On growth and indeterminacy: some theory and evidence

A comment

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1 Introduction

Jess Benhabib and Jordi Gali have written a masterful survey of economic growth models with indeterminacy. In terms of economic theory, they have persuasively demonstrated that the presence of externalities and multiple equilibria in growth paths can represent a significant source of cross-country volatility in growth. In terms of empirical work, they have shown how indeterminacy in growth models will break any necessary cross-section correlations between initial conditions and subsequent growth patterns of the type associated with the Solow-Cass-Koopmans model. Further, they have developed some interesting and new empirical evidence on cross-country growth patterns which appears inconsistent with the neoclassical model.

This discussion will focus on Benhabib and Gali's analysis of the empirics of cross-country growth, with particular emphasis on the issue of interpreting cross-country growth regressions and tests of the convergence hypothesis in the context of neoclassical versus endogenous growth theory.

2 Regression tests of convergence

One way to interpret the substantive economic question underlying controversies in growth theory is the following. Suppose one wished to forecast long-run differences in log per capita output between countries $i$ and $j$, $Y_{i,T} - Y_{j,T}$...
at horizon $T$, given information on those economies at time 0, $F_{i,0}$ and $F_{j,0}$. Formally, this is equivalent to computing

$$\lim_{T \to \infty} \text{Prob}(Y_{i,T} - Y_{j,T} | F_{i,0} \cup F_{j,0}).$$  \hspace{1cm} (1)

Observe that $F_{i,0} \cup F_{j,0}$ can include all initial conditions and expectations, as well as any aspect of microeconomic parameterizations of individual economies that one wishes to consider.

Different growth theories suggest that different elements of $F_{i,0} \cup F_{j,0}$ are relevant in computing (1), in the sense that different values of these elements will change the limit in (1). Neoclassical growth theory, as defined by the models of Solow, Cass, and Koopmans, identifies differences in technology, preferences, and population growth rates in explaining long-run deviations across aggregate economies. On the other hand, the new growth theory as developed by Romer (1986), Azariadis and Drazen (1990), and many others would argue that due to nonconvexities in production, an economy’s initial stocks of human and physical capital matter as well in explaining long-run behavior. Note that the models studied by Jones and Manuelli (1990), which employ aggregate production functions which are convex, have a similar prediction.

The class of growth models exhibiting indeterminacy which Benhabib and Gali survey (and to whose development they have been important contributors) add an additional set of variables to this list. Expectations matter due to the presence of multiple equilibria in the growth paths of economies. The mechanism by which one of several self-fulfilling equilibria is selected thus becomes a critical element of $F_{i,0} \cup F_{j,0}$, from the perspective of identifying the determinants of the limit in equation (1).

Rather than directly attempt to discriminate between different growths on the basis of estimating equation (1) nonparametrically, the empirical growth literature has focused on the regression

$$g_{Y_{i,T}} = \alpha + \beta Y_{i,0} + \Pi X_i + \epsilon_i$$  \hspace{1cm} (2)

across $i$, where $g_{Y_{i,T}}$ measures per capita output growth in country $i$ between 0 and a fixed time $T$ and $X_i$ denotes a vector of country specific control variables such as population growth and human and physical capital savings rates. This cross-country regression is in fact the law of motion associated with either the Solow or the Cass-Koopmans model (at least to a local approximation) under suitable choice of the control variables. The empirical growth literature has focused on the sign of the coefficient $\beta$. When $\beta$ is negative, this implies that initial incomes are negatively correlated with subsequent growth rates, so that the poor are on average catching up to the rich, assuming equal $X_i$’s. This negative $\beta$ has been dubbed evidence of
"conditional convergence" and treated as evidence in favor of the neoclassical model.

One important problem in the empirical growth literature is the choice of the control variables $X_i$. When constant term differences are allowed in aggregate production functions, any variable can be rationalized as entering $X_i$. In a very clever derivation, Benhabib and Gali show that by considering a time period $T' < 0$, one can eliminate the need for these control variables by testing the implication of the neoclassical model that the change in the growth rate in per capita output $0$ to $T$ versus the growth rate in per capita output from $T'$ to $0$ should be negatively correlated with the growth in the capital labor ratio, $K/Y$, from $T'$ to $0$. This condition can be reexpressed in the cross-country regression framework as requiring that in the regression

$$gY_{i,t} = \alpha + \beta gK/Y_{i,T'} + \gamma gY_{i,T'} + \epsilon_i$$

(3)

$\beta < 0$ and $\gamma = -1$. (Notice here that $g_{i,T'}$ measures growth from $T'$ to $0$.) Benhabib and Gali call this a "weak conditional convergence test."

In my view, this is a valuable addition to the empirical growth literature. Differences in the choice of control variables have made the body of cross-country growth regressions extremely difficult to interpret, due to the questionable economic justification underlying many of them. Further, the inclusion of ad hoc variables in the cross-country regression runs the risk that they will proxy for some aspect of a new growth model which differentiates it from the neoclassical null. For example, the equilibrium selection rule in a model with indeterminacy might correlate with a variable measuring political stability. When this happens, a conditional convergence result maybe spuriously generated. The Benhabib-Gali test should be a valuable robustness check in this literature.

At the same time, it is important to recognize that this new convergence test may have some weaknesses relative to the conventional regression and testing framework. First, since the null hypothesis in this literature is typically no convergence, the failure to include explicit control variables seems likely to reduce the power of the test relative to standard frameworks. In particular, since Mankiw, Romer, and Weil (1992) reject no convergence for a set of control variables directly suggested by the Solow model, this issue of power needs to be understood before one can replace Mankiw-Romer-Weil's conclusion in favor of Banhabib and Gali's. Similarly, the capital series necessary for constructing the control variables for the Benhabib-Gali test may be less accurately measured than those needed in other regressions, which again affect test power. Second, the weak conditional convergence test requires that the vector of controls $X_i$ does not vary between the time intervals $T'$ to $0$ and $0$ to $T$. Since $X_i$ presumably contains savings and population growth rates, this requirement is unlikely to hold. Neither of these concerns
invalidates the value of the Benhabib-Gali test, but each indicates the importance of developing ways to reconcile disparate hypothesis test results based on non-nested models.

3 Observational equivalence

As Benhabib and Gali observe, the finding that $\beta$ is not negative in (2) and/or (3) is inconsistent with the neoclassical model. This relationship between the neoclassical model and $\beta$, however, leaves unresolved the implications of regressions of this type in terms of alternatives to the neoclassical model. This is especially important given the many papers which have found negative $\beta$'s and concluded that conditional convergence holds. Specifically, it is important to identify in what sense a negative $\beta$ supports the neoclassical framework as opposed to various new growth alternatives.

As shown formally by Bernard and Durlauf (1995), a negative $\beta$ does not discriminate between growth models, as such a result can easily be generated under a number of new growth alternatives. To see this intuitively, consider Figures 1-5, which represent various laws of motion for per capita output across time. By assuming that all control variables $X_i$ are constant across countries, we can use these diagrams to see when a negative $\beta$ will occur for a cross-section. Figure 1 presents the transition dynamics of the neoclassical model. As is clear from the figure, there is a negative correlation between initial income and growth for any distribution of initial incomes, represented by the various $x$'s in the figure. However, consider Figures 2 and 3, which represent the Azariadis-Drazen (1990) model of threshold externalities. In each figure, the limiting per capita income of an economy will depend on its initial income, so clearly there is no convergence in an economically interesting sense, i.e., following eq. (1) above, initial income needs to be included when forecasting long-run income disparities. However, the regression estimate of $\beta$ when each economy exhibits a threshold externality may or may not be negative, depending on the initial distribution of incomes. In Figure 2, this distribution will produce a negative $\beta$, whereas the distribution of initial incomes as shown in Figure 3 will not.

Figures 4 and 5 indicate how this same finding will hold for the sort of models which Benhabib and Gali examine. In each figure, the $x$'s indicate both initial conditions as well as the resolution of the indeterminacy in the mapping from $Y_t$ to $Y_{t+1}$ for each country. As before, different initial conditions can yield evidence in favor of convergence, as shown in Figure 4, or can yield evidence against convergence, as shown in Figure 5, depending on the initial distribution of incomes and the selection rule which resolve the indeterminacy within each economy.

Why can models such as those with threshold externalities or indetermi-
Figure 1
Neoclassical Transition Function Exhibiting $\beta$ Convergence
For Every Time $t$ Distribution

$y_{t+1}$

$45^\circ$

$y_t$

$\phi(y_t)$
Figure 2
Threshold Externalities Transition Function
Exhibiting β Convergence for Some Time t Distribution

\[ y_{t+1} \sim (y_t) \sim \phi(y_t) \sim y_{\text{thresh}} \]
Figure 3
Threshold Externalities Transition Function
Failing to Exhibit β Convergence for Some Time t Distribution
Figure 4
Transition Function for Model with Indeterminacy
Exhibiting β Convergence for Some Time t Distribution
Figure 5
Transition Function for Model with Indeterminacy
Failing to Exhibit β Convergence for Some Time t Distribution
nacy fail to have any necessary implications for the $\beta$ coefficient? Both the threshold externality and indeterminate growth models (as represented in the figures) constitute nonlinear alternatives to the standard cross-country growth regression. Under either of these alternative models, the functional form of the regression equation (2) is misspecified due to both the use of a linear functional form and omitted variables. The behavior of the estimated $\beta$ in (2) under either of these alternatives is in fact difficult to ascertain and cannot be extrapolated in any way which does not depend on the initial income distribution. Finally, observe that direct evidence of nonlinearity in cross-country growth behavior has been developed by Durlauf and Johnson (1995) and Quah (1995) using very different statistical methodologies, so this is more than a theoretical concern.

In short, the models studied by Benhabib and Gali would be no less interesting had they found convergence in a statistical sense. Their models are still interesting because of the observational equivalence we have described above. In this sense, the theoretical analysis provided by Benhabib and Gali provides an even more powerful critique of standard convergence tests than does their empirical work, as this theoretical analysis shows that the standard tests have failed to focus on a robust difference between the neoclassical and new growth models. On the other hand, the nonlinear nature of the particular models Benhabib and Gali study means that it will be extremely difficult to develop compelling empirical evidence in favor of those models from linear regressions.

4 Conclusions

The major empirical challenge for work on growth is to derive ways of breaking the observational equivalence between the neoclassical model and interesting new growth alternatives of the type studied by Benhabib and Gali in ways which provide positive evidence for a particular class of models. As it stands, the theoretical and empirical growth literatures have been evolving at cross-purposes. My own belief (which is unsurprisingly the one taken in Durlauf and Johnson (1995)) is that this will involve nonparametric statistical methods such as classification and regression tree analysis which attempt to identify groups of observations which obey a common linear law of motion rather than estimate a common linear law imposed on a heterogeneous cross-section. The important work of Quah (1995) on evolving cross-sections can be interpreted in a similar spirit. In addition, better microeconomic evidence on the sorts of spillovers which drive the new growth literature will allow resolution of a number of disputes without need to rely on aggregate data.
References


