned}$$

Dynamics:

$$\Delta c = 0 \;\Rightarrow\; f'(K) = \rho + \delta$$

$$\Delta k = 0 \;\Rightarrow\; f(K) = C + \delta K + G$$

1.2 Distorting Taxes

Now consider linear tax τ_t^N on labor income, τ_t^K on capital income:

$$T_t = \tau_t^N w_t N_t + \tau_t^K (r_t - \delta) K_t$$

Government budget constraint as before with this definition of revenue.

Define after-tax gross return $R_t^K = 1 + (1 - \tau_t^K)(r_t - \delta)$.

Household budget constraint, using capital law of motion:

$$\sum_{t=0}^{\infty} q_t[C_t + K_{t+1}] = \sum_{t=0}^{\infty} q_t[(1 - \tau_t^N)w_t N_t + R_t^K K_t]$$

Firm problem unaffected: $w_t = F_N$, $r_t = F_K$.

Households: now allow elastic labor supply, so solve

$$\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t) \quad \text{s.t.} \quad \text{BC}$$

Equilibrium conditions:

$$\begin{aligned}
u_C(C_t, 1 - N_t) &= \beta u_C(C_{t+1}, 1 - N_{t+1})[1 + (1 - \tau_{t+1}^K)(F_K(K_{t+1}, N_{t+1}) - \delta)] \\
\frac{u_L(C_t, 1 - N_t)}{u_C(C_t, 1 - N_t)} &= (1 - \tau_t^N)F_N(K_t, N_t) \\
K_{t+1} &= F(K_t, N_t) + 1 - \delta K_t - C_t - G_t
\end{aligned}$$

2 Ramsey Optimal Taxation

2.1 Setup

Look for linear taxes that fund given $\{G_t\}$ and maximize household welfare

First, find implementability constraint summarizing equilibria. Rewrite HH BC:

$$\sum_{t=0}^{\infty} q_t[C_t - (1 - \tau_t^N)w_t N_t] = \sum_{t=0}^{\infty} q_t[R_t^K K_t - K_{t+1}] = q_0 R_0^K K_0$$

Then use HH first order conditions to substitute out for prices

$$\sum_{t=0}^{\infty} \beta^t [U_C(C_t, 1-N_t)C_t - U_L(C_t, 1-N_t)N_t] = U_C(C_0, 1-N_0)K_0[1+(1-\tau_0^K)(F_K(K_0, N_0)-\delta)]$$

Primal approach: solve for allocation first, back out supporting taxes from equilibrium conditions:

$$\begin{aligned}\left(\frac{u_C(C_t, 1-N_t)}{\beta u_C(C_{t+1}, 1-N_{t+1})} - 1\right)\frac{1}{F_K(K_{t+1}, N_{t+1}) - \delta} &= 1 - \tau_{t+1}^K \\ \frac{u_L(C_t, 1-N_t)}{u_C(C_t, 1-N_t)F_N(K_t, N_t)} &= 1 - \tau_t^N\end{aligned}$$

τ_0^K only affects period zero: initial capital is inelastic, tax it as much as possible. To make problem interesting, restrict $\tau_0^K \leq \overline{\tau}^K$.

Define $U(C, N) = U(C, 1-N)$ so $U_N = -U_L$. Also define:

$$W(C_t, N_t; \lambda) = U(C_t, N_t) + \lambda[U_C(C, N)C + U_N(C, 1-N)N]$$

Then Ramsey problem can be written:

$$\max_{\{C_t, K_{t+1}, N_t\}} \sum_{t=0}^{\infty} \beta^t W(C_t, N_t; \lambda) - \lambda U_C(C_0, N_0)K_0[1 + (1-\tau_0^K)(F_K(K_0, N_0) - \delta)]$$

subject to (multiplier μ_t):

$$C_t + G_t + K_{t+1} = F(K_t, N_t) + 1 - \delta K_t$$

2.2 Characterization

First order conditions for $t \geq 1$:

$$\begin{aligned}W_C(t) &= \mu_t \\ W_N(t) &= -\mu_t F_N(t) \\ \mu_t &= \beta\mu_{t+1}(F_K(t+1) + 1 - \delta)\end{aligned}$$

These imply modified Euler equation and intra-temporal optimality condition:

$$\begin{aligned} W_C(C_t, N_t) &= \beta W_C(C_{t+1}, N_{t+1})[1 + F_K(K_{t+1}, N_{t+1} - \delta)] \\ -\frac{W_N(C_t, N_t)}{W_C(C_t, N_t)} &= F_N(K_t, N_t) \end{aligned}$$

Implications:

- zero long-run capital tax: if $G_t \to \bar{G}$, allocation converges to steady state, then $\tau_t^K \to 0$.
- smoothing of tax rates: allocation smooths tax response to changes in G_t.

Example: $U(C, N) = \frac{C^{1-\gamma}}{1-\gamma} - \frac{N^{1+\theta}}{1+\theta}$

Then $W_C = U_C(1 + \lambda - \lambda\gamma)$, $W_N = U_N(1 + \lambda - \lambda\phi)$.

Implies $\tau_t^K = 0$ for $t \geq 2$, and τ_t^N constant for $t \geq 1$.